

MTH632

Complex Analysis and Differential Geometry

Formula Sheet

1 Chapters, 975 Formulas

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$$(x_1, y_1)(x_2, y_2)$$

$$(x_1, y_1)(x_2, y_2) = (x_1x_2 - y_1y_2, y_1x_2 + x_1y_2).$$

Any complex number z

Any complex number $z = (x, y)$ can be written $z = (x, 0) + (0, y)$, and it is

z

$$z = x + iy.$$

Because (x, y)

Because $(x, y) = x + iy$, definitions (3) and (4) become

$$(x_1 + iy_1)(x_2 + iy_2)$$

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(y_1x_2 + x_1y_2).$$

for any z

for any $z = x + iy$.

$$(x, y) + (u, v)$$

$$(x, y) + (u, v) = (0, 0)$$

$$yu + xv$$

$$yu + xv = 0$$

$$z_1 - z_2$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$= x_1x_2 + y_1y_2$$

$$= (x_1 + iy_1)(x_2 - iy_2)$$

$$z(z_1 + z_2 + z_3)$$

$$z(z_1 + z_2 + z_3) = zz_1 + zz_2 + zz_3.$$

11. Solve the equation $z^2 + z + 1$

11. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0)$$

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$$

$$= (z_1 + z_2)z - 1$$

$$2 - 3i$$

$$2 - 3i = (4 + i)(2 + 3i)$$

$$(2 - 3i)(2 + 3i)$$

$$(2 - 3i)(2 + 3i) = 5 + 14i$$

$$n$$

$$n = m + 1.$$

$$(z_1 + z_2)^{m+1}$$

$$(z_1 + z_2)^{m+1} = z^{m+1}$$

$$z$$

$$z = x + iy$$

$$z_1 + z_2$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$|z|^2$$

$$|z|^2 = (\operatorname{Re} z)^2 + (\operatorname{Im} z)^2.$$

$$|z - 2| \leq |z| + 2$$

$$|z - 2| \leq |z| + 2 = 3$$

$$|z - 2| \geq ||z| - 2|$$

$$|z - 2| \geq ||z| - 2| = 1.$$

$$z_1 + z_2$$

$$z_1 + z_2 = z_1 + z_2.$$

$$\operatorname{Re} z$$

$$\operatorname{Re} z = z + z$$

$$= (-1 + 3i)(2 + i)$$

$$|2 - i|^2$$

$$|2 - i|^2 = -5 + 5i$$

$$(b) |2z + i|$$

$$(b) |2z + i| = 4.$$

$$|z|^2 - 2 \operatorname{Re}(zz^0) + |z^0|^2$$

$$|z|^2 - 2 \operatorname{Re}(zz^0) + |z^0|^2 = R^2.$$

$$z^2 + z^2$$

$$z^2 + z^2 = 2.$$

$$z^1 z^2 + z^1 z^2$$

$$z^1 z^2 + z^1 z^2 = 2 \operatorname{Re}(z^1 z^2) \leq 2|z^1||z^2|.$$

$$z$$

$$z = r(\cos + i \sin).$$

$$\arg z$$

$$\arg z = \operatorname{Arg} z + 2n$$

$$z$$

$$z = x + iy$$

$$\operatorname{Arg}(-1 - i)$$

$$\operatorname{Arg}(-1 - i) = -3$$

$$\text{argument, it is not true that } \operatorname{Arg}(-1 - i)$$

$$\text{argument, it is not true that } \operatorname{Arg}(-1 - i) = 5/4.$$

$$\arg(-1 - i)$$

$$\arg(-1 - i) = -3$$

$$\arg(-1 - i)$$

$$\arg(-1 - i) = 5$$

$$e^{-i4}$$

$$e^{-i4} = 1.$$

R. As the parameter increases from

R. As the parameter increases from $= 0$ to $= 2$, the point z starts from the

$$z$$

$$z = z^0 + Re^{i}$$

$$= 0 \text{ to } = 2.$$

$$e^{i1}e^{i2}$$

$$\begin{aligned} e^{i1}e^{i2} &= (\cos 1 + i \sin 1)(\cos 2 + i \sin 2) \\ &= (\cos 1 \cos 2 - \sin 1 \sin 2) + i(\sin 1 \cos 2 + \cos 1 \sin 2) \end{aligned}$$

$$r_2 e^{i2}$$

$$r_2 e^{i2} = r_1$$

$$e^{i2}e^{-i2}$$

$$e^{i2}e^{-i2} = r_1$$

$$r e^{i(0-)}$$

$$r e^{i(0-)} = 1$$

$$z^{m+1}$$

$$z^{m+1} = z^m z = r^m e^{im} r e^i = (r^m r) e^{i(m+1)} = r^{m+1} e^{i(m+1)}.$$

$$e^{i(-n)}(-)$$

$$e^{i(-n)}(-) = r^n e^{in}$$

$$(\cos + i \sin)^n$$

$$(\cos + i \sin)^n = \cos n + i \sin n$$

$$\cos$$

$$\cos = \cos 2 + i \sin 2.$$

$$\sin 2$$

$$\sin 2 = 2 \sin \cos.$$

$$z_1 z_2$$

$$z_1 z_2 = (r_1 r_2) e^{i(1+2)}$$

$$\arg(z_1 z_2)$$

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2.$$

$$\arg(z_1 z_2)$$

$$\arg(z_1 z_2) = (1 + 2) + 2n$$

$$\arg z_1$$

$$\arg z_1 = 1 + 2n_1$$

$\arg z^2$

$$\arg z^2 = 2 + 2(n - n1)$$

$\text{Arg}(z_1 z_2)$

$$\text{Arg}(z_1 z_2) = \text{Arg}(-i) = -$$

$\text{Arg } z_1 + \text{Arg } z_2$

$$\text{Arg } z_1 + \text{Arg } z_2 = +$$

$\text{Arg}(z_1 z_2) + 2$

$$\text{Arg}(z_1 z_2) + 2 = -$$

$2 + 2$

$$2 + 2 = 3$$

$2)$

$$2) = \arg z_1 + \arg(z-1)$$

$\text{Arg}(-2)$

$$\text{Arg}(-2) =$$

$\text{Arg } z$

$$\text{Arg } z = 2/3.$$

(b) e^i

$$(b) e^i = e^{-i}.$$

$e^{i1} e^{i2} \cdots e^{in}$

$$e^{i1} e^{i2} \cdots e^{in} = e^{i(1+2+\cdots+n)}$$

write z

write $z = re^{i\theta}$ and $m = -n = 1, 2, \dots$. Using the expressions

$$= \exp(i1)$$

$$= \exp(i2).$$

$1 + z + z^2 + \cdots + z^n$

$$1 + z + z^2 + \cdots + z^n = 1 - z^{n+1}$$

$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta$

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta = 1$$

(b) $\sin 3$

$$(b) \sin 3 = 3 \cos^2 \sin - \sin^3.$$

z^1

$$z^1 = r^1 e^{i1}$$

z^2

$$z^2 = r^2 e^{i2}$$

$$= 0 + 2k$$

z^0

$$z^0 = r^0 e^{i(0+2k)}$$

1

$$1 = 1 \exp[i(0 + 2k)]$$

$3 + i$

$$3 + i = 2 \exp$$

2

$$2 = 1 + \cos$$

2

$$2 = 1 - \cos$$

$a^2 + 1$ and

$$a^2 + 1 \text{ and } = \text{Arg}(a + i).$$

$2 e^{i/4}$

$$2 e^{i/4} = 1 + i.$$

$az^2 + bz + c$

$$az^2 + bz + c = 0$$

z

$$z = -b + (b^2 - 4ac)^{1/2}$$

$z^{1/n}$

$$z^{1/n} = (z^{1/m})^{-1}. \text{ (Compare with Exercise 7, Sec. 8.)}$$

(d) z^n

$$(d) z^n = (-1)^n (1+i)^{n-1}$$

$u + iv$

$$u + iv = f(x + iy).$$

$f(z)$

$$f(z) = u(x, y) + iv(x, y).$$

$u + iv$

$$u + iv = f(rei)$$

$f(z)$

$$f(z) = u(r, \theta) + iv(r, \theta).$$

$u(r, \theta)$

$$u(r, \theta) = r^2 \cos 2\theta$$

$v(r, \theta)$

$$v(r, \theta) = r^2 \sin 2\theta.$$

$f(z)$

$$f(z) = |z|^2 = x^2 + y^2 + i0.$$

$P(z)$

$$P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

$z^{1/2}$

$$z^{1/2} = \pm r \exp$$

$f(z)$

$$f(z) = r \exp$$

Ans. $f(z)$

$$\text{Ans. } f(z) = z^2 + 2iz$$

$f(z)$

$$f(z) = z + 1$$

in the form $f(z)$

$$\text{in the form } f(z) = u(r, \theta) + iv(r, \theta).$$

right. Since i

right. Since $i = e^{i\pi/2}$, the mapping

u

$$u = c_1 > 0$$

v

$$v = c_2 > 0$$

$y-u$

$$y-u = -$$

w

$$w = r^2 e^{i2}$$

The transformation w

The transformation $w = z^2$ also maps the upper half plane $r \geq 0, 0 \leq \theta \leq \pi$

$e^{i\theta_1} + e^{i\theta_2}$

$$e^{i\theta_1} + e^{i\theta_2} = e^{i\theta_1} e^{i\theta_2}$$

When w

When $w = e^{i\theta}$, the image of the infinite strip $0 \leq \theta \leq \pi$ is the

real number c increases from c

real number c increases from $c = 0$ to $c = \infty$, the y intercepts of the lines increase

$x^2 - y^2$

$$x^2 - y^2 = c_1 \quad (c_1 < 0)$$

$2xy$

$$2xy = c_2 \quad (c_2 < 0)$$

vector fields represented by (a) w

vector fields represented by (a) $w = iz$; (b) $w = z/|z|$.

$f(z)$

$$f(z) = x + i0$$

$$f(z)$$

$$f(z) = 0 + iy$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$(z$$

$$(z = x + iy)$$

$$w_0$$

$$w_0 = u_0 + iv_0.$$

$$|(u + iv) - (u_0 + iv_0)|$$

$$|(u + iv) - (u_0 + iv_0)| = |(u - u_0) + i(v - v_0)| \leq |u - u_0| + |v - v_0|$$

$$W_0$$

$$W_0 = U_0 + iV_0.$$

$$f(z)F(z)$$

$$f(z)F(z) = (uU - vV) + i(vU + uV)$$

$$P(z)$$

$$P(z) = a_0 + a_1z + a_2z^2 + \cdots + a_nz^n$$

$$z z_0 f(z)$$

$$z z_0 f(z) =$$

$$zf(z)$$

$$zf(z) = w_0$$

$$zf(z)$$

$$zf(z) =$$

$$f(1/z)$$

$$f(1/z) = 0.$$

That is, the point w

That is, the point $w = f(z)$ lies in the neighborhood $|w| > 1/$
of whenever

$$iz + 3$$

$$iz + 3 = 0$$

$$(1/z) + 1$$

$$(1/z) + 1 = \lim$$

$$1 + z$$

$$1 + z = 2.$$

$$z^2 + 1$$

$$z^2 + 1 =$$

$$(2/z^3) - 1$$

$$(2/z^3) - 1 = \lim$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$z^2 O(z^2 + c)$$

$$z^2 O(z^2 + c) = z^2$$

$$z^{1-i} [x + i(2x + y)]$$

$$z^{1-i} [x + i(2x + y)] = 1 + i$$

$$(z$$

$$(z = x + iy).$$

$$z^0 f(z^0 + z)$$

$$z^0 f(z^0 + z) = w^0.$$

$$T(z)$$

$$T(z) = az + b$$

$$zT(z)$$

$$zT(z) =$$

$$zT(z)$$

$$zT(z) = a$$

$$z^{-d/c} T(z)$$

$$z^{-d/c} T(z) =$$

$$z^0(2z + z)$$

$$z^0(2z + z) = 2z$$

z

$$z = z + z - z$$

$$= z + z - z$$

z

$$z = \frac{z}{2} + \frac{z}{2} - \frac{z}{2}$$

$$= (z + z)(z + z) - zz$$

$$= z + z + zz$$

z

$$z = z + z + z$$

$u(x, y)$

$$u(x, y) = x^2 + y^2$$

w

$$w = f(z + z)g(z + z) - f(z)g(z)$$

z

$$z = f(z)g(z + z) - g(z)$$

To find the derivative of $(2z^2 + i)^5$, write w

To find the derivative of $(2z^2 + i)^5$, write $w = 2z^2 + i$ and

(d) $f(z)$

$$(d) f(z) = (1 + z^2)^4$$

$P(z)$

$$P(z) = a_0 + a_1z + a_2z^2 + \cdots + a_nz^n$$

$f(z)$

$$f(z) = u(x, y) + iv(x, y)$$

w

$$w = f(z_0 + z) - f(z_0)$$

z

$$z = u(x_0 + x, y_0) - u(x_0, y_0)$$

$$f'(z_0)$$

$$f'(z_0) = u_x(x_0, y_0) + i v_x(x_0, y_0).$$

$$z$$

$$z = u(x_0, y_0 + y) - u(x_0, y_0)$$

$$= v(x_0, y_0 + y) - v(x_0, y_0)$$

$$f'(z_0)$$

$$f'(z_0) = -i[u_y(x_0, y_0) + i v_y(x_0, y_0)].$$

$$f(z)$$

$$f(z) = u(x, y) + i v(x, y)$$

$$f(z)$$

$$f(z) = z^2 = x^2 - y^2 + i 2xy$$

$$f'(z)$$

$$f'(z) = 2x + i 2y = 2(x + i y) = 2z.$$

$$u(x, y)$$

$$u(x, y) = x^2 + y^2$$

$$f(z)$$

$$f(z) = u(x, y) + i v(x, y)$$

$$f'(z_0)$$

$$f'(z_0) = u_x + i v_x$$

$$w$$

$$w = f(z_0 + z) - f(z_0).$$

$$u$$

$$u = u(x_0 + x, y_0 + y) - u(x_0, y_0)$$

$$v$$

$$v = v(x_0 + x, y_0 + y) - v(x_0, y_0).$$

$$u$$

$$u = u_x(x_0, y_0) x + u_y(x_0, y_0) y + 1 x + 2 y$$

$$w$$

$$w = u_x(x_0, y_0) x + u_y(x_0, y_0) y + 1 x + 2 y$$

z

$$z = u(x_0, y_0) + iv(x_0, y_0) + (1 + i3) x$$

$u(x, y)$

$$u(x, y) = x^2 + y^2$$

y

$$y = r \sin$$

z

$$z = x + iy$$

u

$$u = -u_x r \sin + u_y r \cos.$$

v

$$v = -v_x r \sin + v_y r \cos.$$

v

$$v = u_y r \sin + u_x r \cos$$

$f(z)$

$$f(z) = u(r, \theta) + iv(r, \theta)$$

$r e^{-i}$

$$r e^{-i} = 1$$

$u(r, \theta)$

$$u(r, \theta) = \cos$$

$v(r, \theta)$

$$v(r, \theta) = -\sin$$

$r u_r$

$$r u_r = -\cos$$

u

$$u = -\sin$$

$f'(z)$

$$f'(z) = e^{-i}$$

$$= -e^{-i} e^{-i}$$

$$(rei)^2$$

$$(rei)^2 = -1$$

$$f'(z)$$

$$f'(z) = e^{-i}$$

$$(a) f(z)$$

$$(a) f(z) = 1/z^4$$

$$(b) f(z)$$

$$(b) f(z) = rei/2$$

$$(c) f(z)$$

$$(c) f(z) = e^{-\cos(\ln r)} + ie^{-\sin(\ln r)}$$

$$f'(z)$$

$$f'(z) = ux + ivx = 3x^2$$

$$ux$$

$$ux = ur \cos \theta - u$$

$$uy$$

$$uy = ur \sin \theta + u$$

$$f'(z_0)$$

$$f'(z_0) = ux + ivx$$

10. (a) Recall (Sec. 5) that if z

10. (a) Recall (Sec. 5) that if $z = x + iy$, then

$$2[(ux - vy) + i(vx + uy)]$$

$$2[(ux - vy) + i(vx + uy)] = 0.$$

$$\text{grad } u$$

$$\text{grad } u = ux i + uy j.$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$f(z)$$

$$f(z) = U(x, y) + iV(x, y)$$

$$g(z)$$

$$g(z) = re^{i/2}$$

$$g(z)$$

$$g(z) = \ln r + i$$

$$T_{xx} + T_{yy}$$

$$T_{xx} + T_{yy} = 0$$

$$u_{xx} + u_{yy}$$

$$u_{xx} + u_{yy} = 0$$

$$v_{xx} + v_{yy}$$

$$v_{xx} + v_{yy} = 0.$$

$$|z|^4$$

$$|z|^4 = 2xy + i(x^2 - y^2)$$

$$v(x, y)$$

$$v(x, y) = -3xy^2 + (x)$$

$$v(x, y)$$

$$v(x, y) = -3xy^2 + x^3 + C$$

$$f(z)$$

$$f(z) = (y^3 - 3x^2y) + i(-3xy^2 + x^3 + C).$$

$$(d) u(x, y)$$

$$(d) u(x, y) = y/(x^2 + y^2).$$

$$(d) v(x, y)$$

$$(d) v(x, y) = x/(x^2 + y^2).$$

$$F(z)$$

$$F(z) = U(x, y) + iV(x, y)$$

$$u(x, -y) - iv(x, -y)$$

$$u(x, -y) - iv(x, -y) = u(x, y) + iv(x, y).$$

$$z + 1$$

$$z + 1 = z + 1$$

$$f_1(z)$$

$$f_1(z) = re^{i/2}$$

$$f_2(z)$$

$$f_2(z) = re^{i/2}$$

$$f_3(z)$$

$$f_3(z) = re^{i/2}$$

$$f_4(z)$$

$$f_4(z) = re^{i/2}$$

$$e^{iy}$$

$$e^{iy} = \cos y + i \sin y$$

ne of e is assigned to e^x when x

ne of e is assigned to e^x when $x = 1/n$

According to definition (1), $e^{xe^{iy}}$

According to definition (1), $e^{xe^{iy}} = e^{x+iy}$; and, as already pointed out in

$$e^{x_1}e^{x_2}$$

$$e^{x_1}e^{x_2} = e^{x_1+x_2}$$

$$z_1$$

$$z_1 = x_1 + iy_1$$

$$z_2$$

$$z_2 = x_2 + iy_2.$$

$$e^{iy_1}e^{iy_2}$$

$$e^{iy_1}e^{iy_2} = e^{i(y_1+y_2)}.$$

$$e^{z+2i}$$

$$e^{z+2i} = e^ze^{2i}$$

$$e^{z+2i}$$

$$e^{z+2i} = e^z.$$

$$(c) \exp(z + i)$$

$$(c) \exp(z + i) = -\exp z$$

$$2 + ni \quad (n$$

$$2 + ni \quad (n = 0, \pm 1, \pm 2, \dots).$$

and w are written z

and w are written $z = rei \quad (-\infty < \infty)$ and $w = u + iv$, equation (1) becomes

v

$$v = 2n$$

w

$$w = \ln r + i(2n)$$

$\log z$

$$\log z = \ln r + i(2n)$$

$3i$, then r

$3i$, then $r = 2$ and $\theta = -2/3$. Hence

$3i$)

$$3i) = \ln 2 + i$$

$$= \ln 2 + 2$$

$\log z$

$$\log z = \ln |z| + i \arg z$$

$\arg(ez)$

$$\arg(ez) = y + 2n \quad (n = 0, \pm 1, \pm 2, \dots)$$

$\log(ez)$

$$\log(ez) = \ln |ez| + i \arg(ez) = \ln(ex) + i(y + 2n) = (x + iy) + 2ni$$

$\log(ez)$

$$\log(ez) = z + 2ni$$

$\text{Log } z$

$$\text{Log } z = \ln r + i\theta$$

$\log z$

$$\log z = \text{Log } z + 2ni$$

and that $\text{Log}(-1)$
and that $\text{Log}(-1) = i$.

$$\log z$$

$$\log z = \ln r + i(+2n)$$

$$\log z$$

$$\log z = \ln r + i.$$

$$\log z$$

$$\log z = \ln r + i$$

$$u(r,)$$

$$u(r,) = \ln r$$

$$dz \log z$$

$$dz \log z = e^{-i}(ur + ivr) = e^{-i}$$

$$\text{Log } z$$

$$\text{Log } z = \ln r + i$$

(6) consists of the origin and the ray
(6) consists of the origin and the ray $=$. The origin is
evidently a branch point

$$\text{Log}(i3)$$

$$\text{Log}(i3) = \text{Log}(-i) = \ln 1 - i$$

$$(a) \text{Log}(-ei)$$

$$(a) \text{Log}(-ei) = 1 -$$

$$(a) \log e$$

$$(a) \log e = 1 + 2ni$$

$$3i)$$

$$3i) = \ln 2 + 2$$

$$(b) \text{Log}(-1 + i)^2$$

$$(b) \text{Log}(-1 + i)^2 = 2 \text{Log}(-1 + i).$$

$$\log z$$

$$\log z = \ln r + i$$

$$\log z$$

$$\log z = \ln r + i$$

7. Find all roots of the equation $\log z$

7. Find all roots of the equation $\log z = i/2$.

$$f(z)$$

$$f(z) = \text{Log}(z + 4)$$

$$\log(z_1 z_2)$$

$$\log(z_1 z_2) = \log z_1 + \log z_2.$$

$$\arg(z_1 z_2)$$

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2$$

$$\ln |z_1 z_2|$$

$$\ln |z_1 z_2| = \ln |z_1| + \ln |z_2|.$$

$$\log 1$$

$$\log 1 = 2\pi i$$

$$\text{Log}(-1)$$

$$\text{Log}(-1) = i$$

$$\text{Log } z_1 + \log z_2$$

$$\text{Log } z_1 + \log z_2 = 2i$$

$$z^{1/n}$$

$$z^{1/n} = \exp$$

$$\text{Log}(z_1 z_2)$$

$$\text{Log}(z_1 z_2) = \text{Log } z_1 + \text{Log } z_2.$$

$$\text{Log}(z_1 z_2)$$

$$\text{Log}(z_1 z_2) = \text{Log } z_1 + \text{Log } z_2 + 2\pi i$$

(b) showing that $\log(1/z)$

(b) showing that $\log(1/z) = -\log z$ ($z \neq 0$), in the sense that $\log(1/z)$ and $-\log z$

6. Let z denote any nonzero complex number, write

6. Let z denote any nonzero complex number, written $z = re^{i\theta}$ ($-\pi < \theta \leq \pi$), and let

$$\log(z^{1/n})$$

$$\log(z^{1/n}) = 1$$

$$\log i$$

$$\log i = \ln 1 + i$$

$$i-2i$$

$$i-2i = \exp[(4n+1)]$$

$$i2i$$

$$i2i = \exp[(4n+1)]$$

$$\log z$$

$$\log z = \ln r + i$$

$$P.V. (-i)i$$

$$P.V. (-i)i = \exp$$

$$1$$

$$1 = e^{i \log(1+i)} = e^{i(\ln}$$

$$2-i/4)$$

$$2-i/4) = e^{1/4} e^{i(\ln 2)/2}$$

$$2-i3/4)$$

$$2-i3/4) = e^{3/4} e^{i(\ln 2)/2}$$

$$(a) (1+i)i$$

$$(a) (1+i)i = \exp$$

$$(b) (-1)^{1/}$$

$$(b) (-1)^{1/} = e^{(2n+1)i}$$

$$3i)^{3/2}$$

$$3i)^{3/2} = \pm 2$$

$$e^{ix}$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{ix} + e^{-ix}$$

$$e^{ix} + e^{-ix} = 2 \cos x$$

$\cos x$

$$\cos x = e^{ix} + e^{-ix}$$

$\cos z$

$$\cos z = e^{iz} + e^{-iz}$$

e^{iz}

$$e^{iz} = \cos z + i \sin z$$

$\cos(z_1 + z_2)$

$$\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2.$$

$\sin^2 z + \cos^2 z$

$$\sin^2 z + \cos^2 z = 1.$$

$\cos(z + \pi)$

$$\cos(z + \pi) = -\cos z$$

$\cosh y$

$$\cosh y = e^y + e^{-y}$$

$f(z)$

$$f(z) = u(x, y) + iv(x, y)$$

$f'(z)$

$$f'(z) = u_x(x, y) + iv_x(x, y).$$

$\cos^2 z / 2$

$$\cos^2 z / 2 = \cos^2 x + \sinh^2 y.$$

$\sin^2 x + \sinh^2 y$

$$\sin^2 x + \sinh^2 y = 0.$$

z

$$z = n \quad (n = 0, \pm 1, \pm 2, \dots).$$

$2 + n$

$$2 + n \quad (n = 0, \pm 1, \pm 2, \dots).$$

$\tan(z + \pi)$

$$\tan(z + \pi) = \tan z$$

$$\sin(z_1 + z_2)$$

$$\sin(z_1 + z_2) = 1$$

$$\sin(z_1 + z_2)$$

$$\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

$$\cos(z_1 + z_2)$$

$$\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$

$$f(z)$$

$$f(z) = \sin^2 z + \cos^2 z - 1$$

$$(b) 1 + \cot^2 z$$

$$(b) 1 + \cot^2 z = \csc^2 z$$

$$2 \sin(z_1 + z_2) \sin(z_1 - z_2)$$

$$2 \sin(z_1 + z_2) \sin(z_1 - z_2) = \cos 2z_2 - \cos 2z_1.$$

$$z$$

$$z = ni \ (n = 0, \pm 1, \pm 2, \dots).$$

$$z$$

$$z = 2n + i \cosh^{-1} 2$$

$$\cosh z$$

$$\cosh z = e^z + e^{-z}$$

$$\cos z$$

$$\cos z = e^{iz} + e^{-iz}$$

$$\cosh(z_1 + z_2)$$

$$\cosh(z_1 + z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$$

$$z$$

$$z = ni \ (n = 0, \pm 1, \pm 2, \dots)$$

$$(c) \tanh(z + i)$$

$$(c) \tanh(z + i) = \tanh z$$

$$(b) \sinh z + \cosh z$$

$$(b) \sinh z + \cosh z = e^z$$

e^{iw}

$$e^{iw} = iz + (1 - z^2)^{1/2}$$

$\sin^{-1} z$

$$\sin^{-1} z = -i \log[iz + (1 - z^2)^{1/2}].$$

(d) n_i (n

$$(d) n_i (n = 0, \pm 1, \pm 2, \dots).$$

$\frac{d}{dt} e^{z_0 t}$

$$\frac{d}{dt} e^{z_0 t} = (e^{x_0 t} \cos y_0 t)' + i(e^{x_0 t} \sin y_0 t)'$$

$\frac{d}{dt} e^{z_0 t}$

$$\frac{d}{dt} e^{z_0 t} = (x_0 + iy_0)e^{x_0 t} e^{iy_0 t}.$$

never zero, while $w(2) - w(0)$

$$\text{never zero, while } w(2) - w(0) = 0.$$

$w(t)$

$$w(t) = u(t) + iv(t)$$

$W(t)$

$$W(t) = U(t) + iV(t)$$

$$= [U(b) + iV(b)] - [U(a) + iV(a)].$$

we write a

$$\text{we write } a = 0, b = 2 \text{ and use the same function } w(t) = e^{it} (0 \leq t \leq 2) \text{ as in}$$

$w(t)$

$$w(t) = u(t) + iv(t)$$

$z(t)$

$$z(t) = x(t) + iy(t).$$

z

$$z = z_0 + Re^{i\theta}$$

$Z()$

$$Z() = z[()].$$

$$z'(t)$$

$$z'(t) = x'(t) + iy'(t)$$

$$Z'()$$

$$Z'() = z'[()]'()$$

$$z$$

$$z = z() = 2ei$$

$$\text{Ans. } ()$$

$$\text{Ans. } () = b - a$$

Suggestion: Write $f(z)$

Suggestion: Write $f(z) = u(x, y) + iv(x, y)$ and $z(t) = x(t) + iy(t)$, so that

$$w(t)$$

$$w(t) = u[x(t), y(t)] + iv[x(t), y(t)].$$

$$z$$

$$z = x + iy(x)$$

$$C$$

$$C = C1 + C2$$

$$2ei(2ei)'d$$

$$2ei(2ei)'d = 4$$

$$ei$$

$$ei = e^{-i}$$

$$z dz$$

$$z dz = 4i$$

$$(z$$

$$(z = x + iy).$$

$$-i3x^2(1+i) dx$$

$$-i3x^2(1+i) dx = 3(1-i)$$

$$I1 - I2$$

$$I1 - I2 = -1 + i$$

$$f(z)$$

$$f(z) = z^{1/2} = \exp$$

$$f[z()]$$

$$f[z()] = \exp$$

$$3 e^{i/2} 3 e^{i/2}$$

$$3 e^{i/2} 3 e^{i/2} = 3$$

$$3 e^{i/2} 3/2$$

$$3 e^{i/2} 3/2 = -3$$

at

at = 0 exist, those limits being 0 and i3

$f[z()]z'()$ is continuous on the closed interval

$f[z()]z'()$ is continuous on the closed interval $0 \leq \leq$ when its value at = 0

$$e^{i/2} d$$

$$e^{i/2} d = 2$$

$$1. f(z)$$

$$1. f(z) = (z + 2)/z \text{ and } C \text{ is}$$

(c) the circle z

(c) the circle $z = 2 e^{i\theta} (0 \leq \theta \leq 2\pi)$.

$$z^{-1+i}$$

$$z^{-1+i} = \exp[(-1+i)\log z]$$

z

$$z = z_0 + Re^{i\theta} (-\infty \leq \theta \leq \infty)$$

where $Z()$

$$\text{where } Z() = z[()].$$

$$w(t) dt$$

$$w(t) dt = r_0 e^{i\theta}.$$

$$|f[z(t)]z'(t)|$$

$$|f[z(t)]z'(t)| = |f[z(t)]| |z'(t)| \leq M |z'(t)|$$

$$|z + 4| \leq |z| + 4$$

$$|z + 4| \leq |z| + 4 = 6$$

$$|z^3 - 1| \geq ||z|^3 - 1|$$

$$|z^3 - 1| \geq ||z|^3 - 1| = 7.$$

$$= |z + 4|$$

Writing M

Writing $M = 6/7$ and observing that $L =$ is the length of C , we may now use

$$z^{1/2}$$

$$z^{1/2} = \exp$$

$$z^2 + 1 \, dz$$

$$z^2 + 1 \, dz = 0.$$

$$|z^{1/2}|$$

$$|z^{1/2}| = 1$$

$$|z^2 + 1| \geq ||z|^2 - 1|$$

$$|z^2 + 1| \geq ||z|^2 - 1| = R^2 - 1.$$

$$3(1+i)^3$$

$$3(1+i)^3 = 2$$

$$\log z$$

$$\log z = \ln r + i$$

$$\log z$$

$$\log z = \ln r + i$$

$$z$$

$$z = i + i = 2i.$$

$$f(z)$$

$$f(z) = z^{1/2} = \exp$$

$$f^{1/2}(z)$$

$$f^{1/2}(z) = re^{i/2}$$

$$3(e^{i0} - e^{i3/2})$$

$$3(e^{i0} - e^{i3/2}) = 2$$

$$f_2(z)$$

$$f_2(z) = re^{i/2}$$

$$f(z) dz$$

$$f(z) dz = F(z_{n+1}) - F(z_1).$$

$$z^i dz$$

$$z^i dz = 1 + e^{-}$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$(z^2 + 9)^5 dz$$

$$(z^2 + 9)^5 dz = 0.$$

$$(f) f(z)$$

$$(f) f(z) = \text{Log}(z + 2).$$

$$f(z)$$

$$f(z) = re^{i/2}$$

$$2$$

$$2 = 2.$$

$$(z - z_0)^{n+1}$$

$$(z - z_0)^{n+1} = 2i$$

$$(z - 0)^{3+1}$$

$$(z - 0)^{3+1} = 2i$$

$$3! f'''(0)$$

$$3! f'''(0) = 8i$$

$$(z - z_0)^{n+1}$$

$$(z - z_0)^{n+1} = 0$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$e^{a \cos \theta} \cos(a \sin \theta) d\theta = \dots$$

$$P(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n$$

$$P(z) = (a_n + w)z^n.$$

$$|P_n(z)| = |a_n + w|/|z|^n > |a_n|$$

$$z = z_0 + ei$$

$$|f(z_0)|/d = 2|f(z_0)|.$$

$$[|f(z_0)| - |f(z_0 + ei)|]/d = 0.$$

$$|f(z_0 + ei)| = |f(z_0)|$$

and noting that the term $\sin^2 x$ is greatest when $x = \pi/2$ and that the increasing

values (Sec. 54) to the function $g(z) = 1/f(z)$.

$$\text{Ans. } z = 1, z = 1 + i.$$

$$P(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n$$

$$0 = (z - z_0)(z^{k-1} + z^{k-2}z_0 + \dots + z^{k-2}z_0^{k-1})$$

$$nzn = z.$$

$n y_n$

$$n y_n = y.$$

$$|x_n - x| \leq |(x_n - x) + i(y_n - y)|$$

$$|x_n - x| \leq |(x_n - x) + i(y_n - y)| = |(x_n + i y_n) - (x + i y)|$$

$$n(x_n + i y_n)$$

$$n(x_n + i y_n) = \lim$$

$n 1$

$$n 1 = 0 + i \cdot 1 = i.$$

z_n

$$z_n = -2 + i(-1)^n$$

$n z_n$

$$n z_n = \lim$$

$$= -2 + i \cdot 0 = -2.$$

$n r_n$

$$n r_n = \lim$$

$n 2_n$

$$n 2_n =$$

$n 2_{n-1}$

$$n 2_{n-1} = -$$

z_n

$$z_n = z_1 + z_2 + \cdots + z_n + \cdots$$

z_n

$$z_n = z_1 + z_2 + \cdots + z_N$$

$N Y_N$

$$N Y_N = Y.$$

z_n

$$z_n = x_n + i y_n$$

$n z_n$

$$n z_n = \lim$$

$$(z^n)$$

$$(z^n = x_n + i y_n)$$

$$1 + z + z^2 + \cdots + z^n$$

$$1 + z + z^2 + \cdots + z^n = 1 - z^{n+1}$$

$$z^n$$

$$z^n = 1 + z + z^2 + \cdots + z^{N-1}$$

$$z^n$$

$$z^n = 2 + i(-1)^n$$

$$n/|z^n|$$

$$n/|z^n| = |z|.$$

$$(z^n + w^n)$$

$$(z^n + w^n) = S + T.$$

$$|z^n|$$

$$|z^n| = |z + (z^n - z)| < |z| + 1$$

the convergence of x_n and y_n (n

the convergence of x_n and y_n ($n = 1, 2, \dots$) implies that $|x_n| \leq M_1$ and $|y_n| \leq M_2$

$$f(z)$$

$$f(z) = f(z_0) + f'(z_0)$$

$$NN(z)$$

$$NN(z) = 0.$$

$$|s - z| \geq ||s| - |z||$$

$$|s - z| \geq ||s| - |z|| = r_0 - r.$$

$$2r_0$$

$$2r_0 = Mr_0$$

$$1 - (-1)^{2n+1}$$

$$1 - (-1)^{2n+1} = 2$$

Observe that the Taylor series for $\cosh z$ about the

Observe that the Taylor series for $\cosh z$ about the point $z_0 = -2i$, for

$$f(z)$$

$$f(z) = 1 + 2z^2$$

$$z^3 + z^5$$

$$z^3 + z^5 = 1$$

$$1 + z^2$$

$$1 + z^2 = 1 - z^2 + z^4 - z^6 + z^8 - \dots$$

$$z^3 (2 - 1 + z^2 - z^4 + z^6 - z^8 + \dots)$$

$$z^3 (2 - 1 + z^2 - z^4 + z^6 - z^8 + \dots) = 1$$

$$z^4 + 9$$

$$z^4 + 9 = z$$

$$f(2n+1)(0)$$

$$f(2n+1)(0) = (-1)^n$$

$$\cos z$$

$$\cos z = e^{iz} + e^{-iz}$$

$$f(2n+1)(0)$$

$$f(2n+1)(0) = 0$$

$$f(2n+1)(0)$$

$$f(2n+1)(0) = 0$$

9. Use the identity $\sinh(z+i)$

9. Use the identity $\sinh(z+i) = -\sinh z$, verified in Exercise 7(a), Sec. 35, and the

the point z_0

the point $z_0 = i$.

$$(z - z_0)^{n+1}$$

$$(z - z_0)^{n+1} = f^{(n)}(z_0)$$

$$NN(z)$$

$$NN(z) = 0$$

$$NN(z)$$

$$NN(z) = 0.$$

$n!$

$$n! = 1 + z$$

$n! z^n$

$$n! z^n = 1 + 1$$

$e^{1/z} dz$

$$e^{1/z} dz = 2i.$$

The function $f(z)$

The function $f(z) = 1/(z-i)^2$ is already in the form of a

$(z+1)^2$

$$(z+1)^2 = 1$$

$f(z)$

$$f(z) = z + 1$$

origin that includes the unit circle z

origin that includes the unit circle $z = e^{it}$ ($- \leq t \leq$). By taking that circle

$S(z)$

$$S(z) = SN(z) + N(z)$$

(5) are true when N

(5) are true when $N = N + 1$, we now find that

$(z - z_0)^{n+1}$

$$(z - z_0)^{n+1} = f(n)(z_0)$$

$2 + (z - 2)$

$$2 + (z - 2) = 1$$

c_1

$$c_1 = f(z_0)g'(z_0) + f'(z_0)g(z_0)$$

c_2

$$c_2 = f(z_0)g''(z_0) + 2f'(z_0)g'(z_0) + f''(z_0)g(z_0)$$

$$= a_0b_2 + a_1b_1 + a_2b_0.$$

$$f(z)g(z)$$

$$f(z)g(z) = a_0b_0 + (a_0b_1 + a_1b_0)(z - z_0)$$

$$1 + z$$

$$1 + z = e^z$$

$$1 + z$$

$$1 + z = 1 + 1$$

$\sinh z$ are the numbers z

$\sinh z$ are the numbers $z = ni$ ($n = 0, \pm 1, \pm 2, \dots$). So the quotient

$$1 + z^2/3! + z^4/5! + \dots$$

$$1 + z^2/3! + z^4/5! + \dots = 1 - 1$$

$$1 + z^2/3! + z^4/5! + \dots$$

$$1 + z^2/3! + z^4/5! + \dots = 1 - 1$$

$$z(z^2 + 1)$$

$$z(z^2 + 1) = 1$$

$$z^2 \sinh z$$

$$z^2 \sinh z = -i$$

$$z^4 + \dots$$

$$z^4 + \dots = 0$$

$$(fg)(m+1)$$

$$(fg)(m+1) = (fg')(m) + (f'g)(m)$$

$$f(z)$$

$$f(z) = z + a_2z^2 + a_3z^3 + \dots$$

$$f[f(z)]$$

$$f[f(z)] = z + 2a_2z^2 + 2(a_2$$

$$\log z$$

$$\log z = \ln r + i$$

$$\log z$$

$$\log z = \ln r + i$$

have an isolated singular point at z_0

have an isolated singular point at $z_0 = \infty$. Such a singular point will be used in

the punctured disk $0 < |z - z_0| < R$ (Fig. 84). When

the punctured disk $0 < |z - z_0| < R$ (Fig. 84). When $n = 1$, this expression for b_n

$$f(z) dz$$

$$f(z) dz = 2\pi i$$

$$f(z) dz$$

$$f(z) dz = 2\pi i \operatorname{Res}$$

$$e^z$$

$$e^z = 1 + z$$

$2\pi i$ times the residue of its integrand at z

$2\pi i$ times the residue of its integrand at $z = 2$. To determine that residue, we recall

$$z(z-2)^4$$

$$z(z-2)^4 = 2i$$

$$f(z) dz$$

$$f(z) dz = 2i$$

$$f(z) dz$$

$$f(z) dz = 2\pi i \operatorname{Res}$$

$$1 - z$$

$$1 - z = 1 + z + z^2 + \dots$$

$$z(z-1)$$

$$z(z-1) = 5(z-1) + 3$$

when $0 < |z-1| < 1$, it is clear that B_2

when $0 < |z-1| < 1$, it is clear that $B_2 = 3$. Thus

$$f(z) dz$$

$$f(z) dz = 2\pi i \operatorname{Res}$$

$$f(z) dz$$

$$f(z) dz = -2i \operatorname{Res}$$

$$f(z) dz$$

$$f(z) dz = -2i \operatorname{Res}$$

$$z$$

$$z = f(z) = -\operatorname{Res}$$

$$f(z) dz$$

$$f(z) dz = 2i \operatorname{Res}$$

$$z$$

$$z = z_1 f(z) + \operatorname{Res}$$

$$z$$

$$z = z_2 f(z) + \cdots + \operatorname{Res}$$

$$z$$

$$z = z_n f(z) + \operatorname{Res}$$

$$z$$

$$z = f(z) = 0.$$

$$P(z)$$

$$P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

$$Q(z)$$

$$Q(z) = b_0 + b_1 z + b_2 z^2 + \cdots + b_m z^m$$

$$b_{m+1}$$

$$b_{m+1} = b_{m+2} = \cdots = 0.$$

$$= z(z-2) + 3$$

$$z^2(1+z)$$

$$z^2(1+z) = 1$$

$$a_n(z-z_0)^n$$

$$a_n(z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots$$

$$z^n$$

$$z^n = 1 + 1$$

z

$$z = (2n + 1)i$$

$$(z - ai)^3$$

$$(z - ai)^3 = -i/2$$

z

$$z = z_0 \quad f(z) = (z_0)$$

z

$$z = z_0 \quad f(z) = (m-1)(z_0)$$

$$(z)$$

$$(z) = (z_0) + '(z_0)$$

$$(z)$$

$$(z) = b_m + b_{m-1}(z - z_0) + \cdots + b_2(z - z_0)^{m-2} + b_1(z - z_0)^{m-1}$$

$$f(z)$$

$$f(z) = z + 1$$

$$f(z)$$

$$f(z) = (z)$$

$$(z)$$

$$(z) = z + 1$$

$$B_1$$

$$B_1 = (3i) = 3i + 1$$

$$B_2$$

$$B_2 = 3 + i$$

$$f(z)$$

$$f(z) = z^3 + 2z$$

$$(z)$$

$$(z) = z^3 + 2z$$

$$B$$

$$B = "(i)$$

$\log z$

$$\log z = \ln r + i$$

$f(z)$

$$f(z) = (z)$$

(z)

$$(z) = (\log z)^3$$

(i)

$$(i) = (\log i)^3$$

$$= (\ln 1 + i/2)^3$$

B

$$B = (i) = -3$$

$f(z)$

$$f(z) = (z)$$

(z)

$$(z) = \sinh z$$

e^z

$$e^z = 1 + z$$

$f(z)$

$$f(z) = (z)$$

$(c) m$

$$(c) m = 1, B = \pm i/2.$$

$z + 1$

$$z + 1 = 1 + i$$

$(z^2 + 1)^2$

$$(z^2 + 1)^2 = + 2i$$

$(z^2 + 1)^2$

$$(z^2 + 1)^2 = 1 - i$$

$(c) f(z)$

$$(c) f(z) = z^3 e^{1/z}$$

z

$$z = z_0 \quad f(z) = (m-1)(z_0)$$

$g(z)$

$$g(z) = g(z_0) + g'(z_0)$$

$f(z)$

$$f(z) = g(z_0)(z - z_0)^m + g'(z_0)$$

(z)

$$(z) = p(z)$$

$q(z)$

$$q(z) = (z)$$

(z)

$$(z) = p(z)$$

each singular point z

each singular point $z = n$ of f is a simple pole, with residue

B_n

$$B_n = p(n)$$

$q'(n)$

$$q'(n) = (-1)^n$$

at the zero z

at the zero $z = i/2$ of $\cosh z$ (see Sec. 35) is readily found by writing

we find that z

we find that $z = i/2$ is a simple pole of f and that the residue there is

B

$$B = p(i/2)$$

$q'(i/2)$

$$q'(i/2) = -4$$

$$2e^{i/4}$$

$$2e^{i/4} = 1 + i$$

Writing $p(z)$

Writing $p(z) = z$ and $q(z) = z^4 + 4$, we find that

$\sinh z$

$$\sinh z = -2 \cos(t).$$

z

$$z = z_n(z \sec z) = (-1)^{n+1} z_n \text{ where } z_n =$$

$$z^2 \sin z$$

$$z^2 \sin z = 2i$$

$$z^2 f(z)$$

$$z^2 f(z) = .$$

$$-n+1 z$$

$$-n+1 z = M_n$$

$$f(z) dz$$

$$f(z) dz = 2i$$

$$f(x) dx$$

$$f(x) dx = 2i$$

$$f(x) dx$$

$$f(x) dx = 2i$$

$$f(x) dx$$

$$f(x) dx = 2i$$

$$f(x) dx$$

$$f(x) dx = i$$

$$z^6 + 1$$

$$z^6 + 1 = c^2$$

$$2i(B_0 + B_1 + B_2)$$

$$2i(B_0 + B_1 + B_2) = 2i$$

$$f(x) dx$$

$$f(x) dx =$$

$$|z/6 + 1| \geq |z/6 - 1|$$

$$|z/6 + 1| \geq |z/6 - 1| = R/6 - 1.$$

$$x^6 + 1 dx$$

$$x^6 + 1 dx =$$

$$x^6 + 1 dx$$

$$x^6 + 1 dx =$$

$$x^6 + 1 dx$$

$$x^6 + 1 dx =$$

$$x^3 + 1$$

$$x^3 + 1 = 2$$

$$x^{2n} + 1 dx$$

$$x^{2n} + 1 dx =$$

$$z^{2n} + 1$$

$$z^{2n} + 1 = -1$$

$$= 2m + 1$$

$$q(z)$$

$$q(z) = (z^2 - a)^2 + 1$$

$$[q'(z_0)]^3$$

$$[q'(z_0)]^3 = a - i(2a^2 + 3)$$

$$|\sin az/2|$$

$$|\sin az/2| = \sin^2 ax + \sinh^2 ay$$

$$|\cos az/2|$$

$$|\cos az/2| = \cos^2 ax + \sinh^2 ay.$$

$$|eiaz|$$

$$|eiaz| = |e^{ia(x+iy)}| = |e^{-ay}e^{iax}| = e^{-ay}$$

$$(x^2 + 1)^2 dx$$

$$(x^2 + 1)^2 dx = 2$$

$$(x^2 + 1)^2 dx$$

$$(x^2 + 1)^2 dx = 2iB_1 -$$

$$B_1$$

$$B_1 = 'i) = 1$$

$$(x^2 + 1)^2 dx$$

$$(x^2 + 1)^2 dx = 2$$

$$RMR$$

$$RMR = 0.$$

$$e^{-2R/d}$$

$$e^{-2R/d} =$$

$$x^2 + 2x + 2$$

$$x^2 + 2x + 2 = 2iB_1 -$$

$$x^2 + 2x + 2$$

$$x^2 + 2x + 2 = \text{Im}(2iB_1) - \text{Im}$$

$$f(z) dz$$

$$f(z) dz = -B_0 i.$$

$$\leq M_L$$

$$\leq M_L = M.$$

$$z$$

$$z = x_0 + ei$$

$$z$$

$$z = rei\theta = r (\leq r \leq R)$$

$$z dz$$

$$z dz = -i.$$

$$(x^2 + 4)^2 dx$$

$$(x^2 + 4)^2 dx =$$

$$f(z) dz$$

$$f(z) dz = 2i \operatorname{Res}$$

$$f(z) dz$$

$$f(z) dz = 2i \operatorname{Res}$$

$$z$$

$$z = re^{i0} = r \quad (\leq r \leq R)$$

$$z$$

$$z = re^{i\pi} = -r \quad (\leq r \leq R)$$

$$(2i)$$

$$(2i) =$$

$$(r^2 + 4)^2$$

$$(r^2 + 4)^2 =$$

$$(r^2 + 4)^2 dr$$

$$(r^2 + 4)^2 dr =$$

$$(r^2 + 4)^2 dr$$

$$(r^2 + 4)^2 dr =$$

$$|\log z|$$

$$|\log z| = |\ln + i| \leq |\ln| + |i| \leq -\ln +$$

$$|z^2 + 4| \geq |z|^2 - 4$$

$$|z^2 + 4| \geq |z|^2 - 4 = 4 - 2.$$

$$(4 - 2)^2$$

$$(4 - 2)^2 = -\ln$$

$$(R^2 - 4)^2 R$$

$$(R^2 - 4)^2 R =$$

$$(x^2 + 4)^2$$

$$(x^2 + 4)^2 =$$

$$(r^2 + 4)^2$$

$$(r^2 + 4)^2 = 2$$

$$= \exp[-a(\ln r + i)]$$

$$f(z)$$

$$f(z) = \exp[-a(\ln r + i0)]$$

$$f(z)$$

$$f(z) = \exp[-a(\ln r + i2)]$$

$$= r^{-a} e^{-i2a}$$

$$(z)$$

$$(z) = z^{-a} = \exp(-a \log z) = \exp[-a(\ln r + i)]$$

$$(-1)$$

$$(-1) = \exp[-a(\ln 1 + i)] = e^{-ia} = 0.$$

$$z$$

$$z^{-1} f(z) = e^{-ia}$$

$$r + 1 \, dr$$

$$r + 1 \, dr = 2ie^{-ia} -$$

$$R^{-1} 2R$$

$$R^{-1} 2R = 2R$$

$$r + 1 \, dr$$

$$r + 1 \, dr = 2i$$

$$-1 < a < 3 \text{ and } xa$$

$$-1 < a < 3 \text{ and } xa = \exp(a \ln x).$$

$$f(z)$$

$$f(z) = z^{1/3} \log z$$

$$= e^{(1/3) \log z} \log z$$

$$x^2 + 1 \, dx$$

$$x^2 + 1 \, dx = 2$$

$$x^2 + 1 \, dx$$

$$x^2 + 1 \, dx =$$

$$x^2 + 1 \, dx$$

$$x^2 + 1 \, dx = 3$$

$$x^2 + 1 \, dx$$

$$x^2 + 1 \, dx = 0.$$

$$(x + a)(x + b) \, dx$$

$$(x + a)(x + b) \, dx = 2$$

$$x(x^2 + 1)$$

$$x(x^2 + 1) =$$

$$f(z)$$

$$f(z) = z^{-1/2}$$

$$z^2 + 1$$

$$z^2 + 1 = e^{(-1/2) \log z}$$

$$f_1(z) \, dz$$

$$f_1(z) \, dz = 2i \, \text{Res}$$

$$d$$

$$d = ie^i = iz$$

$$\sin$$

$$\sin = e^i - e^{-i}$$

$$\cos$$

$$\cos = e^i + e^{-i}$$

$$\sin$$

$$\sin = z - z^{-1}$$

$$\cos$$

$$\cos = z + z^{-1}$$

$$f(z)$$

$$f(z) = (z)$$

$$\cos 2$$

$$\cos 2 = e^{i2} + e^{-i2}$$

$$= (ei)^2 + (ei)^{-2}$$

$$= z^2 + z^{-2}$$

$$C \arg f(z)$$

$$C \arg f(z) = 1 - 0.$$

$$2 C \arg f(z)$$

$$2 C \arg f(z) = Z - P.$$

$$f[z(t)]$$

$$f[z(t)] = (t)ei(t)$$

$$dt [(t)ei(t)]$$

$$dt [(t)ei(t)] = '(t)ei(t) + i(t)ei(t) '(t).$$

$$'(t) dt$$

$$'(t) dt = \ln(t)$$

$$(b) - (a)$$

$$(b) - (a) = C \arg f(z).$$

$$f(z) dz$$

$$f(z) dz = 2i(Z - P).$$

is analytic and nonzero on the unit circle z

is analytic and nonzero on the unit circle $z = ei$ ($0 \leq \leq 2$). If we let C denote

noting that has the parametric representation w

noting that has the parametric representation $w = e^{-i2}$ ($0 \leq \leq 2$).

$$C \arg[f(z) + g(z)]$$

$$C \arg[f(z) + g(z)] = C \arg$$

$$= C \arg f(z) + C \arg$$

$$Zf + g$$

$$Zf + g = Zf + 1$$

$$F(z)$$

$$F(z) = 1 + g(z)$$

$$z^7 - 4z^3 + z - 1$$

$$z^7 - 4z^3 + z - 1 = 0$$

$$g(z)$$

$$g(z) = z^7 + z - 1.$$

$$P(z)$$

$$P(z) = a_0 + a_1z + a_2z^2 + \cdots + a_nz^n$$

$$g(z)$$

$$g(z) = a_0 + a_1z + a_2z^2 + \cdots + a_{n-1}z^{n-1}$$

$$(c) f(z)$$

$$(c) f(z) = (2z - 1)^7/z^3.$$

$$f(z) dz$$

$$f(z) dz = 2i$$

$$2z^5 - 6z^2 + z + 1$$

$$2z^5 - 6z^2 + z + 1 = 0$$

$$s$$

$$s = +Rei$$

$$estF(s) ds$$

$$estF(s) ds = 2i$$

$$| \exp(t + Rtei) |$$

$$| \exp(t + Rtei) | = e^{teRt} \cos$$

$$s$$

$$s=s_0 [estF(s)] + Res$$

$$s$$

$$s=s_0 [estF(s)] + Res$$

$$[estF(s)]$$

$$[estF(s)] = 2et Re$$

$$s$$

$$s=s_0 [estF(s)] + Res$$

s

$$s = s_0 [\text{est}F(s)] + \text{Res}$$

s

$$s = +Rei$$

$|s|$

$$|s| = | +Rei| \leq +R$$

$|s|$

$$|s| = | +Rei| \geq | -R| = R - > a.$$

s_n

$$s_n = (2n - 1)$$

s_n

$$s_n = -(2n - 1)$$

s

$$s = s_n [\text{est}F(s)] + \text{Res}$$

$$= -i \cos n = (-1)^{n+1} i = 0.$$

$F(s)$

$$F(s) = \sinh(xs^{1/2})$$

$F(s)$

$$F(s) = xs^{1/2} + (xs^{1/2})^3/3! + \dots$$

$$s[s^{1/2} + (s^{1/2})^3/3! + \dots]$$

$$s[s^{1/2} + (s^{1/2})^3/3! + \dots] = x + x^3s/6 + \dots$$

occur if $s^{1/2}$

occur if $s^{1/2} = \pm ni$ ($n = 1, 2, \dots$). The points

s_n

$$s_n = -n^2$$

$p(s)$

$$p(s) = \sinh(xs^{1/2})$$

$q(s)$

$$q(s) = s \sinh(s^{1/2}).$$

$$p(s_n)$$

$$p(s_n) = \sinh(xs_1/2)$$

$$f(t)$$

$$f(t) = x + 2$$

$$\text{Ans. } f(t)$$

$$\text{Ans. } f(t) = e^{-t}(\sin 2t + \cos 2t - 1).$$

$$\text{Ans. } f(t)$$

$$\text{Ans. } f(t) = (1 + a^2 t^2) \sin at - at \cos at.$$

$$\text{Ans. } f(t)$$

$$\text{Ans. } f(t) = x + 8$$

$$\text{Ans. } f(t)$$

$$\text{Ans. } f(t) = 1 + 4$$

$$8. F(s)$$

$$8. F(s) = \coth(s/2)$$

$$9. F(s)$$

$$9. F(s) = \sinh(xs_1/2)$$

in the punctured disk $0 < |s - s_0| < R_2$. Also, assu

in the punctured disk $0 < |s - s_0| < R_2$. Also, assume that $F(s) = F(s)$ at points s

$$s$$

$$s = s_0 [\text{est} F(s)] + \text{Res}$$

$$[\text{est} F(s)]$$

$$[\text{est} F(s)] = 2 \text{et Re}$$

$$s_0$$

$$s_0 = +i (= 0) \text{ and } s_0 \text{ contains a term of the type}$$

$$(u, v)$$

$$(u, v) = (x + b_1, y + b_2)$$

$$w$$

$$w = Az + B$$

w

$$w = Z + B.$$

w

$$w = (1 + i)z + 2$$

Z

$$Z = (1 + i)z$$

w

$$w = Z + 2.$$

w

$$w = (1 + i)z + 2.$$

formation w

$$\text{formation } w = (1 + i)z$$

$z_0 T(z)$

$$z_0 T(z) =$$

$zT(z)$

$$zT(z) = 0$$

which is clearly true when z_0

which is clearly true when $z_0 = 0$ and when $z_0 = \infty$, is also true for those two

$$A(x^2 + y^2) + Bx + Cy + D$$

$$A(x^2 + y^2) + Bx + Cy + D = 0$$

A horizontal line y

A horizontal line $y = c_2$ ($c_2 = 0$) is mapped by $w = 1/z$ onto

When w

When $w = 1/z$, the half plane $x \geq c_1$ ($c_1 > 0$) is mapped

where z

where $z = x + iy$.

$$2Dww + (B + Ci)w + (B - Ci)w + 2A$$

$$2Dww + (B + Ci)w + (B - Ci)w + 2A = 0.$$

w

$$w = az + b$$

$$Azw + Bz + Cw + D$$

$$Azw + Bz + Cw + D = 0$$

z

$$z = -dw + b$$

of z

of $z = w$ when $c = 0$. We first write

$$T(z)$$

$$T(z) = az + b$$

$$T^{-1}(w)$$

$$T^{-1}(w) = -dw + b$$

$$T^{-1}()$$

$$T^{-1}() =$$

$$T^{-1}()$$

$$T^{-1}() = -d$$

w

$$w = az + b$$

$$ic - ib$$

$$ic - ib = -a + b$$

$$ic + ib$$

$$ic + ib = a + b.$$

w

$$w = ibz + b$$

$$-ibz + b$$

$$-ibz + b = b(iz + 1)$$

w

$$w = iz + 1$$

w

$$w = az + b$$

w

$$w = (i + 1)z + (i - 1)$$

$$Azw + Bz + Cw + D$$

$$Azw + Bz + Cw + D = 0.$$

$$(w - i)(1 + i)$$

$$(w - i)(1 + i) = (z + 1)(0 - 1)$$

$w1$

$$w1 = i, w2 = , w3 = 1.$$

$w - 1$

$$w - 1 = (z - 1)(0 + 1)$$

w

$$w = (i + 1)z + (i - 1)$$

Ans. w

$$\text{Ans. } w = 3z + 2i$$

points $w1$

$$\text{points } w1 = 0, w2 = i, w3 = .$$

Ans. w

$$\text{Ans. } w = -1/z$$

$T(z)$

$$T(z) = a_1z + b_1$$

$S(z)$

$$S(z) = a_2z + b_2$$

$S[T(z)]$

$$S[T(z)] = a_3z + b_3$$

$T(z)$

$$T(z) = az + b$$

$$(a + d)[cz^2 + (d - a)z - b]$$

$$(a + d)[cz^2 + (d - a)z - b] = 0.$$

z

$z =$ on the line and determining conditions on a linear fractional transformation

w

$$w = az + b$$

z

$z =$ is a finite number only if $c = 0$, that finite number being $w = a/c$. So the

requirement that $|w|$

requirement that $|w| = 1$ when $z =$ means that $|a/c| = 1$, or

w

$$w = \exp(i); \text{ hence } z_1 = z_0.$$

v

$$v = \operatorname{Im} w = \operatorname{Im} (z - 1)(z + 1)$$

$\log Z$

$$\log Z = \ln R + i$$

we see that as a point Z

we see that as a point $Z = R \exp(i\theta)$ ($0 < \theta < \pi$) moves outward from the origin

the choice of θ varies between 0

the choice of θ varies between $\theta = 0$ to $\theta = \pi$, the mapping of the half plane

4. Transformation (6), Sec. 95, maps the point z

4. Transformation (6), Sec. 95, maps the point $z =$ onto the point $w = \exp(i)$, which

5. Note that when

5. Note that when $\theta = \pi/2$, the transformation in Exercise 4 becomes

w

$$w = iz + \exp(i/4)$$

w

$$w = iz + \exp(i/4)$$

formation is to examine images of vertical lines x

formation is to examine images of vertical lines $x = c_1$. If $0 < c_1 < 1/2$, points on

$$\sin^2 c_1 + \cos^2 c_1$$

$$\sin^2 c_1 + \cos^2 c_1 = \pm 1.$$

branch. If $-1/2 < c_1 < 0$, the line x

branch. If $-1/2 < c_1 < 0$, the line $x = c_1$ is mapped onto the left-hand branch of

image of the line segment BA there is found by wri

image of the line segment BA there is found by writing $x = 1/2$ in equations (1)

line x

line $x = c_1$ ($0 < c_1 < 1/2$) is thought of as moving to the left toward the positive

y

$$y = c_2 > 0$$

z

$$z = iz +$$

mation w

mation $w = \sin z$ maps the semi-infinite strip $-1/2 \leq x \leq 1/2, y \geq 0$ onto the

$$z^{1/2}$$

$$z^{1/2} = r \exp i(+2k)$$

$$z^{1/2}$$

$$z^{1/2} = \exp$$

$$2 (\ln r + i)$$

$$2 (\ln r + i) = \ln r + i$$

$$F_0(z)$$

$$F_0(z) = r \exp i$$

inclination 1 ($0 \leq \theta \leq 2\pi$), its image w

inclination 1 ($0 \leq \theta \leq 2\pi$), its image $w = r \exp(i\theta/2)$ moves outward from

$$\log z$$

$$\log z = \ln r + i(\theta + 2\pi)$$

$$F_1(z)$$

$$F_1(z) = r \exp i(\theta + 2\pi)$$

extend the domain of definition of F_0 to include the

extend the domain of definition of F_0 to include the ray $\theta = 2\pi$ and if we write

$$f(z)$$

$$f(z) = r \exp i$$

$$z^{1/n}$$

$$z^{1/n} = \exp$$

domain $r > 0$, $-\pi < \theta < \pi$. The principal branch occurs

domain $r > 0$, $-\pi < \theta < \pi$. The principal branch occurs when $k = 0$, and further

$$v^2$$

$$v^2 = 4b^2(u + b^2).$$

Z plane except for the origin and points on the ray

Z plane except for the origin and points on the ray $\arg Z = \pi$.

The transformation

$$(z^2 - 1)^{1/2}$$

$$(z^2 - 1)^{1/2} = \exp$$

$$(z^2 - 1)^{1/2}$$

$$(z^2 - 1)^{1/2} = (z - 1)^{1/2}(z + 1)^{1/2}$$

is a branch of $(z + 1)^{1/2}$ defined on a domain D_2 , t
is a branch of $(z + 1)^{1/2}$ defined on a domain D_2 , the product f
 $(z) = f_1(z)f_2(z)$ is

$$f_1(z)$$

$$f_1(z) = r_1 \exp i\theta_1$$

$$f_2(z)$$

$$f_2(z) = r_2 \exp i\theta_2$$

$$r_2$$

$$r_2 = |z + 1|$$

$$\theta_2$$

$$\theta_2 = \arg(z + 1).$$

$$f(z)$$

$$f(z) = r_1 r_2 \exp i(\theta_1 + \theta_2)$$

$$F(z)$$

$$F(z) = r_1 r_2 \exp i(\theta_1 + \theta_2)$$

$$G(z)$$

$$G(z) = r_1 r_2 \exp i(\theta_1 + \theta_2)$$

$$\theta_2$$

$$\theta_2 = \text{Arg}(z + 1)$$

$$= \exp i(\theta_1 + \theta_2)$$

$$r_1$$

$$r_1 = r_2 > 1$$

$$(w^2 + 1)^{1/2}$$

$$(w^2 + 1)^{1/2} = (w - i)^{1/2}(w + i)^{1/2}$$

and $w + i$

and $w + i = 2 \exp(i\theta_2)$. (See Fig. 128.) With the restrictions

$$H(w)$$

$$H(w) = 12 \exp i(\theta_1 + \theta_2)$$

are the two values of $(z^2 - 1)^{1/2}$ for a point in D_z

are the two values of $(z^2 - 1)^{1/2}$ for a point in D_z , we see that $w = F(z)$ or

the ray v

the ray $v = u$ ($u > 0$).

z

$$z = 1 + w^2$$

$\log z$

$$\log z = \ln r + i$$

the angle changes from 0 to 2π . As it moves across

the angle changes from 0 to 2π . As it moves across the ray $\theta = 2\pi$, the point passes

in Fig. 130, its image w moves upward across the l

in Fig. 130, its image w moves upward across the line $v = 2\pi$, as indicated in that

$f(z)$

$$f(z) = \ln r + i$$

$z^{1/2}$

$$z^{1/2} = re^{i\theta/2}$$

$|w|$

$|w| = 1$ under the transformation $w = z^{1/2}$.

$z^{1/2}$

$$z^{1/2} = re^{i\theta/2}$$

$z^{1/2} dz$

$$z^{1/2} dz = 0.$$

$g(z)$

$$g(z) = z + (z^2 - 1)^{1/2}.$$

$g^1(z)$

$$g^1(z) = z + f_0(z).$$

$$f_0(z)$$

$$f_0(z) = r_1 r_2$$

$$z + 1$$

$$z + 1 = r_2 \exp(i2).$$

$$\arg w'(t_0)$$

$$\arg w'(t_0) = \arg f'[z(t_0)] + \arg z'(t_0).$$

$$0$$

$$0 = 0 + 0.$$

$$0$$

$$0 = 0 + 0.$$

$$1$$

$$1 = 0 + 1$$

$$2$$

$$2 = 0 + 2$$

$$w$$

$$w = 1 + Z.$$

$$w$$

$$w = f(z) = x^2 - y^2 + i2xy$$

$$y$$

$$y = x \ (x \geq 0)$$

$$x$$

$$x = 1 \ (y \geq 0)$$

$$du$$

$$du = dv/dy$$

by the conformality of the mapping at z

by the conformality of the mapping at $z = 1 + i$. The angle of rotation $/4$ at the

$$\arg[f'(1+i)]$$

$$\arg[f'(1+i)] = \arg[2(1+i)] =$$

$$|f'(1+i)|$$

$$|f'(1+i)| = |2(1+i)| = 2$$

If we write w

If we write $w = u + iv$ and $w_0 = u_0 + iv_0$, as well as

$$u + iv$$

$$u + iv = u(x, y) + iv(x, y)$$

$$\log w$$

$$\log w = \ln + i$$

$$g(1)$$

$$g(1) = \ln 1 + i2 = 2i$$

$$\text{Log } w$$

$$\text{Log } w = \ln + i$$

$$\text{Ans. (a) } w^{1/2}$$

$$\text{Ans. (a) } w^{1/2} = e^{i/2}$$

$$(c) w^{1/2}$$

$$(c) w^{1/2} = e^{i/2}$$

$$0$$

$$0 = m_0 + \arg f(m)(z_0).$$

$$f(z)$$

$$f(z) = u(x, y) + iv(x, y)$$

$$v_{xx} + v_{yy}$$

$$v_{xx} + v_{yy} = 0.$$

$$w$$

$$w = f(z) = u(x, y) + iv(x, y)$$

$$(w)$$

$$(w) = h(u, v) + ig(u, v)$$

reveals that as a point z

reveals that as a point $z = r \exp(i\theta)$ ($-\pi/2 < \theta < \pi/2$) travels outward from

w

$$w = \log z = \ln r + i$$

w

$$w = f(z) = u(x, y) + iv(x, y)$$

w

$$w = iz^2 = -2xy + i(x^2 - y^2)$$

$H(x, y)$

$$H(x, y) = x^2 - y^2 + 2$$

3. The transformation w

3. The transformation $w = \exp z$ maps the horizontal strip $0 < y <$
onto the upper half

4. Under the transformation w

4. Under the transformation $w = \exp z$, the image of the segment 0
 $\leq y \leq$ of the y

axis is the semicircle $u^2 + v^2$

axis is the semicircle $u^2 + v^2 = 1$, $v \geq 0$ (see Sec. 14). Also, the
function

$h(u, v)$

$$h(u, v) = \operatorname{Re}(-2i w + e^{-w}) = 2v + e^{-u} \cos v.$$

$p_{uu}(u, v) + p_{vv}(u, v)$

$$p_{uu}(u, v) + p_{vv}(u, v) = (u, v)$$

w

$$w = f(z) = u(x, y) + iv(x, y)$$

$P_{xx}(x, y) + P_{yy}(x, y)$

$$P_{xx}(x, y) + P_{yy}(x, y) = [u(x, y), v(x, y)] / f'(z) / 2$$

$T_{xx}(x, y) + T_{yy}(x, y)$

$$T_{xx}(x, y) + T_{yy}(x, y) = 0.$$

$T_{xx}(x, y) + T_{yy}(x, y)$

$$T_{xx}(x, y) + T_{yy}(x, y) = 0$$

$$z - 1$$

$$z - 1 = r_1 \exp(i\theta_1)$$

where $0 \leq \theta_1 \leq 2\pi$

where $0 \leq r_1 \leq \infty$ ($k = 1, 2$). The transformation

$$z + 1$$

$$z + 1 = \ln r_1$$

strip and unity on the edge v

strip and unity on the edge $v = 1$ is clearly

The isotherms $T(x, y)$

The isotherms $T(x, y) = c_1$ ($0 < c_1 < 1$) are arcs of the circles

x

$x = \pm 1/2$ and $y = 0$ when the first two surfaces are kept at temperature zero and

$$T_{xx}(x, y) + T_{yy}(x, y)$$

$$T_{xx}(x, y) + T_{yy}(x, y) = 0$$

The flux outward through the surface lying in the plane

The flux outward through the surface lying in the plane $x = 1/2$ is

$$T_{xx}(x, y) + T_{yy}(x, y)$$

$$T_{xx}(x, y) + T_{yy}(x, y) = 0$$

$$dh/dn$$

$dh/dn = 0$ in the theorem of Sec. 106.

$$\sin^2 u + \cos^2 u$$

$$\sin^2 u + \cos^2 u = 1$$

$$(x-1)^2 + y^2$$

$$(x-1)^2 + y^2 = 2 \sin^2 u.$$

u

$u = \pi/2$. In view of equation (5), then, the required temperature function is

It can be seen from equation (5) that the isotherm

It can be seen from equation (5) that the isotherms $T(x, y) = c_1$
($0 < c_1 < 1$)

$$H_{xx}(x, y) + H_{yy}(x, y) = 0$$

$$H(1/2, y) = 0$$

13. According to Sec. 54 and Exercise 6 of that se

13. According to Sec. 54 and Exercise 6 of that section, if $f(z) = u(x, y) + iv(x, y)$

$$V_{xx}(x, y) + V_{yy}(x, y) = 0.$$

$$\log w = 1$$

$$\arctan t = .$$

by the half planes

by the half planes $\theta = 0$ and $\theta = \pi/4$ and the portion $0 \leq r \leq 1$ of the cylindrical

$$\text{and } x^2 + y^2 = r^2$$

$$\text{Ans. } V = \ln(x^2 + y^2)$$

$$V_{xx}(x, y) + V_{yy}(x, y) = 0$$

$$n = (2n - 1)$$

$$V = p + iq$$

$$y(x, y)$$

$$y(x, y) = q(x, y).$$

$$xx(x, y) + yy(x, y)$$

$$xx(x, y) + yy(x, y) = 0$$

$$V$$

$$V = p(x, y) + iq(x, y)$$

$$F(z)$$

$$F(z) = (x, y) + i(x, y)$$

$$F'(z)$$

$$F'(z) = x(x, y) + ix(x, y)$$

$$F'(z)$$

$$F'(z) = x(x, y) - iy(x, y).$$

$$(x, y)$$

$$(x, y) = Ax$$

complex potential for the flow in the strip between

complex potential for the flow in the strip between the lines $y = 0$ and $y = \pi$, both

$$F[f(z)]$$

$$F[f(z)] = [u(x, y), v(x, y)] + i[u(x, y), v(x, y)]$$

$$w$$

$$w = z^2 = x^2 - y^2 + i2xy$$

$$\sin$$

$$\sin^2 = c^2$$

$$|z|/V$$

$$|z|/V = Ae^{i\theta}$$

$$(z^2 - 4)^{1/2}$$

$$(z^2 - 4)^{1/2} = r^1 r^2 \exp$$

$$2 [z + (z^2 - 4)^{1/2}]$$

$$2 [z + (z^2 - 4)^{1/2}] = 1$$

F

$$F = A[z \cos -i(z^2 - 4)^{1/2} \sin]$$

of the line y

of the line $y =$ are mapped onto the half line $v = (u \leq -1)$.

Similarly, the line

$\arg$

$$\arg = \arg f'(z_0) + \arg t.$$

vertex w_n , which is the image of the point z

vertex w_n , which is the image of the point $z =$, can be written

k_n

$$k_n = 2 - (k_1 + k_2 + \cdots + k_{n-1}).$$

$$k_1 + k_2 + \cdots + k_{n-1}$$

$$k_1 + k_2 + \cdots + k_{n-1} = 2.$$

$$(z - x_j)^{-k_j}$$

$$(z - x_j)^{-k_j} = \exp[-k_j \log(z - x_j)] = \exp[-k_j(\ln |z - x_j| + ij)]$$

$$(z - x_j)^{-k_j}$$

$$(z - x_j)^{-k_j} = |z - x_j|^{-k_j} \exp(-ik_j j)$$

$$f'(z)$$

$$f'(z) = (z - x_1)^{-k_1}(z)$$

$$zF(z)$$

$$zF(z) = W_n$$

$$zF(z)$$

$$zF(z) = W_n$$

$$\arg(x-1)$$

$$\arg(x-1) = .$$

when z

when $z = -1$. With the substitution $x = t$, the last integral here reduces to a

$$t^{-1/2}(1-t)^{-2/3} dt$$

$$t^{-1/2}(1-t)^{-2/3} dt = B$$

$$w1$$

$$w1 = b \exp i$$

$$w3$$

$$w3 = w1 + \exp$$

$$= b \exp i$$

$$w3$$

$$w3 = b \exp i$$

$$\arg(x+a)$$

$$\arg(x+a) = \arg(x+1) = 0$$

$$|g(x)| dx$$

$$|g(x)| dx = -b + ic.$$

$$w4$$

$$w4 = b + ic.$$

$$k1$$

$$k1 = k2 =$$

$$w$$

$$w = A \log z + B.$$

$$i$$

$$i = A \log x1 = A \ln |x1| + A i.$$

$$A$$

$$A = \exp 3i$$

same half line v

same half line $v = i$ ($u \geq 1$). Note the changes in direction of the motion of w at

$\text{Im } z \geq 0$. (See Fig. 13, Appendix 2.) Let Z_j be poin

$\text{Im } z \geq 0$. (See Fig. 13, Appendix 2.) Let Z_j be points on the circle $|Z| = 1$ whose

v

$v = 0$ and $v =$ when the fluid is entering through a narrow slit along the line

As $\text{Re } w$ tends to $-\infty$, the image of w approaches the

As $\text{Re } w$ tends to $-\infty$, the image of w approaches the point $z = 0$. A sink of

curve connecting the boundary lines v

curve connecting the boundary lines $v = 0$ and $v =$ of the left-hand part of the

u/V

$$u/V = Q$$

dz

$$dz = Az^{-1/2}(z-x^2)^{-1/2}(z-1)^{1/2}.$$

F

$$F = V_0 \text{Log } z = V_0 \ln r + iV_0$$

V_0 over each semicircle z

V_0 over each semicircle $z = Re^{i\theta} (0 \leq \theta \leq \pi)$ as θ varies from 0 to π . [Compare

At the limiting position of w_4 , which corresponds

At the limiting position of w_4 , which corresponds to $z = \infty$, let the real number V_4

w

$$w = h \text{Log } 1 + s$$

s^2

$$s^2 = \exp(F/V_0) - h^2$$

k_3

$$k_3 = k_4 = \frac{1}{2}.$$

$u + iv$

$$u + iv = A_1 + iA_2$$

$$\arg(x-1)$$

$$\arg(x-1) = .$$

Thus x

Thus $x = 0$ is the point whose image is the vertex $w = i/2$; and, upon substituting

z^2

$$z^2 = 1 + e^{-2w}.$$

$\tan V$

$$\tan V = 1$$

$T(u, v)$

$$T(u, v) =$$

and, using the parametric representation s

and, using the parametric representation $s = r_0 \exp(i) \ (0 \leq \leq 2)$ for C_0 , we

re^{-i}

$$re^{-i} = ss$$

$f(rei)$

$$f(rei) = r^2$$

$u(r,)$

$$u(r,) = 1$$

$P(r_0, r, -)$

$$P(r_0, r, -) = r^2$$

$P(r_0, r, -)$

$$P(r_0, r, -) = Re$$

$u(r,)$

$$u(r,) = 1$$

When $f(z)$

When $f(z) = u(r,) = 1$, equation (10) shows that P has the property

$$P(r_0, r, -) d$$

$$P(r_0, r, -) d = 1$$

$$U(r,)$$

$$U(r,) = 1$$

$$U(r,)$$

$$U(r,) = F()$$

$$V(r,)$$

$$V(r,) = 1$$

$$V(r,)$$

$$V(r,) = \arctan$$

$$V(r,)$$

$$V(r,) = 1$$

$$r^2 P_{rr} + r P_r + P$$

$$r^2 P_{rr} + r P_r + P = 0$$

$$U(r,) - F()$$

$$U(r,) - F() = 1$$

$$U(r,) - F()$$

$$U(r,) - F() = I_1(r) + I_2(r)$$

$$P(r_0, r, -) d$$

$$P(r_0, r, -) d =$$

$$U(0,)$$

$$U(0,) = 1$$

$$P(r_0, r, -)$$

$$P(r_0, r, -) = 1 + 2$$

$$U(r,)$$

$$U(r,) = 1$$

$$2$$

$$2 = 1 + \cos$$

$$\tan(-)$$

$$\tan(-) = \tan - \tan$$

$$I(h, -0) d$$

$$I(h, -0) d = 1.$$

$$P(r_0, r, -) I(h, -0) d$$

$$P(r_0, r, -) I(h, -0) d = P(r_0, r, c -)$$

$$P(r_0, r, -) I(h, -0) d$$

$$P(r_0, r, -) I(h, -0) d = P(r_0, r, -0)$$

$$r \cos$$

$$r \cos = A$$

$$U(r,)$$

$$U(r,) = 1$$

$$U(r,)$$

$$U(r,) = F()$$

$$\text{If } F(2 -)$$

$$\text{If } F(2 -) = F(), \text{ then}$$

$$U(r,)$$

$$U(r,) = 1$$

$$= 2 -.$$

$$P(r_0, R, -) d$$

$$P(r_0, R, -) d = -1$$

$$RH(R,)$$

$$RH(R,) = 1$$

$$r_2 \text{urr}(r,) + r \text{ur}(r,) + u(r,)$$

$$r_2 \text{urr}(r,) + r \text{ur}(r,) + u(r,) = 0$$

$$\text{If } f(z)$$

$$\text{If } f(z) = u(x, y) + iv(x, y), \text{ it follows from formulas (5) and (6) that the}$$

$$G(x, y, \theta)$$

$$G(x, y, \theta) = F(x + y \tan \theta) - F(x)$$

$$G(x, y, \theta) d$$

$$G(x, y, \theta) d = I_1(y) + I_2(y) + I_3(y)$$

$$T(x, y)$$

$$T(x, y) = T_1(x, y) + T_2(x, y)$$

$$V(x, y) + iU(x, y)$$

$$V(x, y) + iU(x, y) = 1$$

where z

$$\text{where } z = x + iy.$$

$$s$$

$$s = r_0 \exp(i\theta)$$

$$z$$

$$z = r \exp(i\theta)$$

$$Q(r_0, r, -)$$

$$Q(r_0, r, -) = -r_0$$

$$U(r, \theta)$$

$$U(r, \theta) = 1$$

$$U_r(r, \theta)$$

$$U_r(r, \theta) = 1$$

$$P(r_0, r, -) G(\theta) d$$

$$P(r_0, r, -) G(\theta) d = G(\theta).$$

$$U_r(r, \theta)$$

$$U_r(r, \theta) = G(\theta)$$

$$U(r, \theta)$$

$$U(r, \theta) = -r_0$$

$$U(r, \theta)$$

$$U(r, \theta) = 1$$

$$U(r, 0)$$

$$U(r, 0) = U(r, \infty) = 0$$

$$U_r(r, \infty)$$

$$U_r(r, \infty) = G()$$

$$U(r, \infty)$$

$$U(r, \infty) = 1$$

$$U(r, 0)$$

$$U(r, 0) = U(r, \infty) = 0$$

$$U_r(r, \infty)$$

$$U_r(r, \infty) = G()$$

$$G() d$$

$$G() d = 0.$$

$$(b - 1/b)^2$$

$$(b - 1/b)^2 = 1.$$

$$z + 1; z$$

$$z + 1; z = -\coth w$$

$$w$$

$$w = \log z + 1$$

$$z$$

$$z = ez + 1$$

$$w$$

$$w = i + z - \log z.$$

$$w$$

$$w = 2(z + 1)^{1/2} + \log (z + 1)^{1/2} - 1$$

$$w$$

$$w = 1/z, 313-317$$

Good Luck!

Keep practicing!