Practice Questions for Lecture No. 23 to 45

Question 1

Find the equation of the line that passes through the point *(6, 7)* and is perpendicular to line *2x + 3y = 8.*

Solution

The line *L* can be written as

y = -2x/3 + 8/3.

So its slope is -2/3.

Slope of the line perpendicular to *L* is 3/2.

The equation of perpendicular line is

y - 7 = 3/2(x - 6)

Question 2

Find the slope of the line through the points (-5, -2) and (-6, 7).

Solution

$$\text{slope } m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-2)}{-6+5} = \frac{9}{-1}$$

Question 3

Find slope of the line $6x - 8y = 26$.

Solution

$$6x - 8y = 26$$
$$-8y = -6x + 26.$$
$$y = \frac{6}{8}x - 26/8$$

Slope is 6/8 or ¾.

Question 4

Find the point of intersection of the lines $2x + 3y = 7$ and $3x + 5y = 3$.

Solution

To find the point of intersection we need *(x, y)* which lie on both the lines, i.e. which satisfy the two equations simultaneously. We need to solve the equations simultaneously. Solving the equations simultaneously we get $x = 26$ and $y = -15$.

Question 5

Find the equation of the perpendicular bisector of the line joining the points A(2, 8) and B(6, 2).

Solution

Let *M* be the mid-point of *AB*. Then M(4,5)

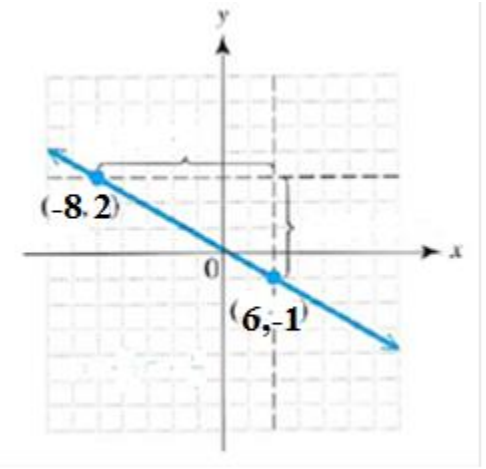
Gradient of *AB* is *(8 - 2)/(2-6) = 6/-4=-3/2* .

So slope of required line is 2/3 . Equation of required line is

y - 5 = 2/3(x - 4)

Question 6

Find the slope for the following graph:



Solution

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1-2}{(6)-(-8)} = \frac{-3}{14}$$

Question 7

Find the center and radius of the given equation of circle:

$$(x+5)^2+(y-3)^2=36.$$

Solution

$\because$ The equation of a circle with center (h, k) and radius r in standard form

$(x-h)^2+(y-k)^2=r^2$

$\Rightarrow$ Comparing the given equation with standard form, we see that

$(x-(-5))^2+(y-(3))^2=6^2$

$\therefore$ The center is $(-5, 3)$ and radius is 6.

Question 8

Are the lines $4x+5y=6$ and $5x-4y=2$ parallel, perpendicular, or neither?

Solution

$\because$ $y=mx+c$; where $c=y_1-mx_1$

For Line:1 $4x+5y=6$

$\Rightarrow$ $5y=-4x+6$

$\Rightarrow$ $y=-\frac{4}{5}x+\frac{6}{5}$

$\Rightarrow$ slope $=m_1=-\frac{4}{5}$

For Line:2 $5x-4y=2$

$\Rightarrow$ $-4y=-5x+2$

$\Rightarrow$ $y=\frac{5}{4}x-\frac{2}{4}$

$\Rightarrow$ slope $=m_2=\frac{5}{4}$

$\because$ Two lines with gradients (slopes) m_1 and m_2 are perpendicular if $m_1.m_2$ = -1, or $m_1=-\frac{1}{m_2}$.

$\therefore m_1.m_2=\left(-\frac{4}{5}\right)\left(\frac{5}{4}\right)=-1$

Question 9

Given the equations of the lines $3x+y=-4$ and $3x+3y=1$. Do these lines intersect? If they do, find the point of intersection.

Solution

$\because \quad y = mx + c \quad$; where $\quad c = y_1 - mx_1$

For Line:1 $\quad 3x + y = -4$

$\Rightarrow \qquad y = -3x - 4$

$\Rightarrow \qquad y = -3x - 4$

$\Rightarrow \quad \text{slope} = m_1 = -3$

For Line:2 $\quad 3x + 3y = 1$

$\Rightarrow \qquad 3y = -3x + 1$

$\Rightarrow \qquad y = -x + \frac{1}{3}$

$\Rightarrow \quad \text{slope} = m_2 = -1$

Since these two lines have different gradients (slopes), they must intersect.

To find the point of intersection we need (x, y) which lie on both the lines,

i.e. which satisfy the two equations simultaneously.

We need to solve the equations simultaneously.

Solving the equations simultaneously we get x = -13/6 and y = 5/2

So the point of intersection is $(-13/6, 5/2)$.

Question 10

What is the domain and range of y = arcsin x and *arccosx?*

Solution

The domain of y = arcsin x is [-1,1]

Range of y = arcsin x = [-pi/2, pi/2]

Domain of y = arccos x = [-1,1]

Range of y = arccos x = [0, pi]

Question 11

Prove the Pythagorean Identities.

- $\tan^2\theta + 1 = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Solution

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

For $1 + \cot^2\theta = \csc^2\theta$

Use $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and solve as above.

Question 12

Expand the sum and difference formula

Sin (A+B) and Sin (A-B)

Solution

Given in handouts

Question 13

Use trigonometric identities to prove that.

$$\frac{\tan y}{\sin y} = \sec y$$

Solution

$$\frac{\tan y}{\sin y} = \frac{\sin y}{\cos y}.\frac{1}{\sin y} = \frac{1}{\cos y} = \sec y$$

Question 14

Without using the calculator, find $arc\sin\frac{1}{\sqrt{2}}$ in radians.

Solution

$arc\sin\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ since $\frac{\pi}{4}$ is the angle whose sine is $\frac{1}{\sqrt{2}}$.

Question 15

Without using the calculator, find the exact value of Cos 210º.

Solution

$$\begin{aligned}\cos(210°) &= \cos(180° + 30°)\\ &= \cos 180°\cos 30° - \sin 180°\sin 30°\\ &= (-1)\left(\frac{\sqrt{3}}{2}\right) - (0)\left(\frac{1}{2}\right)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

Question 16

Given that $\sin\theta = \frac{\sqrt{3}}{2}$, find all possible values of θ in the range *-180 ≤ □ ≤ 180.*

Solution

$$\theta = \sin^{-1}(\frac{\sqrt{3}}{2}) = 60^{\circ}$$

Also, by using symmetry property:

$\because \sin(180-\theta)=\sin(\theta) \Rightarrow \sin(180-(-60))=\sin(240)$ to get 240 as a solution.

Note that 240 must be a solution but it does not lie in our range *-180 ≤ □ ≤ 180.*

Now use the periodic property: *sin(240) = sin(240 - 360).* So another solution is *-120*degrees.

All the solutions are ± 60 and ± 120 degrees.

Question 17

Verify that the following:

$$\frac{1-\tan\theta}{1-\cot\theta}=-\tan\theta.$$

Solution

$$\because \frac{1-\tan\theta}{1-\cot\theta}=-\tan\theta$$

$$LHS=\frac{1-\tan\theta}{1-\cot\theta}$$

$$=\frac{1-\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}$$

$$=\frac{\cos\theta-\sin\theta}{\cos\theta}\times\frac{\sin\theta}{-(\cos\theta-\sin\theta)}$$

$$=-\frac{\sin\theta}{\cos\theta}$$

$$\frac{1+\tan\theta}{1+\cot\theta}=-\tan\theta=RHS \quad (\text{verified})$$

Question 18

Verify that $\cos 4A = \cos^2 2A - \sin^2 2A$.

Solution

$$\begin{aligned}\cos 4A &= \cos(2A + 2A) \\ &= \cos 2A \cos 2A - \sin 2A \sin 2A \\ &= \cos^2 2A - \sin^2 2A \quad \text{(verified)}\end{aligned}$$

Question 19

Write the double angle formula for $\tan$ function.

$\tan 2A$

Solution

$$\tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

Question 20

Write and memorize Pythagorean identities.

Question 21

Without using calculator find the value of Cos 15^o and Sin 15°

Question 22

Prove that

$$\frac{\cot\theta - \tan\theta}{\sin\theta\cos\theta} = \operatorname{cosec}^2\theta - \sec^2\theta$$

Question 23

If first and fourth term of arithmetic progression is the first two terms of a geometric progression then find the common ratio of geometric progression.

Take first term of arithmetic progression as 'a' and d = 4/3a.

Question 24

Given that $\cos\theta = \frac{1}{2}$, find all possible values of θ in the range $0 \le \theta \le 360.$

Question 25

If first and second term of arithmetic progression is 2s and 3/2s respectively, then find the 20th term.

Question 26

Find the first two terms in the expansion of $(1-2x)^5$, in ascending powers of x.

Question 27

If a circle has center (4, 4) and touches the origin then find the equation of circle.

Question 28

Find the slope of the line which is passing through the points $(3, -1)$ and $(-2, -3)$.

Question 29

Find the slope of the given line $2x-4y=10$.

Question 30

Are the lines $x+2y=10$ and $6x-3y=7$ parallel, perpendicular, or neither?

Question 31

Without using the calculator, find the exact value of Cos 150^{o}.

Question 32

Find the Relative frequencies for the frequency distribution given below:

X	30 - 39	40 -49	50 - 59	60 - 69	70 - 79	80 - 89
F	6	11	15	18	17	21

Question 33

Following is the group frequency distribution of 87 passengers. Find the arithmetic mean.

Class Interval	**No. Of Passengers**
10-20	5
20-30	11
30-40	18
40-50	22
50-60	14
60-70	10
70-80	07

Question 34

Salary of an employee increased by 2%, 2.25%, and 3% in three consecutive years. What is the average growth rate of salary?

Question 35

Find the mean, median, mode, and geometric mean of the following data.

16, 9, 7, 11, 8, 9, 16, 19, 14, 13, 17, 16

Question 36

Find the mean, mode and median of the given grouped data

Values	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4

Question 37

Find the mean, mode and median of the given grouped data

Length (mm)	Frequency
150 - 154	5

155 - 159	2
160 - 164	6
165 - 169	8
170 - 174	9
175 - 179	11
180 - 184	6
185 - 189	3

Question 38

Give a five-number summary of the data. i.e., Minimum, First Quartile, Median, Third Quartile, and maximum value.

10, 19, 25, 17, 33, 29, 11, 40, 38, 15, 35

Question 39

Find mean absolute deviation and standard deviation for the following data

25, 22, 27, 29, 35, 39, 41, 37, 45, 51, 43

Question 40

If the mean is 285, the median is 330 and the mode is 340, then find that either the distribution positively skewed, negatively skewed, or symmetrical? Give reason.

Question 41

Classify the following as qualitative or quantitative variables.

i. Weight measurements of students
ii. Race of the people of a country
iii. Liking or disliking of 1000 persons for a certain product
iv. Expenditure Government servants of a city
v. Color of eyes of 1000 persons

Solution

i. Quantitative
ii. Qualitative
iii. Qualitative
iv. Quantitative
v. Qualitative

Question 42

Define quantitative and qualitative variables.

Solution

Quantitative variable:

A variable is called a quantitative variable when a characteristic can be expressed numerically such as age, weight, income or number of children.

Qualitative variable:

If the characteristic is non-numerical such as education, sex, eye-color, quality, intelligence, poverty, satisfaction, etc. the variable is referred to as a qualitative variable. A qualitative characteristic is also called an attribute.

Question 43

Name five fields where statistics can contribute?

Solution

Business and industry

Health and medicine

Natural and social sciences

Natural resources

Education, Government media and law

Question 44

Find the Cumulative frequencies for the frequency distribution given below:

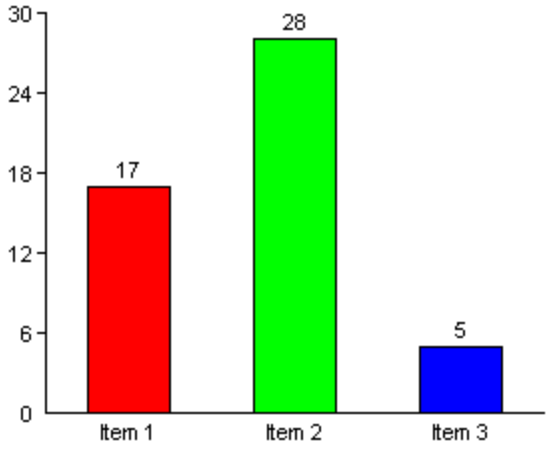
Output in tons	50 - 59	60 -69	70 - 79	80 - 89	90 - 99	100 - 109
f	3	11	19	28	20	15

Solution

Out put in tons	f	Cumulative Frequency (F)
50-59	3	3
60-69	11	14
70-79	19	33
80-89	28	61
90-99	20	81
100-109	15	96
Total	96	….

Question 45

Following is an example of which type of graphs.



Question 46

If for a given date, frequency =15 and frequency density=0.3, then find the class width.

Solution

Frequency density= frequency/class width

Class width=frequency/frequency density=15/0.3=50

Question 47

Calculate the frequency density when class width = 6 and frequency=24

Solution

Frequency density= frequency/class width

=24/6=4

Question 48

Find the frequency density for the distribution given below:

Class intervals	50 - 59	60 -69	70 - 79	80 - 89	90 - 99	100 - 109
f	4	11	18	28	22	15

Solution

Class intervals	Class width	f	Frequency density (F)
50-59	9	4	4 /9
60-69	9	11	11/9
70-79	9	18	18/9
80-89	9	28	28/9
90-99	9	22	22/9
100-109	9	15	15/9

Question 49

Find the Relative frequencies for the frequency distribution given below:

Output in tons	50 - 59	60 -69	70 - 79	80 - 89	90 - 99	100 - 109
f	15	12	10	28	25	10

Solution

Output in tons	f	Relative Frequency (F)
50-59	15	15/100=0.15
60-69	12	12/100=0.12
70-79	10	10/100=0.1
80-89	28	28/100=0.28
90-99	25	25/100=0.25
100-109	10	10/100=0.1
Total	100	….

Question 50

Following is the group frequency distribution of 94 students which shows their test result in a class. Find the arithmetic mean.

Class Interval	**No. Of Students**
10-20	5
20-30	16
30-40	12
40-50	25
50-60	17
60-70	8
70-80	6

Solution

Class Interval	f_1	***Class Boundry***	X_1	***fxi***
10-20	5	9.5-20.5	(9.5+20.5)/2=15	75
20-30	16	19.5-30.5	25	400
30-40	12	29.5-40.5	35	420
40-50	25	39.5-50.5	45	1125
50-60	17	49.5-60.5	55	935
60-70	8	59.5-70.5	65	520
70-80	6	69.5-80.5	75	450

Arithmetic mean = Sum fxi/sumf = 3925/89= 44.10

Class Interval	f_1	***Class Boundry***	X_1	***fxi***
10-20	5	9.5-20.5	(9.5+20.5)/2=15	75
20-30	16	19.5-30.5	25	400
30-40	12	29.5-40.5	35	420
40-50	25	39.5-50.5	45	1125
50-60	17	49.5-60.5	55	935
60-70	8	59.5-70.5	65	520
70-80	6	69.5-80.5	75	450

Arithmetic mean = Sum fxi/sumf = 3925/89= 44.10

Note: Class boundry column is not necessary. If student directly calculate Xi by taking average of class interval then it is also correct.

Question 51

Find the Cumulative frequencies for the frequency distribution given below:

Class limits	50 - 59	60 -69	70 - 79	80 - 89	90 - 99	100 - 109
f	5	18	10	27	19	12

Question 52

Find the Relative frequencies for the frequency distribution given below:

Class limits	50 - 59	60 -69	70 - 79	80 - 89	90 - 99	100 - 109
f	5	18	10	27	19	12

Question 53

Complete the following table by finding angles for the pie chart:

Medium of Institution	Frequency	Angle
Urdu	436	
English	764	
Total	1200	

Solution

Medium of Institution	Frequency	Angle
Urdu	436	130.8
English	764	229.2
	1200	360^o

Question 54

For the grouped data the median is equal to as

$$M_e = L_o + \frac{h}{f_o}\left(\frac{n}{2} - F\right)$$

What the following symbols stands for in the above given formula.

$$L_o,\ \ f_o,\ \ h,\ \ F$$

Solution

L0 = Lower class boundary of the median class

h = Width of the median class

f0 = Frequency of the median class

F = Cumulative frequency of the pre-median class

Question 55

The data of profit (in Rs. lakh) earned by 60 companies is given below:

Profits	Below 10	10-20	20-30	30- 40	40-50	50 and over
No. of companies	5	18	22	16	7	2
Cumulative frequency	5	23	45	61	68	70

Find Median for the above given data.

Solution

Median= size of n/2 th observation which lies in the class 20-30

$$Q_2 = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$Q_2 = 20 + \frac{10}{22}(35 - 23)$$

$$Q_2 = 20 + 5.45$$

$$= 25.45\, lakh$$

Question 56

Consider the data set:

67 25 42 57 38 49 36 29 61 31 71 53 73 84 87

Find the 50th percentile point.

Solution

First arrange the data in ascending order

25 29 31 36 38 42 49 53 57 61 67 71 73 84 87

Location of 50th Percentile is (15+1) (50/100) = 8th value = 53

Question 57

Give a five-number summary of the data given below.

15, 17, 18, 19, 35, 25, 14, 45, 37, 51, 42

Solution

Re arrange data in ascending order

14, 15, 17, 18, 19, 25, 35, 37, 42, 45, 51

Minimum = 14

First Quartile = (11+1)(25/100) = 3rd position = 17

Median = 25

Third Quartile = (11 + 1) (75/100) = 9th position = 42

Maximum = 51

Question 58

Following is the group frequency distribution of 103students which shows their test result in a class. Find the arithmetic mean.

Class Interval	No. Of Students
10-20	9
20-30	10
30-40	15
40-50	25
50-60	32
60-70	5
70-80	7

Question 59

Find the mean, median and mode of the following data.

45, 52, 51, 50, 49, 49, 45, 48, 52, 56

Question 60

Suppose a car travels with 8 stops, each stop after an interval of 10 miles. Suppose that the speeds at which the car travels these 8 intervals are 40, 35, 50, 40, 65, 20, 60, and 30 miles per hours respectively.

Calculate the harmonic mean for the given data.

Question 61

Consider the data set:

25 42 57 38 49 36 29 31 53

Find the 50th percentile point.

Question 62

Find the inter quartile range when third quartile is 45 and first quartile is 33.

Question 63

Find standard deviation for the following data

25, 47, 29, 43, 45, 52, 38, 59, 62

Question 64

What is the essential information which must be given to give a five number summary of a data set.

Question 65

Average sales and standard deviation for a store are 25 and 8.5 respectively. Find the coefficient of variation?

Question 66

Mean and median of frequency distributions are 32 and 25 respectively. Find mode of the distribution.

Question 67

A bag contains 13 balls. Among them 5 are blue and 8 are red. One ball is taken out at random from the bag. Find the probability the ball is blue.

Question 68

Find standard deviation for the following data

36, 42, 47, 29, 41, 45, 52, 37, 59, 63, 29

Solution

Mean of the data = 43.63

x	x - 43.63	(x -43.27)^2
36	-7.63	58.2169
42	-1.63	2.6569
47	3.37	11.3569
29	-14.63	214.0369
41	-2.63	6.9169
45	1.37	1.8769
52	8.37	70.0569
37	-6.63	43.9569
59	15.37	236.2369
63	19.37	375.1969
29	-14.63	214.0369

Total **1234.5459**

Now variance = 1234.5459/(11) = 112.2314

Standard Deviation = 10.5939

Question 69

Find Mean absolute Deviation from Mean of 3, 4 and 2with mean = 3.

Solution

X	$\left\|X-\overline{X}\right\|$
3	0
4	1
2	1
Total	2

Mean =4

$$M.D=\frac{\sum\left|X-\overline{X}\right|}{n}$$

$$=\frac{2}{3}$$

Question 70

What is the essential information which must be given to give a five number summary of the data set?

Solution

First quartile

Median

Third quartile

Lowest value

Highest value

Question 71

Average sale and standard deviation for a store are 18 and 6.5 respectively. Find coefficient of variation.

Solution

$$C.V = \left(\frac{S}{\bar{x}}\right) * 100$$

$$= \left(\frac{6.5}{18}\right) * 100$$

=36.11

Question 72

Mean and median of frequency distributions are 18 and 25respectively. Find mode of the distribution.

Solution

Mode= 3 median – 2 mean

Mode = 3 (25) –2(18)

= 75-36=39

Question 73

An automobile dealership pays to its salespeople, a salary plus a commission on sales. The mean biweekly commission is $1395, the median is $1350 and the mode is $65.

Is the distribution of commissions positively skewed, negatively skewed, or symmetrical? Give reason.

Solution

Since,

Mean > Median >Mode

Hence the distribution of commission is positively skewed.

Question 74

A bag contains 9 balls. Among them 4 are blue and 5 are red. One ball is taken out at random from the bag. Find the probability the ball is red.

Solution

Total number of balls = 9

Probability that the first ball drawn is blue = 4/9.

Question 75

If the mean is 285, the median is 330 and the mode is 340, then find that either the distribution positively skewed, negatively skewed, or symmetrical? Give reason.