MTH401 Final TERM Solved Subjective

Dynamics (Virtual University of Pakistan)



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MTH401 FINAL TERM SOLVED SUBJECTIVE

BY SHINING STAR

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Q1.Write down the system of differential equations

$$\frac{dx}{dt} = 6x + y + 6t \ , \ \frac{dy}{dt} = 4x + 3y - 10t + 4$$

in form of $X' = AX + F(t)$

Solution:

$$X' = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} X + \begin{pmatrix} 6t \\ -10t + 4 \end{pmatrix}$$

Q2. Solve the system of differential equations $\frac{dy}{dt} = x \ , \frac{dx}{dt} = y$ **by systematic elimination.**

Solution:

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$\frac{dy}{dt} = x \Rightarrow Dy - x = 0$ .....(i)

$\frac{dx}{dt} = y \Rightarrow -y + Dx = 0$ .....(ii)

Operate (ii) by D, we get

$-Dy + D^2x = 0$ .....(iii)

Add (i) and (iii), we get

$Dy - x = 0$

$\underline{-Dy + D^2x = 0}$

$D^2x - x = 0$

$\left(D^2 - 1\right)x = 0$

Auxiliary equation is $m^2 - 1 = 0$

$m = \pm 1$

$$\boxed{x(t) = c_1e^t + c_2e^{-t}}$$

Put this in (i), we get

$Dy - \left[c_1e^t + c_2e^{-t}\right] = 0$

$Dy = c_1e^t + c_2e^{-t}$

Integrate both sides, we get

$$\boxed{y(t) = c_1e^t - c_2e^{-t}}$$

Q3. Find a series solution for the differential equation $y'' + y = 0$ about $x_0 = 0$ such that

$a_{n+2} = -\frac{a_n}{(n+2)(n+1)}$ $n = 0, 1, 2\ldots$ $y(x) = \sum_{n=0}^{\infty} a_n x^n$ **and**

Solution:

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$$a_{n+2} = -\frac{a_n}{(n+2)(n+1)}\ ; \qquad n = 0,1,2,...$$

For $n = 0$, $a_2 = -\frac{a_0}{(0+2)(0+1)} = -\frac{a_0}{2}$

For $n = 1$, $a_3 = -\frac{a_1}{(1+2)(1+1)} = -\frac{a_1}{6}$

For $n = 2$, $a_4 = -\frac{a_2}{(2+2)(2+1)} = -\frac{a_2}{12} = -\frac{1}{12}\left(-\frac{a_0}{2}\right) = \frac{a_0}{24}$

For $n = 3$, $a_5 = -\frac{a_3}{(3+2)(3+1)} = -\frac{a_3}{20} = -\frac{1}{20}\left(-\frac{a_1}{6}\right) = \frac{a_1}{120}$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + ...$$

$$y(x) = a_0 + a_1x + \left(-\frac{a_0}{2}\right)x^2 + \left(-\frac{a_1}{6}\right)x^3 + \left(\frac{a_0}{24}\right)x^4 + \left(\frac{a_1}{120}\right)x^5 + ...$$

$$y(x) = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + ...\right) + a_1\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + ...\right)$$

Q4. Write solution $X = \frac{4}{3}Cos3t - \frac{5}{3}Sin3t$ **in the form** $X = A.Sin(wt + \phi)$**.**

$$A = \sqrt{\left(\frac{4}{3}\right)^2 + \left(-\frac{5}{3}\right)^2} = \frac{\sqrt{41}}{3}$$

$$\phi = \tan^{-1}\left(\frac{4/3}{-5/3}\right) = 0.6747 \text{ radians}$$

$$x(t) = \frac{\sqrt{41}}{3}\sin(3t + 0.6747)$$

Q5. If the equation $M(x, y)dx + N(x, y)dy = 0$ is not exact,

Case 1:

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When $\exists$ an integrating factor $u(y)$, a function of y only. This happens if the expression

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}$$

is a function of y

Case2:

If the given equation is homogeneous and

$$xM + yN \neq 0$$

Then find the integrating factor in both cases.

Solution:

$$u = \frac{1}{xM + yN}$$

Q6. Find eigen vector of

$$\begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}$$

A= ,corresponding eigen value $\lambda = -2$.

$$\left(\begin{array}{cc|c} -3-(-2) & 1 & 0 \\ 2 & -4-(-2) & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 2 & -2 & 0 \end{array}\right)$$

Add two times row 1 in row 2

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$-k_1 + k_2 = 0$

$k_1 = k_2$

Choosing $k_2 = 1$, we get $k_1 = 1$

therefore, eigen vector is $V = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Q8. Under which conditions linear independence of the solutions for the differential equation $y'' + P(x)y' + Q(x)y = 0$ .....(1) is guaranteed?

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Solution:

Linear independence is guaranteed in case when the Wronskian of the two solutions is not equal to zero.

Q9. Write the homogenous system of differential equations

$$2\frac{dx}{dt}-5x+\frac{dy}{dt}=5e^{t}$$
$$\frac{dx}{dt}-x+\frac{dy}{dt}=e^{t}$$

lec 36 example 1

in decoupled form.

$$2\frac{dx}{dt}-5x+\frac{dy}{dt}=5e^{t}$$
$$\frac{dx}{dt}-x+\frac{dy}{dt}=e^{t}$$

$$(2D-5)x+Dy=5e^{t}$$
$$(D-1)x+Dy=e^{t}$$

Determinants are $\begin{vmatrix}2D-5 & D\\ D-1 & D\end{vmatrix}$, $\begin{vmatrix}5e^{t} & D\\ e^{t} & D\end{vmatrix}$, $\begin{vmatrix}2D-5 & 5e^{t}\\ D-1 & e^{t}\end{vmatrix}$

Therefore, in decpoupled form, we get

$$\begin{vmatrix}2D-5 & D\\ D-1 & D\end{vmatrix}x=\begin{vmatrix}5e^{t} & D\\ e^{t} & D\end{vmatrix}$$
$$\begin{vmatrix}2D-5 & D\\ D-1 & D\end{vmatrix}y=\begin{vmatrix}2D-5 & 5e^{t}\\ D-1 & e^{t}\end{vmatrix}$$

Q10. When Frobenius' Theorem is used in Differential Equation

$a_2(x)y''+a_1(x)y'+a_0(x)y=0$ **?**

Solution:

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When we have a regular singular point x= x0, then we can find at least one series solution of the form $y=\sum_{n=0}^{\infty} c_n \left(x-x_0\right)^{n+r}$, where r is the constant that we will determine after solving the differential equation.

Q11. Find the eigenvalues of the following system

$$X' = \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

Solution:

$$X' = \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

$$A = \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A-\lambda I|=0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-3-\lambda)+36=0$$

$$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$$

$$-9-3\lambda+3\lambda+\lambda^2+36=0$$

$$-9+\lambda^2+36=0$$

$$\lambda^2+27=0$$

$\lambda=\sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

Q12. Define Legendre's polynomial of degree n

Solution:

Legendre polynomial is an n^{th} degree polynomial and it is given by the formula

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

Q13. What is the ordinary differential equation and give an example?

Solution:

A differential equation which only includes ordinary derivatives is known as ordinary differential equation. Some examples of ordinary differential equations include:

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$$\frac{dy}{dx} = x^2 + y$$

$$\frac{dy}{dx} = \left(x^2 + 1\right)\left(y^2 + 1\right)$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 0$$

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