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HANDOUT

MTH5207

Mechanics I

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MTH5207

Mechanics I

This handout covers the complete course topic by topic in simple, exam-focused language. Each chapter begins with learning objectives and ends with key points and exam pointers. Definitions are boxed in green-friendly format, examples are worked out step by step, and very important exam points are flagged separately. Study the highlighted points carefully before your papers.

Course Information	
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1. Mathematical Preliminaries: Vectors

Learning Objectives

- Perform vector algebra
- Apply dot and cross products

- Vector: magnitude + direction; scalar: magnitude only.
- Addition: triangle/parallelogram law; components: $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$.
- Dot product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$ — tests perpendicularity (zero) and finds projections.
- Cross product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin(\theta) \mathbf{n}$ — gives torques and normals; $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$.

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Example

$\mathbf{a} = (2, 3, 1)$, $\mathbf{b} = (1, -1, 4)$: $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 4 = 3$; the angle between them is $\arccos(3/(\sqrt{14}\sqrt{18}))$.

Key Points to Remember

- Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ keep 3-D work organized.

Exam Pointer

Very important for exams: Find the projection of $\mathbf{a}$ onto $\mathbf{b}$ — a standard starter numerical.

Scalar vs vector examples

- Scalars: mass, temperature, speed.
- Vectors: displacement, force, velocity.
- Speed is scalar; velocity is vector — direction matters.

Unit vectors in 3D

- $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ along x , y , z .
- Position: $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$.
- Dot/cross via components or determinants.

Vector triple products

- $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$: volume of parallelepiped.

- Cyclic permutations preserve value.
- Zero means coplanar vectors.

Dimensional analysis

- Check every equation's units.
- Derive formulas up to constants.
- Catches most algebra slips.

Dot product applications

- Work = $F \cdot d$: projection in action.
- Angle between vectors via cos formula.
- Orthogonality test: dot = 0.
- Component of a along b.
- Practice geometric interpretations.

Cross product applications

- Torque and angular momentum definitions.
- Area of parallelogram from $|a \times b|$.
- Right-hand rule gives direction.
- Anti-commutative: $a \times b = -(b \times a)$.
- Determinant form for computation.

Vector notation

- Bold or arrow for vectors.
- Magnitude always non-negative.
- Unit vectors carry direction only.
- Position vs displacement distinction.
- Consistent notation prevents errors.

2. Kinematics of a Particle

Learning Objectives

- Relate position, velocity and acceleration
- Solve projectile and circular motion

- Position $r(t)$; velocity $v = dr/dt$; acceleration $a = dv/dt$ — differentiate down, integrate up.

- Constant acceleration: $v = u + at$, $s = ut + (1/2)at^2$, $v^2 = u^2 + 2as$.

Projectile motion

- Horizontal: constant velocity; vertical: constant acceleration $-g$.
- Range $R = (u^2 \sin 2\theta)/g$ — maximum at 45 degrees.
- Time of flight $T = (2u \sin \theta)/g$; max height $H = (u^2 \sin^2 \theta)/(2g)$.

Circular motion

- Angular velocity $\omega = d(\theta)/dt$; $v = \omega \times r$.
- Centripetal acceleration $a = v^2/r$ toward the centre.

$$R = u^2 \sin(2\theta) / g$$

Key Points to Remember

- Separate x and y in projectiles — they are independent.
- Centripetal force is a requirement, not a new force.

Exam Pointer

Very important for exams: Projectile range/height numericals are guaranteed.

Relative motion

- Velocity of A relative to B: $v_A - v_B$.
- River-boat problems are relative motion classics.
- Choose the frame that simplifies.

Tangential vs normal acceleration

- Tangential: changes speed (dv/dt).
- Normal: changes direction (v^2/ρ).
- Curved paths need both.

Motion in polar coordinates

- $v_r = dr/dt$; $v_\theta = r d(\theta)/dt$.
- Acceleration has Coriolis-like terms.
- Natural for central forces.

Frames of reference

- Inertial: Newton holds; non-inertial: add pseudo-forces.

- Centrifugal is a pseudo-force.
- Earth is approximately inertial.

Projectile range derivation

- $R = v_0^2 \sin(2\theta)/g$ on level ground.
- Maximum at 45 degrees.
- Complementary angles give equal range.
- Air resistance breaks the symmetry.
- Derive it once, then use freely.

Circular motion formulas

- $a_c = v^2/r$ toward centre.
- Banked curves: $\tan \theta = v^2/rg$.
- Conical pendulum combines both.
- No outward "force" — inertia instead.
- Common misconception to avoid.

Relative velocity problems

- Aircraft in wind: vector triangles.
- Swimmer crossing rivers.
- Choose ground or moving frame.
- Draw triangles tip-to-tail.
- Classic exam geometry.

3. Newton's Laws and Forces

Learning Objectives

- Apply Newton's laws to particle motion
- Handle friction, tension and constraint

- First law: inertia — inertial frames.
- Second law: $F = ma$ (vector equation — resolve into components).
- Third law: action-reaction pairs act on different bodies.

Standard forces

- Weight $W = mg$ downward; normal reaction perpendicular to surfaces.

- Tension: pulls along strings; inextensible strings give equal accelerations.
- Friction: $f \leq \mu_s N$ (static), $f = \mu_k N$ (kinetic), opposes relative motion.
- Spring: $F = -kx$ (Hooke's law).

Example

Block on a 30-degree incline, $\mu = 0.2$: component down the plane $mg \sin 30$, friction up $\mu mg \cos 30$ — net acceleration $g(\sin 30 - \mu \cos 30)$.

Key Points to Remember

- Draw free-body diagrams before writing any equation.
- Friction direction always opposes impending motion.

Exam Pointer

Very important for exams: Incline + friction + pulley combinations are the classic problems.

Free-body diagram rules

- Isolate the body; draw all forces as arrows.
- Label every force; choose axes wisely.
- Never skip this step — it prevents most errors.

Connected particles

- Strings: equal speed along string.
- Pulleys redirect without (ideal) friction.
- Write one equation per mass, link accelerations.

Variable mass systems

- Conveyor belts, rockets, rain-collecting carts.
- $F = m \, dv/dt - u \, dm/dt$.
- Account for incoming momentum.

Impulse-momentum applications

- Airbags extend impact time, cut force.
- Follow-through in sports lengthens contact.
- Design for time, not just strength.

Tension in strings

- Ideal strings: uniform tension, massless.
- Pulleys only redirect tension.
- Atwood machine: classic two-mass problem.
- Tension adjusts to constrain motion.
- Draw FBDs before writing equations.

Normal reaction force

- Perpendicular to contact surface.
- Adjusts to prevent penetration.
- Zero means losing contact.
- Loop-the-loop: $N \geq 0$ condition.
- Weighing scales read N , not mg .

Spring combinations

- Parallel: k adds; series: reciprocals add.
- Equivalent k simplifies systems.
- Car suspensions use combinations.
- Energy splits by stiffness.
- Derive once, apply often.

4. Work, Energy and Power

Learning Objectives

- Apply the work-energy theorem
- Use conservation of mechanical energy

- Work: $W = \mathbf{F} \cdot \mathbf{d}$ (dot product — only the parallel component counts).
- Kinetic energy $T = (1/2)mv^2$; work-energy theorem: net work = change in KE.
- Potential energy: gravitational $V = mgh$; elastic $V = (1/2)kx^2$.
- Conservative forces (gravity, spring): path-independent work, energy conserved; non-conservative (friction): work lost as heat.

$$T_1 + V_1 = T_2 + V_2 \text{ (conservative systems)}$$

$$\text{Power } P = \mathbf{F} \cdot \mathbf{v} = dW/dt$$

Example

A 2 kg block slides from height 5 m: v at bottom = $\sqrt{2gh} = \sqrt{98}$ m/s — mass cancels.

Key Points to Remember

- Energy methods beat force methods when time is not asked.
- Check conservativeness before conserving.

Exam Pointer

Very important for exams: Pendulum/spring energy numericals appear regularly.

Conservative vs non-conservative

- Gravity, spring: path-independent, potential exists.
- Friction, air resistance: dissipate energy.
- Only conservative systems conserve mechanical energy.

Power in machines

- $P = F v$ for constant force.
- Engines rated in horsepower/watts.
- Climbing hills needs power, not just force.

Potential energy curves

- Minima: stable; maxima: unstable equilibrium.
- Turning points where $E = V$.
- Read motion straight off the curve.

Work by variable force

- Integrate $F dx$; area under curve.
- Spring work: $(1/2)k(x_2^2 - x_1^2)$.
- Graphical methods save calculus.

Gravitational potential energy

- Near Earth: mgh ; general: $-GMm/r$.
- Reference point is arbitrary.
- Escape velocity from energy conservation.
- Negative total energy means bound orbits.
- Connect to Kepler's laws.

Conservative force test

- Curl of F equals zero.
- Work around closed loop is zero.
- Potential exists iff conservative.
- Gravity and springs pass; friction fails.
- The mathematical criterion.

Power and efficiency

- Efficiency = useful output / input.
- Always below 100% in practice.
- Machines trade force for speed.
- Engines rated by power output.
- Energy accounting reveals waste.

5. Linear and Angular Momentum

Learning Objectives

- Apply momentum conservation in collisions
- Relate torque to angular momentum

- Linear momentum $p = mv$; impulse $J = \int F dt = \text{change in } p$.
- Conservation: with no external impulse, total p is constant — the key to collision problems.

Collisions

- Elastic: momentum AND kinetic energy conserved.
- Inelastic: momentum conserved, KE lost; perfectly inelastic: bodies stick together.
- Coefficient of restitution e : $1 = \text{elastic}$, $0 = \text{perfectly inelastic}$.

Angular momentum

- $L = r \times p$; torque $\tau = r \times F = dL/dt$.
- Conservation: with no external torque, L is constant — skaters spin faster pulling arms in.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (1\text{-D collisions})$$

Key Points to Remember

- Collisions: always conserve momentum; conserve KE only if elastic.
- Angular momentum conservation explains orbits and spins.

Exam Pointer

Very important for exams: Two-body collision numericals with e given.

Centre of mass

- Weighted average position of mass.
- Translational motion follows $F = M a_{cm}$.
- Symmetry locates it instantly.

Rocket equation intuition

- Momentum conservation with changing mass.
- Exhaust speed decides efficiency.
- Staging sheds dead weight.

Elastic collision formulas

- 1D: exchange velocities if equal masses.
- Derive via momentum + energy.
- Memorize the pattern, not just algebra.

Angular impulse

- Torque $\times$ time = change in L .
- Short hard twists vs long gentle ones.
- Diving twists use it.

Elastic potential energy

- $U = \frac{1}{2}kx^2$ for ideal springs.
- Area under F - x graph.
- Springs in series/parallel combine like capacitors reversed.
- Oscillation frequency from energy methods.
- Real springs deviate at extremes.

Coefficient of restitution

- e = relative speed after/before.
- $e = 1$ elastic; $e = 0$ perfectly inelastic.
- Bouncing ball heights follow e^2 .
- Measured in sports equipment.
- Combines with momentum conservation.

Conservation law summary

- Momentum: no external impulse.
- Energy: no non-conservative work.
- Angular momentum: no external torque.
- Symmetries underlie each (Noether).
- Choose the applicable one first.

6. Central Forces and Orbits

Learning Objectives

- Derive Kepler's laws from Newton's gravitation
- Analyze circular and elliptical orbits

- Newton's gravitation: $F = GmM/r^2$ toward the centre — a central force, so angular momentum is conserved.
- Planar motion; effective potential combines centrifugal and gravitational terms.

Kepler's laws

1. Orbits are ellipses with the Sun at one focus.
2. Equal areas in equal times (follows from angular momentum conservation).
3. T^2 proportional to a^3 (follows from the inverse-square law).

$$T^2 = (4 \pi^2 / GM) a^3$$

Example

Geostationary orbit: $T = 24$ h gives r approx 42,164 km from Earth's centre — the basis of communications satellites over Pakistan.

Key Points to Remember

- Kepler 2 = angular momentum; Kepler 3 = inverse-square law.
- Escape velocity $v = \sqrt{2GM/R}$.

Exam Pointer

Very important for exams: Derive Kepler's third law for circular orbits.

Satellite orbits

- Circular: $v = \sqrt{GM/r}$.
- Geostationary: 24h period over equator.
- Pakistan's PAKSAT sits in GEO.

Effective potential

- Centrifugal barrier + real potential.
- Circular orbits at minima.
- Explains why planets don't fall in.

Precession intuition

- Torque perpendicular to spin causes precession.
- Gyroscopes resist tipping.
- Bicycles and tops exploit it.

Binary stars

- Orbit common centre of mass.
- Period reveals total mass.
- Exoplanets found the same way.

Kepler's three laws

- Elliptical orbits with Sun at focus.
- Equal areas in equal times.
- T^2 proportional to a^3 .
- Newton derived them from gravitation.
- State all three precisely for exams.

Weightlessness

- Orbit: free fall, not zero gravity.
- Scales read zero in free fall.
- Astronauts train in parabolic flights.
- True zero-g only far from masses.
- Explain carefully in exams.

Tidal forces

- Differential gravity across a body.
- Cause ocean tides and moon's lock.
- Roche limit tears satellites apart.

- Weaken with distance cubed.
- Spectacular astrophysics application.

7. Rigid Body Kinematics

Learning Objectives

- Describe rotation of rigid bodies
- Compute moments of inertia

- Rigid body: distances between points fixed; motion = translation of centre of mass + rotation about it.
- Rotation: angular velocity ω , angular acceleration α ; $v = \omega \times r$ for any point.

Moment of inertia

- $I = \int r^2 dm$ — rotational mass; depends on axis.
- Standard results: rod about centre $(1/12)ML^2$; disc about centre $(1/2)MR^2$; hoop MR^2 ; solid sphere $(2/5)MR^2$.
- Parallel axis theorem: $I = I_{cm} + Md^2$.

$$I = I_{cm} + M d^2 \text{ (parallel axis)}$$

Key Points to Remember

- Moment of inertia depends on mass distribution AND axis choice.
- Parallel axis theorem saves many integrals.

Exam Pointer

Very important for exams: Compute I for composite bodies using the parallel axis theorem.

Radius of gyration

- $k: I = M k^2$ — mass concentrated at k .
- Compares mass distributions simply.
- Hoop: $k = R$; disc: $k = R/\sqrt{2}$.

Perpendicular axis theorem

- Planar bodies: $I_z = I_x + I_y$.
- Disc from rod results quickly.
- Only for thin plates.

Compound pendulum

- Any rigid body swinging on a pivot.
- $T = 2\pi \sqrt{I/mgd}$.
- Centre of oscillation interchangeable with pivot.

Flywheels

- Store rotational energy: $(1/2)I \omega^2$.
- Buses and grid stabilize with them.
- High ω beats high mass.

Rolling without slipping

- $v_{cm} = \omega R$ condition.
- Instantaneous contact point at rest.
- Energy splits: $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- Friction does no work (static).
- Race: solid sphere beats hollow.

Moment of inertia tricks

- Parallel axis: $I = I_{cm} + Md^2$.
- Additivity for composite bodies.
- Integration for continuous mass.
- Symmetry halves the work.
- Memorize rod, hoop, disc, sphere.

Gyroscopic precession

- Torque perpendicular to spin.
- Precession rate = τ/L .
- Tops, bikes, Earth's axis.
- Stabilizes ships and satellites.
- Counterintuitive but derivable.

8. Rigid Body Dynamics

Learning Objectives

- Apply torque and rotation equations
- Solve rolling-without-slipping problems

- Torque $\tau = I \alpha$ (rotational $F = ma$).
- Rotational KE = $(1/2) I \omega^2$; work-energy includes both translational and rotational terms.
- Rolling without slipping: $v_{cm} = \omega R$; static friction does no work.

Example

Disc vs hoop rolling down an incline: the disc wins — smaller I/MR^2 means more energy goes to translation. Acceleration $a = g \sin(\theta)/(1 + I/MR^2)$.

Key Points to Remember

- Rolling = translation + rotation combined.
- Energy methods handle rolling elegantly.

Exam Pointer

Very important for exams: Race-the-bodies-down-the-incline numericals.

Toppling vs slipping

- Tall narrow bodies topple; wide ones slip.
- Compare required μ for each.
- Real problems need both checks.

Instantaneous centre

- Point with zero velocity at an instant.
- Velocities radiate from it.
- Speeds up rolling problems.

Static determinacy

- Equations $\geq$ unknowns: solvable.
- More unknowns: indeterminate — needs deformation.
- Trusses: $m + r = 2j$ for internal determinacy.

Friction angles

- Angle of friction: $\tan^{-1}(\mu)$.
- Angle of repose for granular piles.
- Ladders analyzed via both.

Torque and levers

- $\tau = r \times F$; magnitude $rF \sin \theta$.
- Levers trade force for distance.
- Seesaws balance torques.
- Wrenches maximize lever arm.
- Static equilibrium needs $\Sigma \tau = 0$.

Static friction limits

- $0 \leq f_s \leq \mu_s N$ — an inequality.
- It matches the need up to the max.
- Impending motion analysis uses the max.
- Direction opposes relative motion tendency.
- Students often misuse the equality.

Centre of mass motion

- Moves as if all mass concentrated.
- Internal forces cancel out.
- Exploding projectile: CM follows parabola.
- Frame choice simplifies collisions.
- The most underused shortcut.

9. Statics: Equilibrium of Bodies

Learning Objectives

- Apply equilibrium conditions
- Analyze frames, trusses and friction

- Equilibrium: resultant force = 0 AND resultant moment = 0 (about any point).
- Two-force members, three-force members (concurrent) simplify analysis.
- Trusses: method of joints (equilibrium at each pin) and method of sections (cut through members).
- Ladders, beams and wedges combine force and moment balance with friction.

Example

Ladder against a smooth wall: horizontal equilibrium gives friction at the base; moment about the base gives the normal from the wall.

Key Points to Remember

- Always use both force and moment equations.
- Choose the moment centre to eliminate unknowns.

Exam Pointer

Very important for exams: Ladder problems are the signature statics question.

Truss analysis tips

- Zero-force members simplify fast.
- Sections beat joints for single members.
- Assume tension; negative means compression.

Method of joints example

- Start at joints with ≤ 2 unknowns.
- Equilibrium: $\sum F_x = 0$, $\sum F_y = 0$.
- Work systematically across the truss.

Simple vs physical pendulum

- Simple: point mass on string.
- Physical: extended body.
- Simple is the small-length limit.

Virtual work teaser

- Equilibrium: zero virtual work.
- Elegant for complex linkages.
- Beyond this course, but beautiful.

Friction on inclines

- Resolve weight into parallel/perpendicular.
- Static holds until $\mu_s N$ exceeded.
- Kinetic is usually smaller.

- Angle of repose = $\tan^{-1} \mu_s$.
- Classic exam staple — master it.

Simple pendulum derivation

- Small-angle: $\sin \theta \approx \theta$.
- $T = 2\pi\sqrt{L/g}$ independent of mass.
- Amplitude independence needs small angles.
- Used to measure g historically.
- State assumptions explicitly.

Ladder problems

- Friction at base, normal at wall.
- Torque about base eliminates unknowns.
- Person climbing shifts the balance.
- Minimum angle before slipping.
- Template for statics exams.

10. Simple Harmonic Motion

Learning Objectives

- Solve the SHM equation
- Analyze pendulum and spring systems

- SHM: acceleration proportional to displacement, toward equilibrium: $d^2x/dt^2 = -\omega^2 x$.
- Solution: $x = A \cos(\omega t + \phi)$; period $T = 2\pi/\omega$.
- Mass-spring: $\omega = \sqrt{k/m}$; simple pendulum: $\omega = \sqrt{g/l}$ for small angles.

$$T = 2\pi \sqrt{m/k} \quad T = 2\pi \sqrt{l/g}$$

- Energy oscillates between kinetic and potential; total $E = (1/2)kA^2$.
- Damped SHM: amplitude decays; driven SHM: resonance when driven at natural frequency.

Key Points to Remember

- Small-angle approximation makes the pendulum SHM.
- Resonance: small pushes at the right frequency build huge amplitude.

Exam Pointer

Very important for exams: Derive the pendulum period and state the small-angle assumption.

Damped oscillations

- Light: oscillates decaying; critical: fastest return.
- Heavy: slow creep back.
- Car suspensions target near-critical.

Resonance dangers

- Tacoma Narrows: wind matched bridge frequency.
- Soldiers break step on bridges.
- Damping is the engineer's answer.

Coupled oscillators teaser

- Two pendulums exchange energy.
- Normal modes: in-phase and anti-phase.
- Beats reveal the coupling.

Exam revision checklist

- Projectile formulas, FBD method, energy vs momentum choice.
- Collision types, Kepler laws, I theorems.
- SHM derivations with assumptions stated.

Energy in SHM

- Total $E = \frac{1}{2}kA^2$ constant.
- KE max at equilibrium, PE max at extremes.
- Exchange twice per cycle.
- Damping drains total energy.
- Phase plots show the exchange.

Problem-solving strategy

- Read, sketch, choose method (force/energy/momentum).
- Energy for speeds; momentum for collisions.
- Check units and limiting cases.
- Dimensional analysis catches errors.
- Practice beats formula sheets.

Damped vs driven summary

- Damping: amplitude decays.
- Driving: steady state at drive frequency.
- Resonance: maximum response.
- Q factor measures sharpness.
- One framework, many systems.

Practice Questions for Exams

Learning Objectives

- Test your preparation with these exam-style questions

Short questions with model answers

Example

Q: Explain: perform vector algebra Model answer: Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area. Unit vectors i, j, k keep 3-D work organized.

Example

Q: Explain: apply dot and cross products Model answer: Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area. Unit vectors i, j, k keep 3-D work organized.

Example

Q: Explain: relate position, velocity and acceleration Model answer: Separate x and y in projectiles — they are independent. Centripetal force is a requirement, not a new force.

Example

Q: Explain: solve projectile and circular motion Model answer: Separate x and y in projectiles — they are independent. Centripetal force is a requirement, not a new force.

Example

Q: Explain: apply Newton's laws to particle motion Model answer: Draw free-body diagrams before writing any equation. Friction direction always opposes impending motion.

Example

Q: Explain: handle friction, tension and constraint Model answer: Draw free-body diagrams before writing any equation. Friction direction always opposes impending motion.

Example

Q: Explain: apply the work-energy theorem Model answer: Energy methods beat force methods when time is not asked. Check conservativeness before conserving.

Example

Q: Explain: use conservation of mechanical energy Model answer: Energy methods beat force methods when time is not asked. Check conservativeness before conserving.

Example

Q: Explain: apply momentum conservation in collisions Model answer: Collisions: always conserve momentum; conserve KE only if elastic. Angular momentum conservation explains orbits and spins.

Example

Q: Explain: relate torque to angular momentum Model answer: Collisions: always conserve momentum; conserve KE only if elastic. Angular momentum conservation explains orbits and spins.

Example

Q: Explain: derive Kepler's laws from Newton's gravitation Model answer: Kepler 2 = angular momentum; Kepler 3 = inverse-square law. Escape velocity $v = \sqrt{2GM/R}$.

Example

Q: Explain: analyze circular and elliptical orbits Model answer: Kepler 2 = angular momentum; Kepler 3 = inverse-square law. Escape velocity $v = \sqrt{2GM/R}$.

Example

Q: Explain: describe rotation of rigid bodies Model answer: Moment of inertia depends on mass distribution AND axis choice. Parallel axis theorem saves many integrals.

Example

Q: Explain: compute moments of inertia Model answer: Moment of inertia depends on mass distribution AND axis choice. Parallel axis theorem saves many integrals.

Example

Q: Explain: apply torque and rotation equations Model answer: Rolling = translation + rotation combined. Energy methods handle rolling elegantly.

Example

Q: Explain: solve rolling-without-slipping problems Model answer: Rolling = translation + rotation combined. Energy methods handle rolling elegantly.

Example

Q: Explain: apply equilibrium conditions Model answer: Always use both force and moment equations. Choose the moment centre to eliminate unknowns.

Example

Q: Explain: analyze frames, trusses and friction Model answer: Always use both force and moment equations. Choose the moment centre to eliminate unknowns.

Example

Q: Explain: solve the SHM equation Model answer: Small-angle approximation makes the pendulum SHM. Resonance: small pushes at the right frequency build huge amplitude.

Example

Q: Explain: analyze pendulum and spring systems Model answer: Small-angle approximation makes the pendulum SHM. Resonance: small pushes at the right frequency build huge amplitude.

Long questions

- Write a detailed note on Mathematical Preliminaries: Vectors.
- Write a detailed note on Kinematics of a Particle.
- Write a detailed note on Newton's Laws and Forces.
- Write a detailed note on Work, Energy and Power.
- Write a detailed note on Linear and Angular Momentum.
- Write a detailed note on Central Forces and Orbits.
- Write a detailed note on Rigid Body Kinematics.
- Write a detailed note on Rigid Body Dynamics.
- Write a detailed note on Statics: Equilibrium of Bodies.
- Write a detailed note on Simple Harmonic Motion.

Long questions with model answer outlines

Example

Q: Write a detailed note on Mathematical Preliminaries: Vectors. Answer outline: • Vector: magnitude + direction; scalar: magnitude only. • Addition: triangle/parallelogram law; components: $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$. • Dot product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$ — tests perpendicularity (zero) and finds projections. • Cross product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin(\theta) \mathbf{n}$ — gives torques and normals; $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$. • Scalars: mass, temperature, speed.

Example

Q: Write a detailed note on Kinematics of a Particle. Answer outline: • Position $\mathbf{r}(t)$; velocity $\mathbf{v} = d\mathbf{r}/dt$; acceleration $\mathbf{a} = d\mathbf{v}/dt$ — differentiate down, integrate up. • Constant acceleration: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$, $v^2 = u^2 + 2\mathbf{a} \cdot \mathbf{s}$. • Horizontal: constant velocity; vertical: constant acceleration $-g$. • Range $R = (u^2 \sin 2\theta)/g$ — maximum at 45 degrees. • Time of flight $T = (2u \sin \theta)/g$; max height $H = (u^2 \sin^2 \theta)/(2g)$.

Example

Q: Write a detailed note on Newton's Laws and Forces. Answer outline: • First law: inertia — inertial frames. • Second law: $\mathbf{F} = m\mathbf{a}$ (vector equation — resolve into components). • Third law: action-reaction pairs act on different bodies. • Weight $\mathbf{W} = m\mathbf{g}$ downward; normal reaction perpendicular to surfaces. • Tension: pulls along strings; inextensible strings give equal accelerations.

Example

Q: Write a detailed note on Work, Energy and Power. Answer outline: • Work: $W = \mathbf{F} \cdot \mathbf{d}$ (dot product — only the parallel component counts). • Kinetic energy $T = \frac{1}{2}mv^2$; work-energy theorem: net work = change in KE. • Potential energy: gravitational $V = mgh$; elastic $V = \frac{1}{2}kx^2$. • Conservative forces (gravity, spring): path-independent work, energy conserved; non-conservative (friction): work lost as heat. • Gravity, spring: path-independent, potential exists.

Example

Q: Write a detailed note on Linear and Angular Momentum. Answer outline: • Linear momentum $\mathbf{p} = m\mathbf{v}$; impulse $\mathbf{J} = \int \mathbf{F} dt = \text{change in } \mathbf{p}$. • Conservation: with no external impulse, total $\mathbf{p}$ is constant — the key to collision problems. • Elastic: momentum AND kinetic energy conserved. • Inelastic: momentum conserved, KE lost; perfectly inelastic: bodies stick together. • Coefficient of restitution e : $1 = \text{elastic}$, $0 = \text{perfectly inelastic}$.

Example

Q: Write a detailed note on Central Forces and Orbits. Answer outline: • Newton's gravitation: $F = GmM/r^2$ toward the centre — a central force, so angular momentum is conserved. • Planar motion; effective potential combines centrifugal and gravitational terms. • 1. Orbits are ellipses with the Sun at one focus. • 2. Equal areas in equal times (follows from angular momentum conservation). • 3. T^2 proportional to a^3 (follows from the inverse-square law).

Example

Q: Write a detailed note on Rigid Body Kinematics. Answer outline: • Rigid body: distances between points fixed; motion = translation of centre of mass + rotation about it. • Rotation: angular velocity ω , angular acceleration α ; $v = \omega \times r$ for any point. • $I = \int r^2 dm$ — rotational mass; depends on axis. • Standard results: rod about centre $(1/12)ML^2$; disc about centre $(1/2)MR^2$; hoop MR^2 ; solid sphere $(2/5)MR^2$. • Parallel axis theorem: $I = I_{cm} + Md^2$.

Example

Q: Write a detailed note on Rigid Body Dynamics. Answer outline: • Torque $\tau = I \alpha$ (rotational $F = ma$). • Rotational KE = $(1/2) I \omega^2$; work-energy includes both translational and rotational terms. • Rolling without slipping: $v_{cm} = \omega R$; static friction does no work. • Tall narrow bodies topple; wide ones slip. • Compare required μ for each.

Example

Q: Write a detailed note on Statics: Equilibrium of Bodies. Answer outline: • Equilibrium: resultant force = 0 AND resultant moment = 0 (about any point). • Two-force members, three-force members (concurrent) simplify analysis. • Trusses: method of joints (equilibrium at each pin) and method of sections (cut through members). • Ladders, beams and wedges combine force and moment balance with friction. • Zero-force members simplify fast.

Example

Q: Write a detailed note on Simple Harmonic Motion. Answer outline: • SHM: acceleration proportional to displacement, toward equilibrium: $d^2x/dt^2 = -\omega^2 x$. • Solution: $x = A \cos(\omega t + \phi)$; period $T = 2\pi/\omega$. • Mass-spring: $\omega = \sqrt{k/m}$; simple pendulum: $\omega = \sqrt{g/l}$ for small angles. • Energy oscillates between kinetic and potential; total $E = (1/2)kA^2$. • Damped SHM: amplitude decays; driven SHM: resonance when driven at natural frequency.

Exam Pointer

Very important for exams: Practice writing answers within the exam time limit.

MCQ Practice Test

Learning Objectives

- Test your recall of the key points from every chapter

Q1. Which of the following was listed as a key point of 'Mathematical Preliminaries: Vectors' in this handout?

- A. Centripetal force is a requirement, not a new force.
- B. Separate x and y in projectiles — they are independent.
- C. Draw free-body diagrams before writing any equation.
- D. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.

Q2. Which of the following was listed as a key point of 'Mathematical Preliminaries: Vectors' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Centripetal force is a requirement, not a new force.
- C. Draw free-body diagrams before writing any equation.
- D. Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ keep 3-D work organized.

Q3. Which of the following was listed as a key point of 'Kinematics of a Particle' in this handout?

- A. Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ keep 3-D work organized.
- B. Separate x and y in projectiles — they are independent.
- C. Draw free-body diagrams before writing any equation.
- D. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.

Q4. Which of the following was listed as a key point of 'Kinematics of a Particle' in this handout?

- A. Draw free-body diagrams before writing any equation.
- B. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- C. Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ keep 3-D work organized.
- D. Centripetal force is a requirement, not a new force.

Q5. Which of the following was listed as a key point of 'Newton's Laws and Forces' in this handout?

- A. Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ keep 3-D work organized.
- B. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- C. Separate x and y in projectiles — they are independent.
- D. Draw free-body diagrams before writing any equation.

Q6. Which of the following was listed as a key point of 'Newton's Laws and Forces' in this handout?

- A. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- B. Unit vectors i, j, k keep 3-D work organized.
- C. Separate x and y in projectiles — they are independent.
- D. Friction direction always opposes impending motion.

Q7. Which of the following was listed as a key point of 'Work, Energy and Power' in this handout?

- A. Unit vectors i, j, k keep 3-D work organized.
- B. Separate x and y in projectiles — they are independent.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Energy methods beat force methods when time is not asked.

Q8. Which of the following was listed as a key point of 'Work, Energy and Power' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Check conservativeness before conserving.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Unit vectors i, j, k keep 3-D work organized.

Q9. Which of the following was listed as a key point of 'Linear and Angular Momentum' in this handout?

- A. Unit vectors i, j, k keep 3-D work organized.
- B. Collisions: always conserve momentum; conserve KE only if elastic.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Separate x and y in projectiles — they are independent.

Q10. Which of the following was listed as a key point of 'Linear and Angular Momentum' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Unit vectors i, j, k keep 3-D work organized.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Angular momentum conservation explains orbits and spins.

Q11. Which of the following was listed as a key point of 'Central Forces and Orbits' in this handout?

- A. Kepler 2 = angular momentum; Kepler 3 = inverse-square law.
- B. Separate x and y in projectiles — they are independent.
- C. Unit vectors i, j, k keep 3-D work organized.
- D. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.

Q12. Which of the following was listed as a key point of 'Central Forces and Orbits' in this handout?

- A. Escape velocity $v = \sqrt{2GM/R}$.
- B. Unit vectors i, j, k keep 3-D work organized.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Separate x and y in projectiles — they are independent.

Q13. Which of the following was listed as a key point of 'Rigid Body Kinematics' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- C. Unit vectors i, j, k keep 3-D work organized.
- D. Moment of inertia depends on mass distribution AND axis choice.

Q14. Which of the following was listed as a key point of 'Rigid Body Kinematics' in this handout?

- A. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- B. Parallel axis theorem saves many integrals.
- C. Separate x and y in projectiles — they are independent.
- D. Unit vectors i, j, k keep 3-D work organized.

Q15. Which of the following was listed as a key point of 'Rigid Body Dynamics' in this handout?

- A. Unit vectors i, j, k keep 3-D work organized.
- B. Separate x and y in projectiles — they are independent.
- C. Rolling = translation + rotation combined.
- D. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.

Q16. Which of the following was listed as a key point of 'Rigid Body Dynamics' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Energy methods handle rolling elegantly.
- C. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- D. Unit vectors i, j, k keep 3-D work organized.

Q17. Which of the following was listed as a key point of 'Statics: Equilibrium of Bodies' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Dot product $\rightarrow$ angle/work; cross product $\rightarrow$ torque/area.
- C. Unit vectors i, j, k keep 3-D work organized.
- D. Always use both force and moment equations.

Q18. Which of the following was listed as a key point of 'Statics: Equilibrium of Bodies' in this handout?

- A. Dot product -> angle/work; cross product -> torque/area.
- B. Unit vectors i, j, k keep 3-D work organized.
- C. Choose the moment centre to eliminate unknowns.
- D. Separate x and y in projectiles — they are independent.

Q19. Which of the following was listed as a key point of 'Simple Harmonic Motion' in this handout?

- A. Dot product -> angle/work; cross product -> torque/area.
- B. Separate x and y in projectiles — they are independent.
- C. Small-angle approximation makes the pendulum SHM.
- D. Unit vectors i, j, k keep 3-D work organized.

Q20. Which of the following was listed as a key point of 'Simple Harmonic Motion' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Unit vectors i, j, k keep 3-D work organized.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Resonance: small pushes at the right frequency build huge amplitude.

Answer Key

1. D 2. D 3. B 4. D 5. D 6. D 7. D 8. B 9. B 10. D 11. A 12. A 13. D 14. B 15. C 16. B 17. D 18. C 19. C 20. D

Exam Pointer

Very important for exams: Attempt all MCQs without looking back, then check the answer key.

Exam Strategy Guide

Learning Objectives

- Attempt papers with a winning strategy

Before the exam

- Revise the Key Points boxes and glossary first — they carry the highest marks per minute.
- Practice one past paper under timed conditions; examiners repeat question styles.

- Sleep well the night before — an exhausted brain recalls far less.

Attempting the paper

- Read the whole paper first; start with your strongest question to build momentum.
- Allocate time by marks: a 10-mark question deserves roughly double a 5-mark one.
- Structure long answers: definition, explanation, example/diagram, conclusion.

Presentation matters

- Use headings, underline key terms, and draw labelled diagrams where possible.
- Attempt all questions — partial credit beats a blank page.
- Review your paper in the last 10 minutes for missed parts.

Exam Pointer

Very important for exams: Very important for exams: neat, structured answers consistently score higher than longer messy ones.