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STA100

General Mathematics and Biostatistics

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COMPREHENSIVE STUDY HANDOUT

STA100

General Mathematics and Biostatistics

Lecture-wise study notes prepared for Virtual University of Pakistan students.

Covers all major topics with objectives, key definitions, summaries and exam pointers.



10 study units

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Unit 1: Number Systems, Sets and Functions

Learning Objectives

- Objective 1:** Classify numbers and perform set operations
- Objective 2:** Work with functions: domain, range, composition
- Objective 3:** Apply basic counting principles

Mathematics begins with numbers: natural, integer, rational (ratios p/q), irrational, real, and complex. Rational numbers have terminating or repeating decimals; irrationals like π do not. Interval notation and absolute value (distance from zero) organize the real line; inequalities solve by sign analysis.

Sets collect objects: union, intersection, complement, difference, with Venn diagrams visualizing and De Morgan's laws (complement of union = intersection of complements) simplifying. A function assigns each domain element exactly one image; the vertical-line test checks graphs. Composition ($f \circ g$)(x) = $f(g(x))$ chains functions; inverses undo them where they exist.

Counting underlies probability: the multiplication principle (m ways then n ways = mn), permutations (ordered, $P(n,r) = n!/(n-r)!$) and combinations (unordered, $C(n,r) = n!/r!(n-r)!$). Very important:  distinguishing permutations from combinations by whether order matters is the key skill; half of counting errors come from this single confusion.

Key Definitions

Function is defined as a rule assigning each domain element exactly one value.

Permutation is defined as an ordered arrangement; $P(n,r) = n!/(n-r)!$.

Combination is defined as an unordered selection; $C(n,r) = n!/r!(n-r)!$.

Key Points to Remember

- Number types: N , Z , Q (repeating decimals), irrationals, R ; absolute value = distance.
- Set ops: union, intersection, complement; De Morgan's laws; Venn diagrams.
- Functions: vertical-line test, composition, inverses.
- Counting: order matters $\rightarrow$ permutation; order irrelevant $\rightarrow$ combination.

Unit Summary

Numbers, sets and functions provide the language; counting principles (permutations vs combinations) open the door to probability.

Exam Pointer

Very important: solve counting problems choosing permutation vs combination correctly; apply De Morgan's laws.

Unit 2: Algebra: Equations, Inequalities and Matrices

Learning Objectives

Objective 1: Solve linear and quadratic equations

Objective 2: Work with matrices and determinants

Objective 3: Solve systems of equations

Linear equations solve by isolating the variable; quadratics by factoring, completing the square, or the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The discriminant $b^2 - 4ac$ classifies roots: positive (two real), zero (one repeated), negative (complex pair). Inequalities need sign charts, with direction flipping when multiplying by negatives.

Matrices organize linear algebra: addition, scalar multiplication, and multiplication (rows by columns). The determinant of a 2×2 matrix $ad - bc$ tests invertibility; larger ones expand by cofactors. Matrix inverses solve systems via $x = A^{-1}b$.

Systems solve by substitution, elimination, or matrix methods (Gaussian elimination, Cramer's rule). Applied problems (mixture, investment, diet formulations in biostatistics) translate words to systems.

Very important: the discriminant's root classification and 2×2 determinant computation are rapid exam marks; matrix multiplication order matters ($AB \neq BA$).

Key Definitions

Discriminant is defined as $b^2 - 4ac$, classifying quadratic roots.

Determinant is defined as the scalar $ad - bc$ for 2×2 matrices, testing invertibility.

System of equations is defined as multiple equations sharing unknowns, solved simultaneously.

Key Points to Remember

- Quadratics: formula + discriminant classification (two/one/complex roots).
- Matrices: addition, multiplication (order matters), 2×2 determinant and inverse.
- Systems: substitution, elimination, Gaussian elimination, Cramer.
- Word problems: translate carefully, then solve mechanically.

Unit Summary

Algebraic fluency, quadratics, inequalities, matrices and systems, provides the computational base for everything that follows.

Exam Pointer

Very important: classify roots via discriminant; invert 2×2 matrices; solve systems by elimination.

Unit 3: Limits, Differentiation and Applications

Learning Objectives

Objective 1: Evaluate limits

Objective 2: Differentiate using standard rules

Objective 3: Apply derivatives to optimization and rates

Limits capture approaching behavior: direct substitution first, then factoring/rationalizing for 0/0 forms. Continuity means the limit equals the value; most applied functions are continuous. The derivative is the limit of difference quotients: instantaneous rate of change, the slope of the tangent.

Differentiation rules: power, sum, product, quotient, and chain (for compositions). Higher derivatives give concavity via the second derivative. Standard derivatives (polynomials, exponentials, logs, trigonometric) combine through these rules; the chain rule is the most used and most erred.

Applications: optimization (set derivative to zero, test with second derivative), related rates, and marginal analysis in economics (marginal cost = derivative of cost). In biostatistics, growth models differentiate to find maximal growth rates. Very important: optimization word problems follow a fixed pipeline (model -> differentiate -> critical points -> verify); practice it. 

Key Definitions

Limit is defined as the value a function approaches as its input approaches a point.

Derivative is defined as the instantaneous rate of change; limit of the difference quotient.

Critical point is defined as where the derivative is zero or undefined; candidate for extrema.

Key Points to Remember

- Limits: substitute, then resolve 0/0 by factoring/rationalizing.
- Rules: power, product, quotient, chain (most used); second derivative -> concavity.
- Optimization pipeline: model, differentiate, critical points, verify endpoints.
- Marginal analysis and growth-rate applications.

Unit Summary

Limits lead to derivatives, whose rules power optimization and rate analysis across science and economics.

Exam Pointer

Very important: differentiate composite functions (chain rule) and solve optimization word problems completely.

Unit 4: Integration and Its Uses

Learning Objectives

Objective 1: Integrate basic functions

Objective 2: Evaluate definite integrals

Objective 3: Apply integration to areas and totals

Integration reverses differentiation: the indefinite integral adds the constant C. Power rule (reversed), exponential, and $1/x \rightarrow \ln|x|$ cover most needs; substitution reverses the chain rule. The definite integral from a to b gives net accumulation: area under the curve when the function is positive.

The Fundamental Theorem connects the two: differentiate an accumulation and recover the function; evaluate definite integrals via antiderivatives at endpoints. Properties (linearity, additivity over

intervals, reversal flipping sign) simplify computation.

Applications: areas between curves, total from rate (integrate velocity for distance, marginal cost for total cost), and in biostatistics, probabilities as areas under density curves (previewing continuous distributions). Very important: substitution choice (let u be the inner function) is the key technique; area problems need correct top-minus-bottom setup.

Key Definitions

Antiderivative is defined as a function whose derivative is the given function; the indefinite integral.

Definite integral is defined as net accumulation from a to b ; area under positive curves.

Substitution is defined as reversing the chain rule by changing variables.

Key Points to Remember

- Indefinite: reverse the rules + C ; substitution for compositions.
- Definite: Fundamental Theorem, net accumulation/area.
- Setup matters: top minus bottom for areas between curves.
- Biostatistics link: probabilities as areas under curves.

Unit Summary

Integration accumulates: antiderivatives plus the Fundamental Theorem yield areas, totals and (later) probabilities.

Exam Pointer

Very important: integrate by substitution and set up area-between-curves integrals correctly.

Unit 5: Descriptive Biostatistics

Learning Objectives

- Objective 1:** Summarize data with tables and graphs
- Objective 2:** Compute measures of center and spread
- Objective 3:** Choose appropriate summaries for data types

Biostatistics applies statistics to biology and medicine. Data types matter: categorical (blood groups) versus numerical, discrete (counts) versus continuous (heights). Frequency tables, bar charts (categorical), histograms (continuous), and boxplots display distributions; choose the display matching the data type.

Center: mean (balance point, sensitive to outliers), median (middle, robust), mode (most frequent). Spread: range, interquartile range (IQR, robust), variance and standard deviation (average squared distance, then its root). The five-number summary (min, Q_1 , median, Q_3 , max) feeds boxplots; the $1.5 \times \text{IQR}$ rule flags outliers.

In medicine, summaries describe patient cohorts: mean blood pressure with standard deviation, median survival times (skewed data need medians). Very important: match the summary to the shape;

skewed data (income, survival) report median and IQR, symmetric data mean and SD. Examiners test this judgment.

Key Definitions

Standard deviation is defined as the root-mean-square distance of data from the mean.

Interquartile range is defined as $Q3 - Q1$; the spread of the middle 50%, robust to outliers.

Skewness is defined as asymmetry of a distribution; decides mean-vs-median choice.

Key Points to Remember

- Data types: categorical vs numerical, discrete vs continuous; match display to type.
- Center: mean (sensitive), median (robust), mode; spread: range, IQR, SD.
- Five-number summary -> boxplot; $1.5 \times \text{IQR}$ outlier rule.
- Skewed -> median/IQR; symmetric -> mean/SD (core judgment).

Unit Summary

Descriptive biostatistics matches displays and summaries to data types and shapes, with robustness guiding the mean/median choice.

Exam Pointer

Very important: choose mean vs median for skewed data; construct boxplots and interpret them.

Unit 6: Probability Foundations

Learning Objectives

Objective 1: Compute probabilities from sample spaces

Objective 2: Apply addition and multiplication rules

Objective 3: Use conditional probability and Bayes' theorem

Probability quantifies uncertainty: $P(\text{event}) = \text{favorable/possible}$ for equally likely outcomes, axiomatically between 0 and 1 with $P(S) = 1$. The addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (subtract the overlap; mutually exclusive events simply add). Complements simplify: $P(\text{not } A) = 1 - P(A)$.

The multiplication rule: $P(A \text{ and } B) = P(A) \times P(B|A)$; independent events multiply directly. Conditional probability $P(B|A) = P(A \text{ and } B)/P(A)$ updates beliefs with evidence. Medical testing runs on this: sensitivity $P(\text{positive}|\text{disease})$, specificity $P(\text{negative}|\text{healthy})$.

Bayes' theorem inverts conditions: $P(\text{disease}|\text{positive})$ from $P(\text{positive}|\text{disease})$, using prevalence. The base-rate fallacy (ignoring prevalence) makes tests look more informative than they are; a 99%-accurate test for a rare disease still yields mostly false positives. Very important: Bayes numericals on medical testing are the signature application; set up the tree or table carefully.

Key Definitions

Conditional probability is defined as $P(B|A)$: probability of B given that A occurred.

Independence is defined as $P(A \text{ and } B) = P(A)P(B)$; occurrence of one does not affect the other.

Bayes' theorem is defined as inverting conditional probabilities using prior prevalence.

Key Points to Remember

- Addition rule subtracts overlap; complements simplify.
- Multiplication with conditioning; independence multiplies directly.
- Sensitivity/specificity are conditional probabilities.
- Bayes: posterior = likelihood x prior / evidence; beware base rates.

Unit Summary

Probability rules handle unions, intersections and conditioning; Bayes' theorem updates disease probabilities from test results against base rates.

Exam Pointer

Very important: solve medical-testing Bayes numericals; distinguish sensitivity from predictive value.

Unit 7: Probability Distributions

Learning Objectives

- Objective 1:** Work with binomial and Poisson distributions
- Objective 2:** Use the normal distribution and z-scores
- Objective 3:** Apply the central limit theorem

Discrete distributions model counts. The binomial counts successes in n fixed trials with constant p : $P(X=k) = C(n,k) p^k (1-p)^{n-k}$, mean np , variance $np(1-p)$. Requirements: fixed n , independence, constant p , binary outcomes. The Poisson counts rare events in time/space with mean λ , approximating the binomial when n is large and p small.

The normal distribution, bell-shaped and symmetric, is parameterized by mean μ and standard deviation σ ; the empirical rule (68-95-99.7%) gives quick probabilities. Z-scores $z = (x - \mu)/\sigma$ standardize any normal to $N(0,1)$ for table lookup. Biological measurements (heights, blood pressures) are often approximately normal.

The central limit theorem: sample means approach normality as n grows, whatever the population shape, with standard error $\sigma/\sqrt{n}$. This justifies inference from samples. Very important: z-score standardization and the empirical rule are rapid marks; CLT statements must include the $\sigma/\sqrt{n}$ standard error.

Key Definitions

Binomial distribution is defined as counts of successes in n independent trials with constant p .

Normal distribution is defined as the symmetric bell curve parameterized by mean and standard deviation.

Central limit theorem is defined as sample means become approximately normal with standard error $\sigma/\sqrt{n}$.

Key Points to Remember

- Binomial: fixed n , binary, constant p ; mean np . Poisson: rare events, mean λ .
- Normal: empirical 68-95-99.7 rule; $z = (x - \mu)/\sigma$ for table use.
- CLT: means $\rightarrow$ normal; standard error $\sigma/\sqrt{n}$; justifies sampling inference.
- Biological data often approximately normal.

Unit Summary

Binomial and Poisson model counts, the normal models measurements via z-scores, and the central limit theorem legitimizes inference from samples.

Exam Pointer

Very important: standardize with z-scores using the empirical rule; state the CLT with standard error.

Unit 8: Sampling, Estimation and Confidence Intervals

Learning Objectives

- Objective 1:** Distinguish sampling methods
- Objective 2:** Construct confidence intervals for means and proportions
- Objective 3:** Interpret confidence levels correctly

Good inference needs good samples: simple random (every subset equally likely), stratified (random within subgroups), systematic, and cluster sampling. Bias sources: selection bias, nonresponse, measurement bias. Sample size drives precision; representativeness drives validity. Medical trials randomize to kill confounding.

A confidence interval estimates a parameter with stated reliability: for a mean, $\bar{x} \pm z(\sigma/\sqrt{n})$ (large n) or with t for small samples; for a proportion, $\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/n}$. Wider intervals (higher confidence, smaller n , more variability) trade precision for certainty.

Interpretation: 95% confidence means 95% of such intervals capture the true parameter; it is not the probability the parameter lies inside this particular interval. Very important: this correct interpretation is heavily examined because the misconception is so common; also know when t replaces z .

Key Definitions

Confidence interval is defined as a range estimating a parameter with stated long-run capture reliability.

Margin of error is defined as the $\pm$ term; half the interval width.

Sampling bias is defined as systematic distortion from unrepresentative sampling.

Key Points to Remember

- Methods: simple random, stratified, systematic, cluster; randomize to kill confounding.
- CI for mean: $\bar{x} \pm z \cdot \sigma / \sqrt{n}$ (t for small n); for proportion analogous.
- Width drivers: confidence level, n, variability.
- Correct interpretation: long-run capture rate, not probability about this interval.

Unit Summary

Random sampling enables inference; confidence intervals quantify estimate precision, interpreted as long-run capture rates.

Exam Pointer

Very important: construct CIs and interpret confidence correctly; choose z vs t.

Unit 9: Hypothesis Testing

Learning Objectives

- Objective 1:** Set up null and alternative hypotheses
- Objective 2:** Conduct z/t tests and interpret p-values
- Objective 3:** Explain errors, power and chi-square tests

Hypothesis testing asks whether data contradict a null hypothesis H_0 . Steps: state H_0 and H_1 , choose significance α , compute the test statistic (z or t), find the p-value (probability of data this extreme if H_0 true), reject H_0 if $p < \alpha$. The p-value is not $P(H_0 \text{ true})$; it assumes H_0 .

Errors: Type I (rejecting true H_0 , probability α), Type II (missing real effects, probability β); power = $1 - \beta$. Medical testing balances these: a false negative (Type II) may kill, a false positive (Type I) wastes resources. Larger samples raise power.

The chi-square test handles categorical data: goodness-of-fit (does the distribution match?) and independence (are two variables associated?), computed from observed vs expected counts. Very important: the five-step procedure and correct p-value interpretation are the exam core; chi-square suits genetics and survey data.

Key Definitions

P-value is defined as probability of data as extreme as observed, assuming H_0 true.

Type I / Type II error is defined as false positive / false negative relative to H_0 .

Chi-square test is defined as testing categorical fit or independence from observed vs expected counts.

Key Points to Remember

- Five steps: hypotheses, α , statistic, p-value, decision; p assumes H_0 .
- Type I = false alarm (α); Type II = miss (β); power = $1 - \beta$.
- Medical stakes decide which error matters more.
- Chi-square for categorical: goodness-of-fit and independence.

Unit Summary

Hypothesis testing weighs evidence against H_0 via p-values with controlled error rates; chi-square extends testing to categorical biomedical data.

Exam Pointer

Very important: run the five-step test, interpret p-values correctly, and apply chi-square to a contingency table.

Unit 10: Correlation, Regression and Vital Statistics

Learning Objectives

Objective 1: Compute and interpret correlation

Objective 2: Fit and use simple linear regression

Objective 3: Define basic vital and health statistics

Correlation r measures linear association strength and direction (-1 to 1); r^2 is the proportion of variance explained. Correlation is not causation: confounding, reverse causation and coincidence all produce correlations. Scatterplots reveal nonlinearity and outliers that r hides.

Simple linear regression fits $y = a + bx$ by least squares (minimizing squared residuals); $b = r(sy/sx)$ links slope to correlation. Use the line for prediction within the data range (extrapolation is risky); residuals check fit quality. In biostatistics, dose-response relationships are classic regressions.

Vital statistics describe populations: crude birth/death rates, infant mortality rate, maternal mortality ratio, life expectancy, incidence (new cases) versus prevalence (existing cases). Very important: incidence-vs-prevalence and correlation-vs-causation are the two distinctions examiners probe most; state them crisply.

Key Definitions

Correlation coefficient is defined as r in $[-1,1]$ measuring linear association; r^2 = variance explained.

Regression line is defined as the least-squares fit $y = a + bx$ minimizing squared residuals.

Incidence / prevalence is defined as new cases per period / existing cases at a time.

Key Points to Remember

- r : strength+direction of linear association; not causation.
- Regression: least squares, $b = r(sy/sx)$; predict within range; check residuals.
- Vital stats: birth/death rates, IMR, MMR, life expectancy.
- Incidence (new) vs prevalence (existing): crisp distinction.

Unit Summary

Correlation and regression quantify linear relationships without proving causation; vital statistics (incidence, prevalence, mortality rates) describe population health.

Exam Pointer

Very important: interpret r and r^2 ; distinguish incidence from prevalence; warn against causal claims from correlation.

