Course STA301: Statistics and Probability

Practice Questions For Final-Term (Lec # 23to 45)

Lecture No 23 to 27

Multiple Choice Questions:

(Sample questions)

1. When a coin is tossed 4 times, the probability of getting the 4 heads is
- **a)** 1/16
- **b)** 15/16
- **c)** 3/8
- **d)** 1/2

2. For a continuous random variable X, P(X=3) is:

- **a)** 0.5
- **b)** 0
- **c)** 1
- **d)** Undefined

3. The second moment-ratio of a particular continuous probability distribution is 3.44. What does this value tell us about the shape of the distribution?

- **a)** Neither very peaked nor flatter
- **b)** Higher peak than the normal distribution
- **c)** Flatter peak than the normal distribution
- **d)** Normal distribution

4. In a bivariate probability distribution of X and Y, if E (X)=3 and E (Y)=5, then E (4X-2Y) =?

- **a)** 10
- **b)** 5
- **c)** 0
- **d)** 2

5. If the value of p is smaller or lesser than 0.5 then the binomial distribution is classified as:

- **a)** Negatively skewed
- **b)** Positively skewed
- **c)** Symmetric
- **d)** None of the above.

Numerical Questions

(Sample questions)

Q1: Let X be a random variable with probability distribution

x	-1	0	1	2	3
f(x)	0.125	0.50	0.20	0.05	0.125

Find E(X).

Q2: From the following joint probability distribution, find g (2).

x / y	2	4	h(y)
1	0.10	0.15	
3	0.20	0.30	
5	0.10	0.15	
g(x)			

Q3: Suppose X has a p.d. given by

x	-1	0	1
f(x)	3c	3c	6c

Determine the value of c.

Q4:

Let X be a continuous r.v. with p.d.f.

$$f(x) = 6x(1-x), \qquad 0 \le x \le 1.$$
$$= 0, \qquad \text{otherwise.}$$

Check that $f(x)$ *is a p.d.f.*

Q5:

Let X and Y be independent r.v's with joint p.d.f.

$$f(x,y) = x^2 + \frac{1}{3}xy, \qquad 0 \le x \le 1, 0 \le y \le 2$$

$$= 0, \qquad \text{elsewhere.}$$

Find the E(X).

ANSWERS / SOLUTION

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Practice Questions

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Multiple Choice Questions:

(Sample questions)

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3. The second moment-ratio of a particular continuous probability distribution is 3.44. What does this value tell us about the shape of the distribution?
- **a)** Neither very peaked nor flatter.
- **b)** Higher peak than the normal distribution.****
- **c)** Flatter peak than the normal distribution.

d) Normal distribution.

4. In a bivariate probability distribution of X and Y, if E (X)=3 and E (Y)=5, then E (4X-2Y) =?

a) 10
b) 5
c) 0
d) 2****

5. If the value of p is smaller or lesser than 0.5 then the binomial distribution is classified as:

a) Negatively skewed
b) Positively skewed****
c) Symmetric
d) None of the above.

Numerical Questions

(Sample questions)

Q1: Let X be a random variable with probability distribution

x	-1	0	1	2	3
f(x)	0.125	0.50	0.20	0.05	0.125

Find E(X).

Solution:

x	**f (x)**	**xf(x)**
-1	0.125	-0.125
0	0.50	0
1	0.20	0.20
2	0.05	0.10
3	0.125	0.375
		0.55

$$E(X) = \sum xf(x)$$
$$= 0.55.$$

Q2: From the following joint probability distribution, find g (2).

x / y	2	4	h(y)
1	0.10	0.15	
3	0.20	0.30	
5	0.10	0.15	
g(x)			

Solution:

$$g(2) = \sum_{x=1}^{5} f(2, y)$$
$$= 0.10 + 0.20 + 0.10$$
$$= 0.40$$

Q3: Suppose X has a p.d. given by

x	-1	0	1
f(x)	3c	3c	6c

Determine the value of c.

Solution:

f(X) is a probability distribution function, if $\sum f(x) = 1$.

$\sum f(\mathrm{x}) = 1$

$3c + 3c + 6c = 1$

$12c = 1$

$c = \frac{1}{12}$

Q4:

Let X be a continuous r.v. with p.d.f.

$$f(x) = 6x(1-x), \qquad 0 \le x \le 1.$$
$$= 0, \qquad \text{otherwise.}$$

Check that $f(x)$ is a p.d.f.

Solution:

The $f(x)$ will be a p.d.f. if

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 6x(1-x)\,dx = 1$$

$$\int_0^1 6x\,dx - \int_0^1 6x^2\,dx = 1$$

$$\left[\frac{6x^2}{2}\right]_0^1 - \left[\frac{6x^3}{3}\right]_0^1 = 1$$

$$\left[3x^2\right]_0^1 - \left[2x^3\right]_0^1 = 1$$

$$\left[3(1)\right] - \left[2(1)\right] = 1$$

$$3 - 2 = 1$$

$$1 = 1$$

Hence f(x) is the p.d.f.

<u>Q5:</u>

Let X and Y be independent r.v's with joint p.d.f.

$$f(x,y) = x^2 + \frac{1}{3}xy, \qquad 0 \le x \le 1, 0 \le y \le 2$$

$$= 0, \qquad \text{elsewhere.}$$

Find the E(X).

Solution:

$$E(X) = \int_x xg(x)\,dx$$

$$g(x) = \int_y f(x,y)\,dy$$

$$= \int_0^2 \frac{3x^2 + xy}{3}\,dy$$

$$= \frac{1}{3}\int_0^2 (3x^2 + xy)\,dy$$

$$= \frac{1}{3}\int_0^2 3x^2\,dy + \frac{1}{3}\int_0^2 xy\,dy$$

$$= \frac{1}{3}\left[3x^2 y\right]_0^2 + \frac{1}{3}\left[\frac{xy^2}{2}\right]_0^2$$

$$= \frac{1}{3}\left[6x^2\right] + \frac{1}{3}\left[2x\right]$$

$$= \frac{6x^2}{3} + \frac{1}{3}\left[2x\right]$$

$$= 2x^2 + \frac{2x}{3}, \qquad \textit{for } 0 \le x \le 1.$$

$$E(X) = \int_x xg(x)dx$$

$$= \int_0^1 x(2x^2 + \frac{2x}{3})\,dx$$

$$= \int_0^1 2x^3 dx + \int_0^1 \frac{2x^2}{3} dx$$

$$= \left[\frac{2x^4}{4}\right]_0^1 + \frac{2}{3}\left[\frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{2}{4}\right) + \frac{2}{3}\left(\frac{1}{3}\right)$$

$$= \frac{1}{2} + \frac{2}{9} = \frac{9+2}{18} = \frac{11}{18}.$$

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and I'll do the same, In Sha Allah.

— Qalb-e-Momin

Practice Questions

Lecture 28-32

MCQ's

If in a binomial distribution p=1/2 then m3is:

0****

<0

>0

1

In a binomial distribution mean is always

Less than variance

Greater than variance****

Equal to variance

Less than S.D

For a normal distribution, the Z value for an X value is to the right of the mean is always.

Equal to Zero

Negative

Greater than one

Positive****

Which one of these sampling methods is a probability method?

Quota sampling

Judgement sampling

Convenience sampling

Simple random sampling*****

The listing of elements in the population with the identifiable number is classified as

Experiment

Population

Sample

Frame for experiment****

Q1: Show that Mean =2p. And $\sigma = \sqrt{2pq}$ for a binomial distribution in which n=2.

Solution:

Here

$X \sim b(\mathrm{n}, \mathrm{p})$

$X \sim b(2, \mathrm{p})$

x	$P(X=x) = {}^2C_x p^x q^{2-x}$	$xP(X=x)$	$x^2P(X=x)$
0	${}^2C_0 p^0 q^{2-0} = q^2$	0	0
1	${}^2C_1 p^1 q^{2-1} = 2pq$	$2pq$	$2pq$
2	${}^2C_2 p^2 q^{2-2} = p^2$	$2p^2$	$4p^2$
Total	$(q+p)^2$	$2pq+2p^2$	$2pq+4p^2$

$\mu = E(X)$

$\mu = \sum x.P(X=x)$

$\mu = 2pq + 2p^2$

$\mu = 2p(q+p)$

$\mu = 2p(1)$

$\mu = 2p$

Hence proved

Now

$$E(X^2) = \sum x^2.P(X = x)$$

$$E(X^2) = 2pq + 4p^2$$

$$E(X^2) = 2p(q + 2p)$$

$$E(X^2) = 2p(1 - p + 2p)$$

$$E(X^2) = 2p(1 + p)$$

Now

$$\sigma^2 = Var(X)$$

$$\sigma^2 = E(X^2) - \{E(X)\}^2$$

$$\sigma^2 = 2p(1 + p) - (2p)^2$$

$$\sigma^2 = 2p(1 + p - 2p)$$

$$\sigma^2 = 2p(1 - p)$$

$$\sigma^2 = 2pq$$

Now

$$\sigma = S.D(X) = \sqrt{2pq}$$

Hence proved

Q2: A computer shop contains 3 good and 2 defective monitors. Two monitors are selected by without replacement. Find the probability that

i): Both will be good

(ii): One is good and one is defective

Solution:

Here

3 good

2 defective

N=3+2=5

N=2

Let X= good monitors K=3

$x = 0,1,2$

$$P(\mathrm{X} = \mathrm{x}) = \frac{{}^{3}C_{x}\,{}^{2}C_{2-x}}{{}^{5}C_{2}}$$

(*i*)

$$P(\mathrm{X} = 2) = \frac{{}^{3}C_{2}\,{}^{2}C_{0}}{{}^{5}C_{2}}$$

$$P(\mathrm{X} = 2) = \frac{3 \times 1}{10}$$

$$P(\mathrm{X} = 2) = \frac{3}{10}$$

(*ii*)

$$P(\mathrm{X} = 1) = \frac{{}^{3}C_{1}\,{}^{2}C_{1}}{{}^{5}C_{2}}$$

$$P(\mathrm{X} = 1) = \frac{3 \times 2}{10}$$

$$P(\mathrm{X} = 1) = \frac{6}{10}$$

Q3: X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

a) $P(x < 40)$

b) $P(x > 21)$

c) $P(30 < x < 35)$

What is meant here by area is the area under the standard normal curve.

a): For $x = 40$,

the z-value

$$z = \frac{(X - \mu)}{\sigma}$$

$$z = \frac{(40 - 30)}{4}$$

$$z = 2.5$$

Hence

$$P(x < 40) = P(z < 2.5)$$

$$P(x < 40) = [0 < z < 2.5]$$

$$P(x < 40) = 0.9938$$

b)

For $x = 21$,

$$z = \frac{(X - \mu)}{\sigma}$$

$$z = \frac{(21 - 30)}{4}$$

$$z = -2.25$$

Hence

$$P(x > 21) = P(z > -2.25)$$

$$P(x > 21) = 1 - [-2.25 < z < 0]$$

$$P(x > 21) = 1 - 0.0122$$

$$P(x > 21) = 0.9878$$

c)

For $x = 30$,

$$z = \frac{(X - \mu)}{\sigma}$$

$$z = \frac{(30 - 30)}{4}$$

$$z = 0$$

and for

$x = 35,$

$$z = \frac{(X - \mu)}{\sigma}$$

$$z = \frac{(35 - 30)}{4}$$

$$z = 1.25$$

Hence

$$P(30 < x < 35) = P(0 < z < 1.25)$$

$$= (0 < z < 1.25) - (\text{area to the left } 0)$$

$$= 0.8944 - 0.5$$

$$= 0.3944$$

Q4: In throws with a pair of dice, find the probability of throwing doublets at least once.

Solution:

Here

$n = 3$

$Doublets = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$\mathrm{P} = (\text{Doublets})$

$$\mathrm{P} = \frac{6}{36}$$

$$\mathrm{P} = \frac{1}{6}$$

$$q = 1 - p$$

$$q = 1 - \frac{1}{6}$$

$$q = \frac{5}{6}$$

$$P(X = x) = {}^{n}C_{x}(p)^{x}(q)^{n-x}$$

$$P(X = x) = {}^{3}C_{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{n-x}$$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$P(X \geq 1) = 1 - {}^{3}C_{0}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{n-0}$$

$$P(X \geq 1) = 1 - \frac{125}{216}$$

$$P(X \geq 1) = \frac{125}{216}$$

$$P(X \geq 1) = 0.4213$$

Q5: A random sample of size 49 is selected from a Poisson distribution with μ=4. Find using central limit theorem, the probability that the sample mean will be greater than 5.

Solution:

Let X denote the Poisson distribution with μ=4. Then Var (x) = 4

By central limit theorem , $\bar{X}$ is approximately $N\left(4, \frac{4}{49}\right)$

We require $P(\bar{X} > 5)$

$$\sigma_{\bar{X}} = \sqrt{\frac{4}{49}}$$

$$\sigma_{\bar{X}} = \frac{2}{7}$$

$$P(\bar{X} > 5)$$

$$P(\bar{X} > 5) = P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} > \frac{5-4}{\frac{2}{7}}\right)$$

$$P(\bar{X} > 5) = P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} > \frac{7}{2}\right)$$

$$P(\bar{X} > 5) = P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} > 3.5\right)$$

$$P(\bar{X} > 5) = P\left(Z > 3.5\right)$$

$$P(\bar{X} > 5) = 1 - P\left(Z > 3.5\right)$$

$$P(\bar{X} > 5) = 1 - P\left(0 < Z < 3.5\right)$$

$$P(\bar{X} > 5) = 1 - 0.9998$$

$$P(\bar{X} > 5) = 0.0002$$

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— *Qalb-e-Momin*

Practice Question Lecture 33-37

1) If sample of size 2 give mean of 10 then population mean is:

   20

   10

   30

   40

2) If samples of size 2 with replacement give variance of 2.5 then population variance is:

   6
   7
   8
   5

3) For $\alpha=0.05$, the critical value of $z_{0.025}$ is equal to
   1.96
   2.58
   1.645
   2.33

4) A random sample of size n is taken from the population with unknown mean μ and known variance σ^2 . 95% confidence interval for population mean μ is :

$\left(\bar{X}-1.96\frac{\sigma}{\sqrt{n}},\ \bar{X}+1.96\frac{\sigma}{\sqrt{n}}\right)$

$\left(\bar{X}-2.58\frac{\sigma}{\sqrt{n}},\ \bar{X}+2.58\frac{\sigma}{\sqrt{n}}\right)$

$\left(\bar{X}-1.96\frac{S}{\sqrt{n}},\ \bar{X}+1.96\frac{S}{\sqrt{n}}\right)$

$\left(\bar{X}-2.58\frac{S}{\sqrt{n}},\ \bar{X}+2.58\frac{S}{\sqrt{n}}\right)$

5) If population proportion (p) is unknown, the standard error of the sample proportion (P) can be estimated by the formula:

$\sqrt{np(1-p)}$

$\sqrt{nP(1-P)}$

$\sqrt{p(1-p)}$

$\sqrt{P(1-P)}$

6) Define an unbiased estimator.

7) What is meant by confidence interval?

8) Students from school A and B are compared on the basis of their scores on an aptitude test. Two random samples of 90 and 100 students are selected from school A and B respectively. The sample means are 76.4 and 81.2, where as the sample standard deviation is 8.2 and 7.6 respectively. Establish a 98% confidence interval for the difference in population mean scores between students A and B.

9) If the standard deviation of the life times of television tubes is estimated as 100 hours, how large a sample must we take in order to be 95% confident that the error in the estimated mean life time will not exceed 20 hours?

10) Define one tailed and two tailed tests.

Solution:

11) If sample of size 2 give mean of 10 then population mean is:

20

10***

30

40

12) If samples of size 2 with replacement give variance of 2.5 then population variance is:

6
7
8
5***

13) For α=0.05, the critical value of $z_{0.025}$ is equal to

1.96***
2.58
1.645
2.33

14) A random sample of size n is taken from the population with unknown mean μ and known variance σ^2 . 95% confidence interval for population mean μ is :

$\left(\bar{X}-1.96\frac{\sigma}{\sqrt{n}},\ \bar{X}+1.96\frac{\sigma}{\sqrt{n}}\right)$****

$\left(\bar{X}-2.58\frac{\sigma}{\sqrt{n}},\ \bar{X}+2.58\frac{\sigma}{\sqrt{n}}\right)$

$\left(\bar{X}-1.96\frac{S}{\sqrt{n}},\ \bar{X}+1.96\frac{S}{\sqrt{n}}\right)$

$\left(\bar{X}-2.58\frac{S}{\sqrt{n}},\ \bar{X}+2.58\frac{S}{\sqrt{n}}\right)$

15) If population proportion (p) is unknown, the standard error of the sample proportion (P) can be estimated by the formula:

$\sqrt{np(1-p)}$

$\sqrt{nP(1-P)}$

$\sqrt{p(1-p)}$ ***

$\sqrt{P(1-P)}$

16) Define an unbiased estimator.

Solution

An estimator is said to be unbiased if its expected values equals the corresponding population parameter; otherwise it is said to be biased

The statistic $\hat{\theta}$ is said to be an unbiased estimator of θ, if $E\left(\hat{\theta}\right)=\theta$. The sample mean $\bar{X}$ is an unbiased estimator of the population mean μ i.e. $E\left(\bar{X}\right)=\mu$

17) What is meant by confidence interval?

Solution

A range of values used to estimate a population parameter is known as interval estimation or estimation by confidence interval and the interval (a, b) that will include the population parameter with a high probability (e.g. 0.95 or 0.99) is known as confidence interval.

18) Students from school A and B are compared on the basis of their scores on an aptitude test. Two random samples of 90 and 100 students are selected from school A and B respectively. The sample means are 76.4 and 81.2, where as the sample standard deviation is 8.2 and 7.6 respectively. Establish a 98% confidence interval for the difference in population mean scores between students A and B.

Solution

Let the population mean scores of students of Schools A and B be denoted by μ_1 and μ_2 respectively. From the sample data we have

$n_1 = 90, n_2 = 100, \bar{X}_1 = 76.4, \bar{X}_2 = 81.2, S_1 = 8.2, S_2 = 7.6.$

With 1-α=0.98, we have $\alpha/2$=0.01 and $z_{0.01} = 2.33$.

Thus a 98% confidence interval for $\mu_1 - \mu_2$ is

$$\left(\bar{X}_1 - \bar{X}_2\right) \pm z_{0.01}\sqrt{\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}}$$

$$\left(76.4 - 81.2\right) \pm 2.33\sqrt{\frac{\left(8.2\right)^2}{90} + \frac{\left(7.6\right)^2}{100}}$$

$-4.8 \pm \left(2.33\right)\left(1.15\right)$ or -4.8 ± 2.68 or $\left(-7.48, -2.12\right)$

19) If the standard deviation of the life times of television tubes is estimated as 100 hours, how large a sample must we take in order to be 95% confident that the error in the estimated mean life time will not exceed 20 hours?

Solution

Here σ=100, the error, e=20 and z corresponding to the given probability is 1.96

$$\text{n} = \left(\frac{Z_{\alpha/2}\sigma}{e}\right)^2$$

$$n = \left(\frac{1.96 \times 100}{20}\right)^2 = \frac{3.8416 \times \left(100\right)^2}{400} 96.04$$

Hence the required sample size would be 97

20) Define one tailed and two tailed tests.

Solution

When the null hypothesis is tested against an alternative hypothesis of a "greater than" or a "less than" type the critical region is located on one end and the test is called a one tailed or one sided test. When the null hypothesis is tested against an alternative hypothesis of a "not

equal to type", the critical is located on both ends and test is called a two tailed or two sided test.

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— Qalb-e-Momin

Course STA301: Statistics and Probability

Practice Questions

Lecture No 38 to 39

Multiple Choice Questions:

(Sample questions)

6. If a significance level of 5% is used rather than 1%, the null hypothesis is:

e) Less likely to be rejected
f) More likely to be rejected
g) Just as likely to be rejected
h) None of the above

7. In hypothesis testing, suppose the critical value is |Z| > 1.96. This value shows that the applied test is……?

- **e)** One tailed test
- **f)** Two tailed test
- **g)** Right tailed test
- **h)** Left tailed test

8. The term 1- β is called

- **e)** Level of the test
- **f)** Power of the test
- **g)** Size of the test
- **h)** Critical region

9. We can apply t-distribution to test the hypothesis, when

- **e)** Sample size is small
- **f)** Sample size is large
- **g)** Population variance is known
- **h)** Population variance is unknown and sample size is small

10. The t-distribution can never become narrower than:

- **e)** F-distribution
- **f)** Standard normal distribution
- **g)** Chi-square distribution
- **h)** Exponential distribution

Numerical Questions

(Sample questions)

Q1: If n=10 and $\alpha = 0.05,$ then find the value of $t_{\alpha/2v}$.

Q2: Calculate the pooled proportion $\hat{P}_c$ for the given data.

Sample I: $X_1 = 20, n_1 = 100$

Sample II: $X_2 = 30, n_2 = 125$

Q3: If $X = 250, n = 630 \, and \, p_0 = 0.50,$ then find the z-test stati stic for proportion.

Q4: A sample of 100 electric light bulbs of type I showed a mean lifetime of 1190 hours and a standard deviation of 90 hours. A sample of 75 bulbs of type II showed a mean of 1230 hours with a standard deviation of 120 hours. Is there a difference between the mean lifetimes of two types at a significant level of 5%?

Q5: Random samples of 200 bolts manufactured by machine A and 100 bolts manufactured by machine B showed 19 and 5 defective bolts respectively. Test the hypothesis that machine B is performing better than machine A. Use a 0.05 level of Significance.

ANSWERS / SOLUTION

Course STA301: Statistics and Probability

Practice Questions

Lecture No 38 to 39

Multiple Choice Questions:

(Sample questions)

1. If a significance level of 5% is used rather than 1%, the null hypothesis is:

a) Less likely to be rejected
b) More likely to be rejected****
c) Just as likely to be rejected
d) None of the above

2. In hypothesis testing, suppose the critical value is $|Z| > 1.96$. This value shows that the applied test is……?

a) One tailed test
b) Two tailed test****
c) Right tailed test
d) Left tailed test

3. The term 1- β is called

a) Level of the test
b) Power of the test****
c) Size of the test
d) Critical region

4. We can apply t-distribution to test the hypothesis, when

a) Sample size is small
b) Sample size is large
c) Population variance is known
d) Population variance is unknown and sample size is small****

5. The t-distribution can never become narrower than:

a) F-distribution
b) Standard normal distribution****
c) Chi-square distribution
d) Exponential distribution

Numerical Questions

(Sample questions)

Q1: If n=10 and $\alpha = 0.05$, then find the value of $t_{\alpha/2v}$.

Solution:

Since ν = 10 – 1 = 9, and α = 5%, therefore, the right-tail area is 2½% and hence (using the t-table) we obtain

$t_{0.025(9)} = 2.228$

Q2: Calculate the pooled proportion $\widehat{P}_c$ for the given data.

Sample I: $X_1 = 20, n_1 = 100$

Sample II: $X_2 = 30, n_2 = 125$

Solution:

The pooled proportion $\widehat{P}_c$ is given by

$$\hat{P}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

$$= \frac{20+30}{100+125} = \frac{50}{225} = 0.2222$$

Q3: If $X = 250, n = 630 \; and \; p_0 = 0.50,$ then find the z-test stati stic for proportion.

Solution:

Test statistic:

$$Z = \frac{X \pm \frac{1}{2} - n\ p_0}{\sqrt{n\ p_0 (1 - p_0)}}$$

(where ± ½ denotes the continuity correct ion)

Here $np_0 = 630 \times 0.50 = 315.$

and X = 250

Hence $X < np_0$ so use X + ½

$$Z = \frac{250 + \frac{1}{2} - 315}{\sqrt{630(0.50)(0.50)}}$$

$$Z = \frac{-64.5}{\sqrt{157.5}} = \frac{-64.5}{12.55} = -5.139.$$

Q4: A sample of 100 electric light bulbs of type I showed a mean lifetime of 1190 hours and a standard deviation of 90 hours. A sample of 75 bulbs of type II showed a mean of 1230 hours with a standard deviation of 120 hours. Is there a difference between the mean lifetimes of two types at a significant level of 5%?

Solution:

Step 1:

$H_0 : \mu_1 - \mu_2 = 0 \text{ or } (\mu_1 = \mu_2)$

$H_A : \mu_1 - \mu_2 \neq 0 \text{ or } (\mu_1 \neq \mu_2)$

Step 2:

Level of significance = 5%.

Steps 3:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Step 4:

$$z = \frac{1190 - 1230 - 0}{\sqrt{\frac{(90)^2}{100} + \frac{(120)^2}{75}}} = \frac{-40}{16.523} = -2.42.$$

Step 5:
Critical Region:

Reject H_0, if

$|z_{cal}| > z_{\alpha/2}$

$|-2.42| > z_{0.05/2}$

$2.42 > z_{0.025}$

$2.42 > 1.96$

we reject H_0.

Step 6:
Conclusion:

Since the calculated value of $|z|$ = 2.42 is greater than the critical value of $z_{\alpha/2}$ = 1.96, so we reject H_0. Or also we conclude that there is a difference between the two types of bulbs.

Q5: Random samples of 200 bolts manufactured by machine A and 100 bolts manufactured by machine B showed 19 and 5 defective bolts respectively. Test the hypothesis that machine B is performing better than machine A. Use a 0.05 level of Significance.

Solution:

Step 1:

$H_0 : p_1 - p_2 = 0 \text{ or } (\mathrm{p}_1 = p_2)$

$H_A : p_1 - p_2 < 0 \text{ or } (\mathrm{p}_1 \prec p_2)$

Step 2:

Level of significance = 5%.

Steps 3:

$$z = \frac{P_1 - P_2}{\sqrt{\hat{p}_c \hat{q}_c \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where

$$\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

Step 4:

The proportions of non-defective bolts in the two samples are

$$P_1 = \frac{X_1}{n_1} = \frac{181}{200} = 0.905$$

$$P_2 = \frac{X_2}{n_2} = \frac{95}{100} = 0.905$$

$$\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

$$= \frac{181 + 95}{200 + 100} = \frac{276}{300} = 0.92.$$

$$z = \frac{0.905 - 0.95}{\sqrt{(0.92)(0.08)\left(\frac{1}{200} + \frac{1}{100} \right)}}$$

$$= \frac{-0.045}{0.03323} = -1.35.$$

Step 5:
Critical Region:

Reject H_0, if

$z_{cal} < z_{\alpha}$

$-1.35 < z_{0.05}$

$-1.35 < -1.645$

we don't reject H_0.

Step 6:

Conclusion:

Since the calculated value of z=-1.35 is greater than the critical value of z_{α} =-1.645, so we don't reject H_0. Or also we conclude that machine B is not performing better than machine A.

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Best of luck for everything ahead!

May Allah bless you with success, strength, and peace.

Please **remember me in your precious prayers**

and I'll do the same, In Sha Allah.

— *Qalb-e-Momin*

Practice Question

LECTURE NO.42

The F-Distribution

Hypothesis Testing and Interval Estimation in order to compare the Variances of Two Normal Populations (based on F-Distribution)

LECTURE NO.43
Analysis of Variance
Experimental Design

The F-distribution always ranges from:

0 to 1

0 to -∞

-∞ to +∞

0 to +∞

When we want to test the equality of two variances we usually use:

F-test

Chi-square test

ANOVA

Z-test

If s1 =2.3 and s2=1.5 then the value of F-Test statistic is:

2.35

2.45

1.23

2.67

Which is the basic assumption is required for F-Distribution?

The two random samples are independently and randomly selected

The two random samples are dependent.

The two samples are not necessary to follow normal distribution.

None of these.

If F has an F-distribution with v1 and v2 degree of freedom, then 1/F:

Has F distribution with v1 and v2 degree of freedom.

Has F distribution with v2 and v1 degree of freedom

Has t-distribution with v degree of freedom.

Has z distribution.

Analysis of Variance (ANOVA) is a test for equality of:

Variances
Means

Proportions

Only two parameters

When testing for independence in a contingency table with 3 rows and 4 columns, there are error degrees of freedom.

5
6
7
12

In one way ANOVA, the degree of freedom of Mean square between is the

Number of groups minus 1

Number of groups multiplied by, where n is the sample size

Number o f groups

Total number of observations minus 1

The basic entity or unit on which the experiment is performed, is called

Experimental unit

Treatment unit

Factor

Levels of factor

A term referring to the amount of balancing, blocking and grouping of the experimental units is

Local control

Spurious effect

Systematic error

Extraneous factor

Q1: Find Coefficient of association

$AB = 55, \quad \alpha\beta = 75, \quad A\beta = 45, \quad \alpha B = 125$

Q2: Test the independence by a simple approach between intelligences of fathers and sons.

Sons	Intelligent	Not intelligent	Total
Intelligent	300	200	300
Not intelligent	100	400	500
Total	400	600	1000

Q3: From the following table, test the hypothesis that the flower color is independent of flatness of leaf. Use α=0.05

	Flat Leaves	Lean Leaves	Total
White Flowers	99	36	135
Red Flower	20	5	25
Total	119	41	160

Q4: A random sample of 100 people each from 3 cities gave weekly income figures as summarized below, with Xij the income of the ith person from the j^{th} city.

City	Sample Size	$\sum X_{ij}$	$\sum X_{ij}^2$
A	100	10000	1056300

B	100	11200	1596000
C	100	9800	1100000

Set up an Analysis of Variance table. State and test the standard hypothesis from the table:

Q5: Complete an ANOVA table.

Source of Variation	D.f	Sum of Square	Mean Square
Treatment SS	3	----	110
Block SS	----	392	-----
Error SS	----	1212	----
Total	19	-----	

Solution of Practice Questions

Lecture 42 & 43

The F-distribution always ranges from:

0 to 1

0 to -∞

-∞ to +∞*****

0 to +∞

When we want to test the equality of two variances we usually use:

F-test****

Chi-square test

ANOVA

Z-test

If s1 =2.3 and s2=1.5 then the value of F-Test statistic is:

2.35****

2.45

1.23

2.67

Which is the basic assumption is required for F-Distribution?

The two random samples are independently and randomly selected****
The two random samples are dependent.
The two samples are not necessary to follow normal distribution.
None of these.

If F has an F-distribution with v1 and v2 degree of freedom, then 1/F:

Has F distribution with v1 and v2 degree of freedom.
Has F distribution with v2 and v1 degree of freedom*****
Has t-distribution with v degree of freedom.
Has z distribution.

Analysis of Variance (ANOVA) is a test for equality of:

Variances*****
Means
Proportions
Only two parameters

When testing for independence in a contingency table with 3 rows and 4 columns, there are error degrees of freedom.

5
6***
7
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In one way ANOVA, the degree of freedom of Mean square between is the

Number of groups minus 1*****

Number of groups multiplied by, where n is the sample size

Number o f groups

Total number of observations minus 1

The basic entity or unit on which the experiment is performed, is called

Experimental unit ****

Treatment unit

Factor

Levels of factor

A term referring to the amount of balancing, blocking and grouping of the experimental units is

Local control ****

Spurious effect

Systematic error

Extraneous factor

Q1: Find Coefficient of association

$AB = 55, \quad \alpha\beta = 75, \quad A\beta = 45, \quad \alpha B = 125$

Solution:

$AB = 55, \quad \alpha\beta = 75, \quad A\beta = 45, \quad \alpha B = 125$

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(55)(75) - (45)(125)}{(55)(75) + (45)(125)}$$

$$Q = \frac{4125 - 5625}{4125 + 5625}$$

$$Q = \frac{-1500}{9750}$$

$$Q = -0.15$$

This indicates negative association between A and B.

Q2: Test the independence by a simple approach between intelligences of fathers and sons.

Sons	Intelligent	Not intelligent	Total
Intelligent	300	200	300
Not intelligent	100	400	500
Total	400	600	1000

Solution:

Let A denotes" intelligent fathers" and B denotes " Intelligent Sons"

Then

$(AB) = 300,$

$$\frac{(A)(B)}{n} = \frac{(400)(500)}{1000} = 200$$

$$(AB) > \frac{(A)(B)}{n}$$

$$300 > 200$$

Thus there is positive association between intelligent fathers and intelligent sons. It means that intelligent fathers have intelligent sons.

Q3:From the following table, test the hypothesis that the flower color is independent of flatness of leaf . Use α=0.05

	Flat Leaves	Lean Leaves	Total
White Flowers	99	36	135
Red Flower	20	5	25
Total	119	41	160

Solution:

1: Null and alternative Hypothesis:

H_o : the flower color is independent of faltness of leaves.

H_1 : the flower color is not independent of faltness of leaves.

2: Level of significance:

$$\alpha = 0.05$$

3: Test Statistic

$$\chi^2 = \sum\left(\frac{(f_o - f_e)^2}{f_e}\right)$$

4: Critical region:

$$\chi^2 = \chi^2 > \chi^2_{0.05(1)} = 3.84$$

5. Calculations:

f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
99	100.41	-1.41	1.9881	0.020
20	18.59	1.41	1.9881	0.107
36	34.59	1.41	1.9881	0.057
5	6.41	-1.41	1.9881	0.310
160	160	0		χ^2 =0.494

6: Conclusion:

The calculated value of $\chi^2 = 0.494$ lies in acceptance region. Thus Ho: " The flower color is independent of flatness of leaves" is accepted. It means that the color of flowers have no relation with flatness of leaves.

Q4: A random sample of 100 people each from 3 cities gave weekly income figures as summarized below, with Xij the income of the ith person from the jth city.

City	Sample Size	$\sum X_{ij}$	$\sum X_{ij}^2$
A	100	10000	1056300
B	100	11200	1596000
C	100	9800	1100000

Set up an Analysis of Variance table. State and test the standard hypothesis from the table:

Solution:

1: Null and alternative Hypothesis:

$H_o : \mu_1 = \mu_2 = \mu_3$

H_1 : atleast two of the means are not equal.

2: Level of significance:

$\alpha = 0.05$

3: Test Statistic

$$F = \frac{Treatment\ Mean\ square}{Error\ Mean\ Square}$$

4: Critical region:

$$v_1 = k - 1 = 3 - 1, \quad v_2 = n - k = 300 - 3 = 297$$
$$F > 3.03$$

5. Calculations:

	A	B	C	Total
N	100	100	100	300
$\sum X_{ij}$	10000	11200	9800	31000
$\sum X_{ij}^2$	1056300	1596000	1100000	3752300
$\frac{\left(\sum X_{ij}\right)^2}{n_j}$	1000000	1254400	960400	3214800

$$C.F = \frac{\left(\sum X_{ij}\right)^2}{n_j}$$

$$C.F = \frac{(31000)^2}{300}$$

$$C.F = 3203333.33$$

$$\text{Total Sum of Squares} = \sum\sum X_{ij}^2 - C.F$$

$$\text{T}\,SS = 3752300 - 320333.33$$

$$\text{T}\,SS = 548966.67$$

$$Treatment\ \ SS = \sum \frac{\left(\sum X_{ij}\right)^2}{n_j} - C.F$$

$$TrSS = 3214800 - 3203333.33$$

$$TrSS = 11466.67$$

$$ESS = TSS - TrSS$$

$$ESS = 548966.67 - 11466.67$$

$$ESS = 537500$$

ANOVA TABLE

Source of variation	d.f	Sum of Squares	Mean squares	F
TrSS	k-1=3-1=2	11466.67	5733.34	F=3.17
Error SS	n-k=300-3=297	537500	1809.76	
Total	n-1=300-1=299	5487966.67		

6: Conclusion:

The calculated value of F=3.17 falls in rejection region. So we reject our null hypothesis, and conclude that at least two treatment means are not equal.

Q5: Complete an ANOVA table.

Source of Variation	D.f	Sum of Square	Mean Square
Treatment SS	3	----	110
Block SS	----	392	-----
Error SS	----	1212	----
Total	19	-----	

Solution:

Source of Variation	D.f	Sum of Square	Mean Square
Treatment SS	k-1=4-1=3	330	110
Block SS	r-1=5-1=4	392	98
Error SS	(r-1)(k-1)=12	1212	101
Total	19	1934	

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Best of luck for everything ahead!

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and I'll do the same, In Sha Allah.

— *Qalb-e-Momin*