Spring-2022
STA-301 Statistic and Probability
Assignment No. 2
03026263557
Usman Mughal

Question No.1 **Marks = 10**

Let X be a continuous random variable whose probability density function is:

$$f(x)=\frac{x^3}{4}$$

For an interval 0<x<c. What is the value of constant c that makes f(x) a valid probability density function?

Solution:

$$f(x)=\frac{x^3}{4} \quad , \quad 0<x<c$$

For p.d.f

1. $f(x)\geq 0 \qquad \forall x\in R$
2. $\int_{-\infty}^{+\infty} f(x)dx=1$

First condition is satisfies because x is positive in the interval, So f(x) is also positive.
Now according to 2nd condition

$$\int_0^c \frac{x^3}{4}dx=1$$

$$\frac{1}{4}\left[\frac{x^4}{4}\right]_0^c=1$$

$$\frac{1}{16}\left[x^4\right]_0^c=1$$

$$\frac{1}{16}\left[c^4-0^4\right]=1$$

$$\frac{1}{16}\left[c^4\right]=1$$

$$c^4=16$$

$$c^4=2^4$$

$$\Rightarrow c=2$$

So

$0<x<2$ to be a p.d.f function.

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Question No.2 **Marks = 10**

From the following joint p.d of X and Y. Find E(X), Var (Y) and Cov(X, Y).

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x	y	0	1	2	3	g(x)
0		0	0.25	0.05	0.05	0.35
1		0.03	0.10	0.10	0.15	0.38
2		0.07	0.10	0	0.10	0.27
h(y)		0.10	0.45	0.15	0.30	1

Solution:

$$E(X) = \sum x_j g(x_j)$$
$$E(X) = [(0)(0.35)] + [1(0.38)] + [2(0.27)]$$
$$E(X) = 0 + 0.38 + 0.54$$
$$E(X) = 0.92$$

Now

$$Var(Y) = E(Y^2) - [E(Y)]^2$$
$$E(Y) = \sum y_j h(y_j)$$
$$E(Y) = [(0)(0.10)] + [(1)(0.45)] + [2(0.15] + [3(0.30)]$$
$$E(Y) = 0 + 0.45 + 0.30 + 0.90$$
$$E(Y) = 1.65$$
$$E(Y^2) = \sum y_j^2 h(y_j)$$
$$E(Y^2) = [(0)^2(0.10)] + [(1)^2(0.45)] + [2^2(0.15] + [3^2(0.30)]$$
$$E(Y^2) = 0 + 0.45 + 0.60 + 2.70$$
$$E(Y^2) = 3.75$$

Now

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$$Var(Y) = E(Y^2) - \left[E(Y)\right]^2$$

$$Var(Y) = 3.75 - \left[1.65\right]^2$$

$$Var(Y) = 3.75 - 2.7225$$

$$Var(Y) = 1.0275$$

$$Con(X,Y) = \sum_i \sum_j (x_i y_j) f(x_i, y_j)$$

$$E(XY) = [(0)(0)(0)] + [(0)(1)(0.25)] + [(0)(2)(0.05)] + [(0)(3)(0.05)] + [(1)(0)(0.03)] + [(1)(0)(0.03)] + [(1)(1)(0.10)] + [(1)(2)(0.10)] + [(1)(3)(0.15)] + [(2)(0)(0.07)] + [(2)(1)(0.10)] + [(2)(2)(0)] + [(2)(3)(0.10)]$$

$$E(XY) = 0 + 0 + 0 + 0 + 0 + 0 + 0.10 + 0.20 + 0.45 + 0 + 0.20 + 0 + 0.60$$

$$E(XY) = 1.55$$

$$Con(X,Y) = E(XY) - E(X)E(Y)$$

$$Con(X,Y) = 1.55 - (0.92)(1.65)$$

$$Con(X,Y) = 1.55 - 1.518$$

$$Con(X,Y) = 0.032$$

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