

Circuit Theory (PHY 301)

Lecture Handouts

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LECTURE 1

(Introduction-some basic concepts)

International System of Units

It is built upon seven basic units. These are meter, kilogram, second, ampere, Kelvin, mole and candela.

Basic Units
The seven quantities are known to be basic in SI unit System.

Base quantity	Name	symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Meter
Up until 1983, the meter was defined as 1,650,763.73 wavelengths in a vacuum of the orange-red line of the spectrum of krypton-86. Since then, it is equal to the distance traveled by light in vacuum in 1/299,792,45 of a second.

Second
The second is defined as the duration of 9,192,631,770 cycles of the radiation associated with a specified transition of the Cesium-133 atom.

Kilogram
The standard for the kilogram is a cylinder of platinum-iridium alloy kept by International Bureau of Weights and Measures at Paris.
The kilogram is the only base unit still defined by a physical object.

Kelvin
The Kelvin is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water, that is, the point at which water forms an interface of the solid, liquid and vapor.
This is defined as .01oC on the Centigrade scale and 32.02oF on Fahrenheit scale. The temperature 0oK is called the “absolute zero”.

Ampere

The Ampere is defined as that current, if maintained in each of two long parallel wires separated by a distance of one meter, would produce a force, between the two wires, of 2×10^{-7} Newton, for each meter of length.

Candela

The Candela is defined as the luminous intensity of $\frac{1}{600,000}$ of a square meter of a cavity at a temperature of freezing platinum (2,042 K).

Mole

The mole is defined as the amount of substance of a system that contains as many elementary entities as there are as many atoms in 0.012 kilogram of the Carbon-12.

Joule

Joule is defined as the energy consumed in moving an object of one kg through a distance of one meter. One joule is equivalent to the 0.7376 foot-pound-force and .2388 calories.

Watt

Watt is defined as the “Rate of doing work.”
One watt =1 joule/second
One watt is equivalent to 0.7376 ft-lbf/s or equivalently 1/745.7 horsepower.
Decimal System
The SI system uses the decimal system to relate larger and smaller units to basic units. It employs prefixes to signify the various power of 10.

Decimal System (Prefixes)

Factor	Name	Symbol	Factor	Name	Symbol
10 ⁻²⁴	yocto	y	10 ²⁴	yotta	Y
10 ⁻²¹	zepto	z	10 ²¹	zetta	Z
10 ⁻¹⁸	atto	a	10 ¹⁸	exa	E
10 ⁻¹⁵	femto	f	10 ¹⁵	peta	P
10 ⁻¹²	pico	p	10 ¹²	tera	T
10 ⁻⁹	nano	n	10 ⁹	giga	G
10 ⁻⁶	micro	μ	10 ⁶	mega	M
10 ⁻³	milli	m	10 ³	kilo	k
10 ⁻²	centi	c	10 ²	hecto	h
10 ⁻¹	deci	d	10 ¹	Deka	da

Example 1

A laser emits light at a wavelength of 248nm. This is same as

- (a) .0248 millimeter.
- Or
- (b) 2.48 micrometer.
- Or
- (c) 0.248 micrometer.
- Or
- (d) 24800 angstrom.

Answer

- (c) 0.248 micrometer.

Example 2

A logic gate switches from the on state to the off state in 10 neno seconds, this corresponds to

- (a)0.1 micro second
- or
- (b)10 micro second
- or
- (c) .001 micro second
- or
- (d) .01 micro second

Answer

- (d) .01 micro second

Sub-atomic elements

Electron: It is a negatively charged particle.

Proton: It is a positively charged particle.

Neutron: It is a neutral particle and carries no charge.

Electrons and protons in an Atom

An atom is the smallest particle of the basic elements which form the physical substance we know as solid, liquid and gas.

Each stable combination of electrons and protons make one particular type of atom.

To understand the concepts of electronics we must have the understanding what is happening at the atomic level, not why it happens.

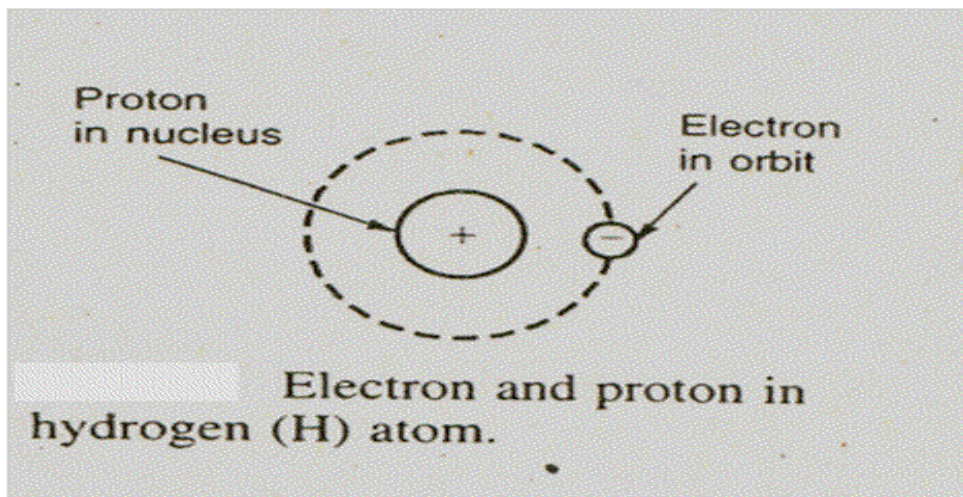
There are number of methods by which electrons and protons might be grouped.

They assemble in specific atomic combination for a stable arrangement.

As a result, the electron stays in its orbit around the nucleus.

In an atom that have more electrons and protons than hydrogen atom, all the protons are in nucleus, while all the electrons are in one or more ings around the nucleus.

The proton in the nucleus makes it heavier and stable part of the atom because it Is 1840 times heavier than the electron.



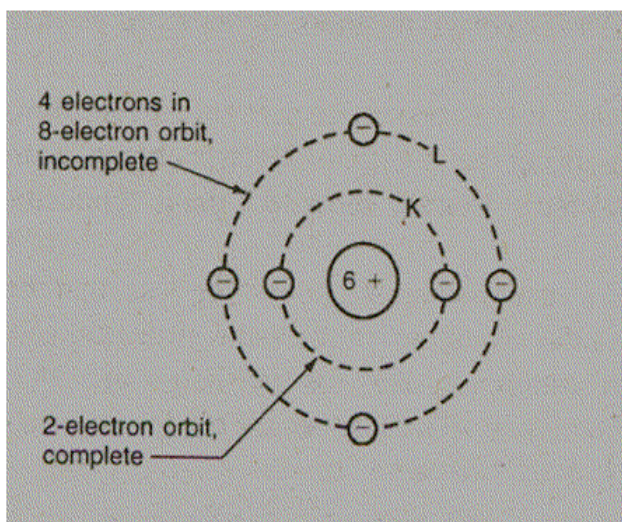
One electron is shown as the orbital ring around the nucleus. In order to account for the atom's stability we can consider electron spinning around the nucleus as planets revolve around the sun.

The electrical force attracting the electron towards proton is balanced by the mechanical force (centrifugal force) directing it outwards.

The total number of electrons in the outer rings must equal to the number of protons in the nucleus in a neutral atom.

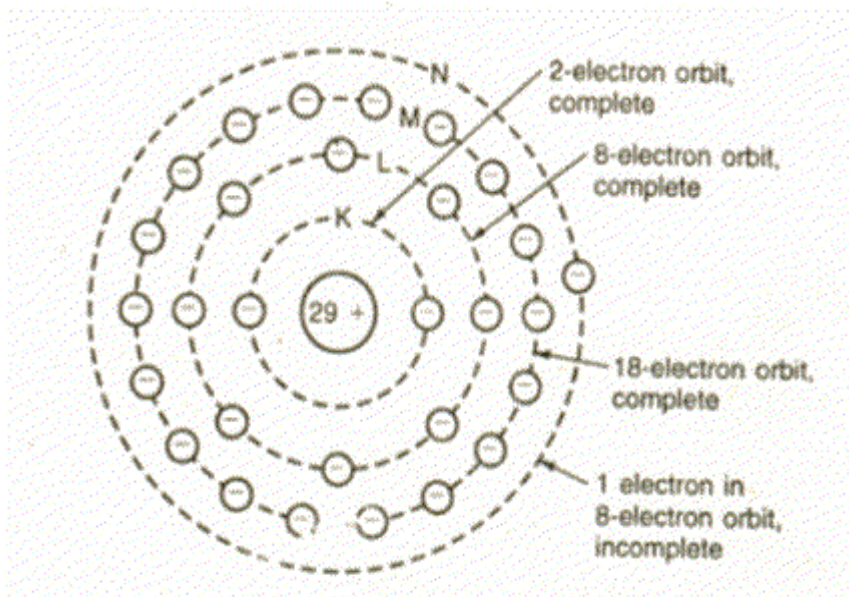
The distribution of electrons in the orbital ring determines the atom's electrical stability. Especially important are the number of electrons farthest from the nucleus.

For example carbon atom illustrated in the figure, 6 protons in the nucleus and 6 electrons in two outside rings.



This outermost ring requires 8 electrons stability, except, when there is only one ring which requires only 2 electrons for its stability.

As another example, the copper atom in figure below has only one electron in the last ring which can include 8 electrons. Therefore the outside ring of copper is less stable than carbon.



When there are many atoms closed in a copper wire, the outermost electrons are not sure from which atom they belong to. They can migrate easily from one atom to another at random. These electrons are called “free electrons”.

Structure of the Atom

Although no body has ever seen an atom, its hypothetical structure fits experimental evidence that has been measured very exactly. The size and electrical charge of the invisible particles are indicated by how much they are affected by the known forces. Our present planetary model of the atom is proposed by Neil Bohar in 1913. His contribution was joined with the new ideas of nuclear atom developed by Lord Ruther Ford. With the quantum theory developed by Max Plank and Albert Einstein.

The nucleus contains protons for all the positive charge in the atom.

The number of protons in the nucleus is equal to the number of planetary electrons.

The positive and negative charges are as the electrons and protons have the equal and opposite charges.

The orbits for the planetary electrons are also called shells or energy levels.

Electron Valence

This value is the number of electrons in an incomplete outermost shell.

A completed outer shell has a valence of zero. Copper for instance, has a Valence of 1, as there is one electron in the outermost shell of the copper Atom.

Sub-shells

Although not shown in the illustrations, all the shells except K shell are divided into sub-shells. This sub-division accounts for the different types of the orbits in the same shell. For instance, electrons in the one sub-shell will have circular orbits while other electrons of the same shell will have elliptical orbit. This sub-division accounts for the magnetic properties of the atom.

Particles in the Nucleus

A stable nucleus, it is not radioactive, but contains protons and neutrons.The neutron is electrically neutral particle, without any charge.Its mass is almost same as that of proton.The proton has the positive charge of the hydrogen nucleus.Table lists the charge and mass for these three basic particles in all atoms.

STABLE PARTICLES IN THE ATOM		
Particle	Charge	Mass
Electrons in orbital shells	0.16x10-18C, negative	9.108x10-28 g
Proton in nucleus	0.16x10-18C, positive	1.672x10-24 g
Neutron, in nucleus	None	1.675x10-24 g

Atomic Number

This gives the number of protons or electrons required in the atom of an element. Hydrogen atom has atomic number 1.

Orbital Rings

The planetary electrons in successive shells are called K, L, M, N, O, P and Q at increasing distance outward from the nucleus.
Each shell has a maximum number of electrons for stability.
As indicated in Table these stable shells correspond to the inert gases as helium and neon etc.

SHELLS OR ORBITAL ELECTRONS IN THE ATOM		
SHELL	MAXIMUM ELECTRONS	INERT GAS
K	2	Helium
L	8	Neon

For example electronic configuration of the copper atom having 29 protons in nucleus and 29 orbital electrons will be

K shell = 2 electrons
L shell = 8 electrons
M shell = 18 electrons
N shell = 1 electron
Total = 29 electrons

Example 1

An element with 16 protons and 16 electrons has atomic number value

- (a) 14
Or
(b) 18
Or
(c) 16
Or
(d) 22

Answer

(c) 16

Example 2

What is the electron valence of an element of atomic number 5

- (a) 2
Or
(b) 3
Or
(c) 4
Or
(d) 5

Answer

(b) 3

Conductors

When electrons can move easily from one atom to another in a material, it is a conductor. In general all the metals are good conductors, with silver the best and copper at second.

Insulators

A material with atoms in which the electrons tend to stay in their own orbits is an insulator and it cannot conduct electricity easily. However, the insulators are able to hold or store electricity better than the conductors. These are also called dielectric materials.

Semiconductors

Carbon can be considered as semiconductor, conducting less than the metal conductors but more than the insulators. In the same group are germanium and silicon which are commonly used for transistor and other semiconductor components.

Coulomb: The Unit of Electric Charge

The mechanical force of attraction and repulsion between the charges is the fundamental method by which electricity makes itself evident. Any charge is an example of static electricity because the electrons and protons are not in motion.

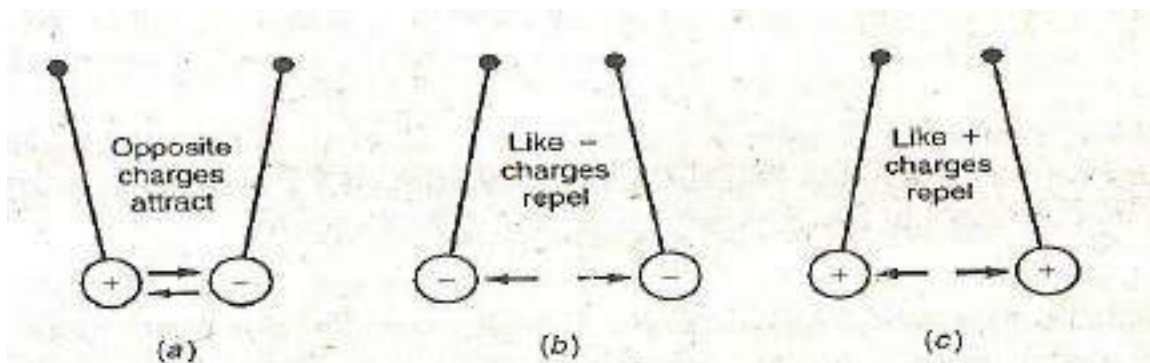
The charge of many billions of electrons or protons is necessary for common applications of electricity. Therefore, it is convenient to define a practical unit called coulomb(C) as equal to charge of 6.25×10^{18} electrons or protons stored in a dielectric. The analysis of static charges and their forces is called electrostatics. The symbol of electric charge is Q or q, standing for quantity. This unit is named after Charles A. Coulomb, a French physicist, who measured force between the charges.

Negative and Positive Polarities

Historically, the negative polarity refers to as static charge produced on rubber, amber, silk and resinous material in general. Positive polarity refers to the static charge produced on glass and other various materials.

On the basis of this, the electrons in all atoms are basic particles of negative charge because their polarity is same as the charge on rubber. Protons have positive charge because the polarity is same as the charge on glass.

Charges of the Same Polarity Repel



Exercises

- (1) What will be the atomic number of an atom having 29 electrons?
- (2) Charge of 1 coulomb and charge of -2 coulomb will attract or repel?
- (3) What will be the electronic distribution of an atom of atomic number 23?

The Atom

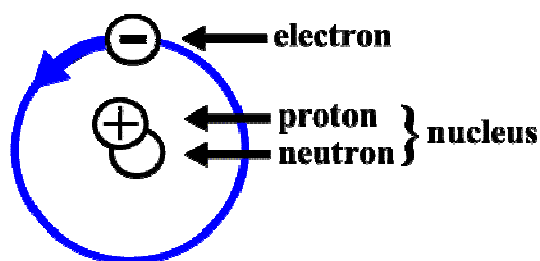
Atoms, the fundamental building blocks of matter, are made of three kinds of particles:

electrons: negatively charged: electrons orbit the nucleus

protons: positively charged: part of the nucleus

neutrons: neutrally charged: part of the nucleus

The Atom



The figure above shows a particularly simple atom (an "isotope" of hydrogen, called deuterium, which just happens to have one particle of each type). All atoms except hydrogen atoms have more than one electron, proton, and usually one or more neutrons. Electron's orbit around the nucleus, which lies at the center and contains the protons and neutrons.

The electron stays in orbit because it is negatively charged, and is therefore attracted to the positively charged proton in the nucleus. This is an example of the Law of electrical charges:

Like charges repel, opposite charges attract.

Most atoms, when in their normal state as components of matter, have the same number of electrons as protons, so that the total, or net charge of the atom is zero. That is, the atom is neutral. The number of protons (which is equal to the number of electrons if the atom is neutral) is called the atomic number of the atom, and this determines what element (hydrogen, helium, iron, etc), that the atom is classified as.

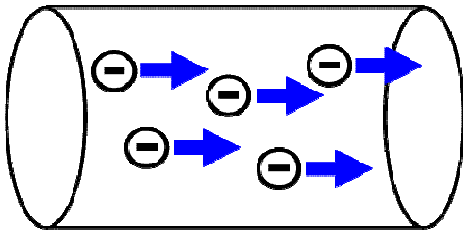
Ions are atoms to which some electrons have been added or taken away, so that the atom has a net charge (positive if electrons have been removed, negative if electrons have been added). Ions that have opposite net charges attract, and those with net charges of the same sign repel.

Atoms of the same atomic number can have different numbers of neutrons. These are the different isotopes of an element. Hydrogen has three isotopes: no neutrons - called "protium" (this is the hydrogen isotope most commonly found), one neutron - called deuterium, and two neutrons - called tritium.

Although he didn't know about electrons and protons, the designations of positive and negative were made by Benjamin Franklin over two hundred years ago! Later on people realized they corresponded to the charges of protons and electrons, respectively.

What is Electricity?

In some materials, particularly metals, the electrons farthest from the nucleus are not bound to a particular atom - they can move freely from one atom to another. Electricity is the flow of these free electrons in a wire:

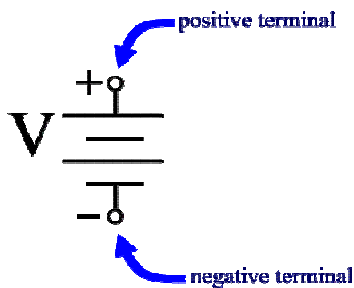


Such a flow of electrons is called a current.

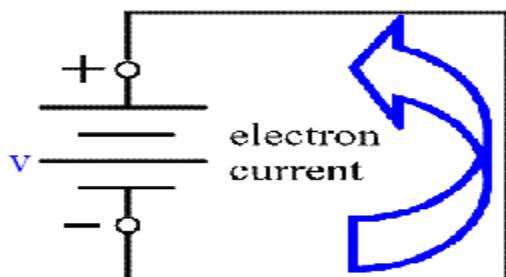
What makes these free electrons move? Suppose we put something that has a net positive charge at the one end of the wire (say, at the right end of the wire pictured above). Let's also suppose that we put something with a net negative charge at the other end (the left end in the wire above). Then the electrons in the wire will be attracted to the positive end and repelled by the negative end. Hence, they will flow from left to right. That's electricity!

Batteries

Batteries are devices which can do exactly what is described just above - make a current flow by creating a positive charge at one end of a wire and a negative charge at the other. A battery has two terminals (wire contacts), called positive and negative, corresponding to the net charges created at the terminals. The symbol for a battery is the following:

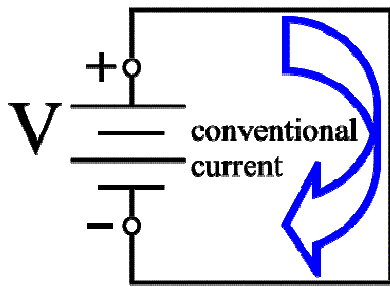


When a wire is connected across these terminals, forming a closed circuit, the positive and negative charges created by the battery cause a current to flow:



Note that the electrons flow from the negative terminal to the positive terminal. Eventually, when enough electrons have flowed, the battery will become drained, and the current will cease.

Even though the electrons flow from the negative to the positive terminals, it is conventional to say that the current flows from positive to negative:



Why is this? This is simply because people can't see electrons, and so they guessed wrong when they settled on a convention. But this is not really a problem, because even today we still don't see electrons in most applications, so it doesn't really matter for most purposes which direction the electrons actually go.

Measuring Current

How do we measure current? Current is measured by literally counting the number of electrons that pass a given point in the wire. Any point will do - it doesn't matter which one because the current will be the same in each point of the wire, unless the wire branches off into a more complicated circuit.

Because there are billions of billions (much more than billions and billions! Carl Sagan would be impressed!) of electrons in even a very little piece of wire, we need to have a unit of measurement that will make it easy to count so many.

The basic unit for counting electrons (that is, charge) is the "coulomb" (pronounced "cool lum"):

1 coulomb = 1.6×10^{19} electrons = 16,000,000,000,000,000
= 16 billion billion electrons!

To measure current, we pick one point along the wire and count the electrons that go by, like watching things go by on an assembly line. If 1 coulomb of electrons go by each second, then we say that the current is 1 "ampere" (pronounced "am - peer"), or 1 amp for short. If 2 coulomb's per second goes by, we say the current is 2 amps, and so on:

1 ampere = 1 coulomb per second

It is traditional to represent the current with the symbol I , as in $I = 1$ amperes, or $I = 15$ amperes, etc.

Measuring Voltage

Some batteries try to push electrons through the wire more strongly than others. How strongly the battery pushes is a measure of its voltage, symbolized with the letter V (as in the diagrams above). You can think of voltage like pressure: the higher the voltage, the higher the pressure is to push electrons through the wire. The lower the voltage, the lower the pressure.

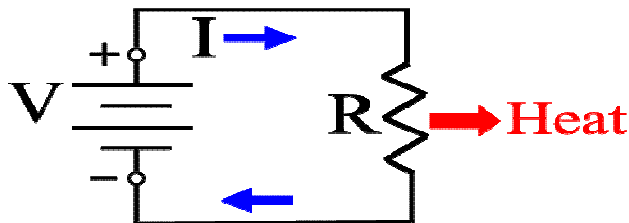
Voltage is measured in volts. For example, common voltages for batteries are 1.5 volts, 6 volts, 9 volts, and 12 volts. Car batteries are typically 12 volts. An electrical outlet in the United States has a voltage of 110 volts (pretty high!).

The voltage of a battery is related to the amount of energy that the battery can deliver. A voltage of $V = 1$ volt means that the battery will deliver 1 "Joule" of energy for each coulomb of charge that flows through the circuit. A voltage of $V = 2$ means that the battery will deliver 2 Joules of energy for each coulomb. A Joule is the basic unit of energy in the metric International system of units - its about the amount of energy it takes to lift two pounds 9 inches. How high the voltage of the battery is depends in detail on the internal construction of the battery, which is an "electro-chemical" energy storage device.

Measuring Resistance

How are amperes (current) and voltage (electrical pressure), related to one another? For a given voltage, some wires let more current flow than others. A wire that doesn't let very much current flow is said to have high resistance. Resistance is symbolized with the letter R.

To simplify matters, we usually assume that the wire itself is an ideal wire with no resistance, and we represent the resistance as a localized component of the circuit symbolized with a broken line:



In fact the resistor now can represent the sum total of the wire's resistance, including that contributed by any additional components in the circuit that have resistance. There are devices called "resistors" whose function is simply to provide additional resistance.

The resistance is related to how much the current I we get from a given applied voltage V by "Ohm's Law" in the following way:

Ohm's Law: $I = V / R$

If I is measured in amperes, and V in volts, then we say that R has units of "ohms" (pronounced "olms"). More specifically, if we have a 1 volt battery, and a wire with a resistance of 1 ohm, then the current that results when the wire is placed across the battery's terminals is given by

$$I = V / R = 1 \text{ volt} / 1 \text{ ohm} = 1 \text{ ampere.}$$

Likewise, if we have a 2 volt battery, and a wire with a resistance of 3 ohms, then the current that results when the wire is placed across the battery's terminals is given by

$$I = V / R = 2 \text{ volt} / 3 \text{ ohm} = 2/3 \text{ amperes.}$$

Thus, knowing the voltage and the resistance, we can now predict the current. Likewise, we can also turn the problem around, and say, calculate the resistance by measuring the voltage and current with a voltmeter (voltmeters are capable of measuring both voltage and current - but not generally at the same time!).

Measuring Power in Watts

What happens to the energy delivered to the wire? For the case of a simple piece of wire plus some additional devices which are purely resistive, the energy is completely converted into heat energy in the wire (as shown in the figure above), which escapes into space.

As discussed above, the voltage of a battery is related to the amount of energy that the battery can deliver. A voltage of $V = 1$ volt means that the battery will deliver 1 Joule of energy for each coulomb of charge that flows through the circuit, a voltage of $V = 2$ yields two Joules per coulomb, and so forth.

In many practical applications, we want to know the rate at which energy is delivered, not how much energy is delivered per coulomb of charge. For example, you might need to insure that a circuit you design will deliver 2 joules per second to make a toy car go fast enough.

The rate at which energy is delivered is called power. Power is thus defined:

Power = Energy / Time.

In the metric International units, the unit of power corresponding to 1 joule per second is called a watt:

1 watt = 1 joule per second.

Because the voltage V tells us the number of joules per coulomb, and the current I tells us the number of coulombs per second, all we have to do to get the current is to multiply them:

Power = number of watts = number of joules/second
= joules/coulomb $\times$ coulombs/second = $I V$,

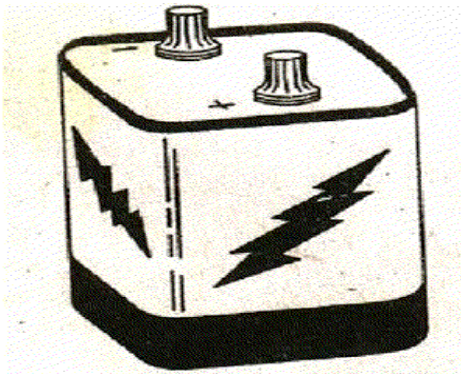
Or **Power Formula: $P = I V$**

Thus, suppose we had a circuit with a battery voltage of 2 volts and a current of 3 amps. Then the power delivered to the resistor would be $P = I V = (3 \text{ amps}) (2 \text{ volts}) = 6 \text{ watts}$.

LECTURE 2
(Introduction-some basic concepts)

NEGATIVE AND POSITIVE POLARITIES OF BATTERY

We see the effects of electricity in a battery, static charge, lighting, radio, television, and many other applications. What do they all have common that is electrical in nature?
The answer is basic particles of opposite polarity.
As we studied in the last lecture, all the materials, including solids, liquids, and gases, contain two basic particles of electric charge: the electrons and protons.
An electron is the smallest amount of electrical charge having characteristics called the negative polarity. The proton is a basic particle with positive polarity.
In order to use the electrical forces associated with the negative and positive charges in all matter, some work must be done to separate the electrons and protons.
Changing the balancing forces produces evidence of electricity.
A battery for instance, can do electrical work because its chemical energy separates electric charges to produce an excess of negative charge at its negative terminal and an excess of protons on its positive terminal.
With separate and opposite charges at the two terminals, electric energy can be supplied to a circuit connected to the battery. In fig below shows a battery with the negative (-) and positive (+) terminals marked to emphasize the two opposite polarities.



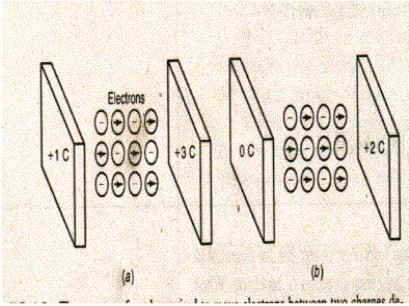
POTENTIAL DIFFERENCE

Potential difference is actually the difference between the electric potentials at the two terminal of a circuit element.

- Potential refers to the possibility of doing work.
- Any charge has the potential to do the work of moving another charge, by either attraction or repulsion.
- When we consider two unlike charges, they have a difference of potential.
- A charge is the result of work done in separating electrons and protons.
- The work of producing the charge causes a condition of stress in protons, which try to attract the electrons and return to the neutral condition and vice versa.

POTENTIAL BETWEEN DIFFERENT CHARGES

For instance, consider a positive charge of 3 C, shown in fig below.

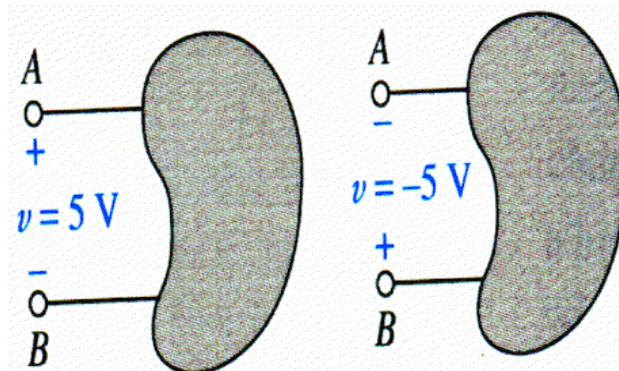


The work to be done in moving some electrons, as illustrated. Assume a charge of 1C can move three electrons. In Fig **a** Then the charge of +3C can attract 9 electrons toward right. However, a charge of +1 C at the opposite side can attract 3 electrons towards left. The net result , then is that 6 electrons can be moved toward the right to the more positive charge.
In Fig **b** , one charge is 2C ,while the other charge is neutral with 0C. For the difference of 2C ,again 2*3 or 6 electrons can be attracted to the positive side.

POTENTIAL DIFFERENCE BETWEEN THE TERMINALS

- A voltage can exist between the terminals of a battery even a current is flowing or not.
- An automobile battery, for example, have 12 volts of voltage across its terminals if nothing whatsoever is connected to the terminals.

VOLTAGE CONVENTIONS

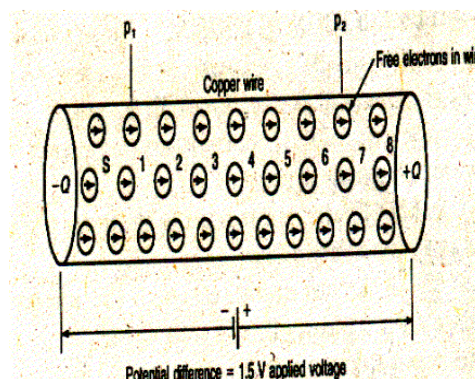


VOLT

- This unit is named after Alessandro Volta.
- Fundamentally, the volt is the measure of the work needed to move an electric charge. When 0.7376 foot-pound of work is required to move 6.25×10^{18} electrons between two points, each with its own charge, the potential difference is one.
- 6.25×10^{18} electrons make up one coulomb.
- 0.7376 ft-lb of work is same as 1 joule, which is the practical unit of work or energy. So we can say that $1V = 1J/1C$
- The symbol of potential difference is V for voltage.
- In fact, the volt unit is used so often that potential difference is called voltage.

CURRENT

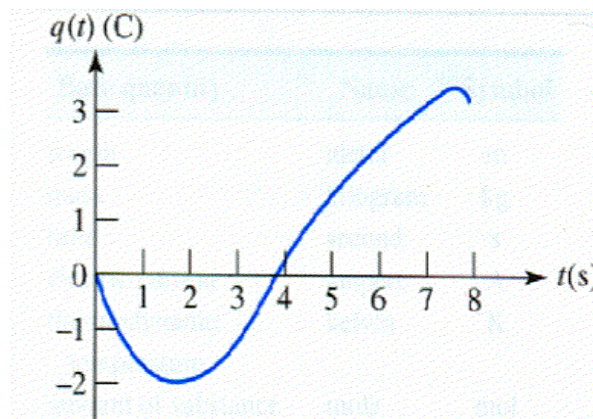
- When the potential difference between two charges forces a third charge to move, the charge in motion is called current.
- To produce current, therefore, charge must be moved by a potential difference.
- In solid materials, such as copper wire, the free electrons are charges that can be forced to move with relative ease by a potential difference, they are required a little work to be moved. As illustrated in fig. if a potential difference is connected across two ends of a copper wire the applied voltage forces the free electron to move.



- This current is drift of electrons, from the point of negative charge at one end, and returning to the positive charge at the other end.
- Each electron in the middle row is numbered, corresponding to a copper to which this electron belongs.
- Considering the case of only one electron moving, note that the electron returning to the positive side of the voltage source is not electron S which left negative side.
- All electrons are same. Therefore, the drift free electrons resulted in the charge of one electron moving through the wire.
- This charge in motion is current.
- Current is the constant flow of electrons.
- Only the electrons move, not the potential difference.
- The current must be the same at all points of the wire at all times.

Another definition of current can be made as

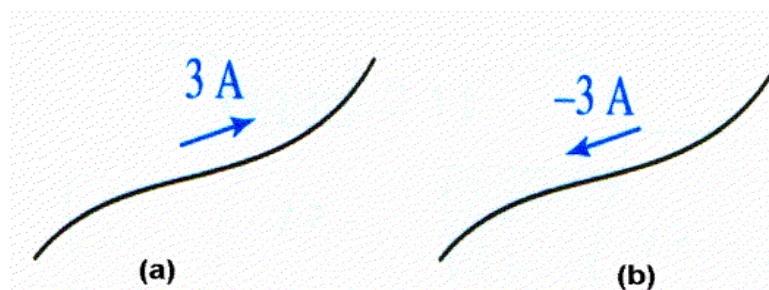
- Let $q(t)$ be the total charge that has passed a reference point since an arbitrary time $t=0$, moving in the defined direction. A contribution to this total charge will be negative if the negative charge is flowing in the reference direction.
- As the figure shows a history of total charge $q(t)$ that has passed a given reference point in a wire.



- The current at a specific point and flowing in a specified direction as the instantaneous rate at which net positive charge is moving past the point in the specified direction.
 - Current is symbolized as I or i .
- Mathematically it can be given as

$$I = dq/dt$$

GRAPHICAL SYMBOLS FOR CURRENT



POTENTIAL DIFFERENCE IS NECESSARY FOR CURRENT

- The number of free electrons that can be forced to drift through the wire to produce the moving charge depends upon the amount of potential difference across the wire, with more applied voltage, the forces of attraction and repulsion can make more free electrons drift, producing more current.
- With zero potential difference across the wire, there will be no current.
- As another case, connecting the same potential across the terminals of a wire will result in no current flow.

THE AMPERE OF CURRENT

- Since current is the movement of charge, the unit for stating the amount of current is defined as the rate of flow charge.

- When charge moves at the rate of 6.25×10^{18} electrons flowing past a given point per second, the value of current is one Ampere.

RESISTANCE

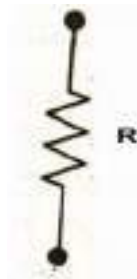
- The fact that a wire conducting a current can become hot is evident that the work done by the applied voltage in producing current is due to accomplishment against some form of opposition.
- This opposition, which limits the current, is called resistance.
- The atoms of a copper wire have a large number of free electrons, which can be moved easily by a potential difference.
- Therefore, the copper wire has little opposition to the flow of free electrons when voltage is applied, corresponding to low value of resistance.
- Carbon, however, has fewer free electrons than copper.
- When the same amount of voltage is applied to the carbon as to the copper, fewer electrons will flow.
- Carbon opposes the current more than copper, therefore, has a higher value of resistance.

OHM

- The practical unit of resistance is the ohm.
- A resistance that develops 0.24 calorie of heat when one ampere of current flows through it, for one second will have opposition of one ohm.

SYMBOL FOR RESISTANCE

- The symbol for resistance is R.
- The abbreviation used for the ohm is the Greek letter OMEGA (Ω).
- In diagrams resistance is indicated by a zigzag line.

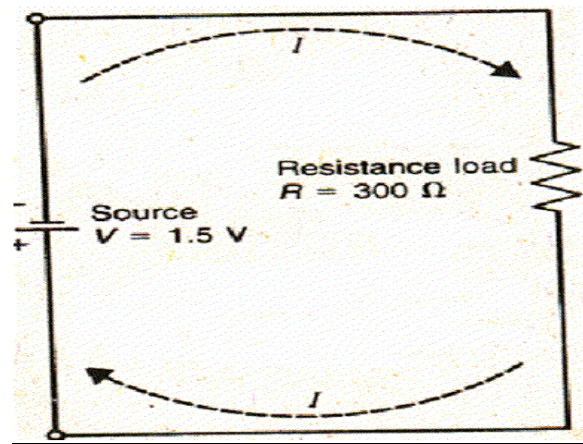


CONDUCTANCE

- The reciprocal of resistance is called conductance.
- The lower the resistance the higher the conductance.
- Its symbol is G and the unit is siemens. (The old unit for conductance is mho, which is ohm spelled backward and written as upside down OMEGA.)

THE CLOSED CIRCUIT

- In application which require the use of current, the components are arranged in the form of circuit.
- As shown in the figure.



- For a closed circuit, these things should always keep in mind.
 - (1) There must be a source of voltage. Without applied voltage current can not flow.
 - (2) There must be a complete path of current flow, from one side of the applied voltage source, through the external circuit, and returning to the other side of the voltage source.
 - (3) The current path normally has resistance. The purpose of a resistance in the circuit is either generating heat or limiting the amount of current.

OPEN CIRCUIT

- When any part of the path is broken, the circuit is open because there is no continuity in the conducting path.
- The resistance of an open circuit is infinitely high.
- The result in no current in the open circuit.

SHORT CIRCUIT

- In this case, the voltage source has a closed path across its terminals, but the resistance is practically zero.
- The result is too much current in the short circuit.
- Usually, the short circuit is a bypass across the load resistance.

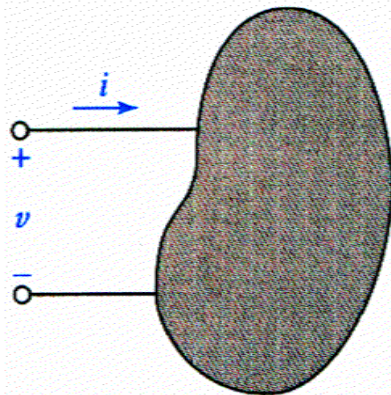
POWER

Rate of doing work is called power.

- The unit of electric power is watt.
- One watt of power is equals the work done in one second by one volt of potential difference in moving one coulomb of charge.
- We know that one coulomb per second is an ampere. Therefore power in watts equals the product of volt times amperes.
- Power in watts = volts x amperes

$$P = V \times I$$

Dimensionally, the right side of this equation is the product of joules per coulomb and coulombs per second, which produces the expected dimension of joule per second or watt.



- The sketch shows that if one terminal of the element is v volts positive with respect to the other terminal, and of current i is entering the element through the terminal then the power is absorbed by the element.
- It is also correct to say that a power $p = vi$ is being delivered to the element.

PASSIVE SIGN CONVENTIONS

- If the current arrow is directed into the + marked terminal of an element, then $p=vi$ yields the absorbed power.

- A negative value indicates the power is actually being generated by the element, it might have been better to define a current flowing out of the + terminal.

Example

Q. A simple circuit is formed using a 12V lead-acid battery and an automobile headlight. If the battery delivers a total energy of 460.8 watt-hours over an 8 hours discharge period.

- How much power is delivered to the headlight?
- What is the current flowing through the bulb (assume the battery voltage remains constant while discharging)

Solution:

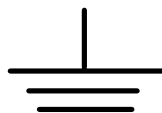
The battery delivers energy of 460.8 W-hr over a period of 8 hrs.

- The power delivered to the headlight is therefore (460.8 W-hrs)
- The current through the headlight is equal to the power it absorbs from the battery divided by the Voltage, at which the power is supplied,
OR $I = (57.6 \text{ W}) / (12\text{V}) = 4.8 \text{ A}$

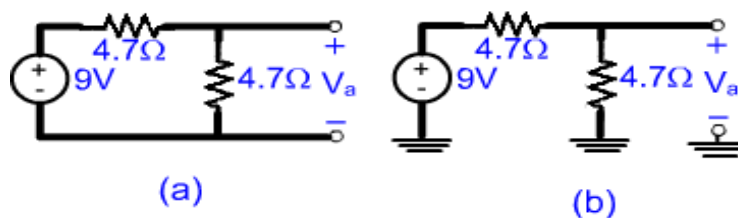
Importance of Earth Ground

Voltage can not be defined at a single point it is by definition the difference in potential between two points. However, many schematics makes use of the convention of taking the earth as defining zero volts, so that all other voltages are implicitly referenced to this potential.

The concept is often referred to as earth ground, and is fundamentally tied to safety regulations designed to prevent fires, fatal electrical shocks, and related mayhem. The symbol for earth ground is shown in the figure below

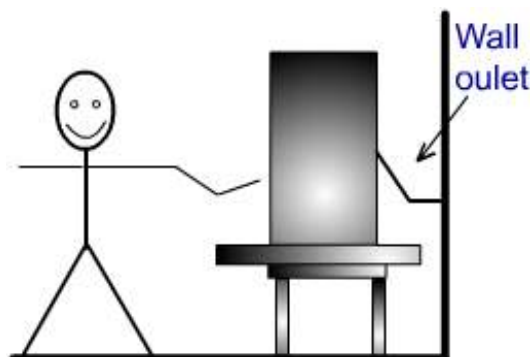


Since earth ground is defined as zero volts, it is often convenient to use this as a common terminal in schematics. The circuit of Fig a is shown redrawn in this fashion in Fig b. Where the earth ground symbol represents a common node. It is important to note that the two circuits are equivalent in terms of our value for V_a .



Example

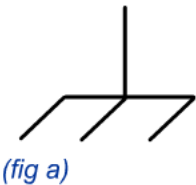
The fact that the common terminal of a circuit may or may not be connected by some low-resistance pathway to earth ground can lead to potentially dangerous situations. Consider the diagram of Fig, which depicts an innocent bystander about to touch a piece of equipment powered by an ac outlet.



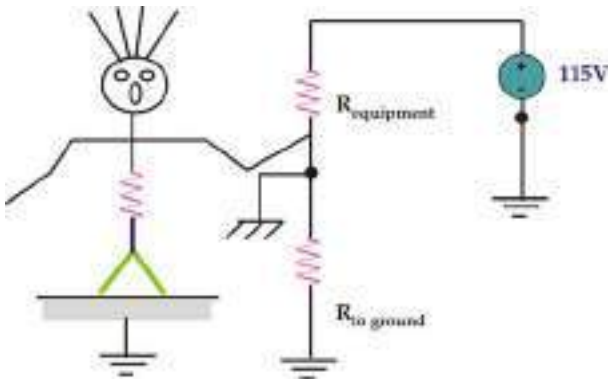
Only two terminals have been used from the wall socket; the round ground pin of the receptacles was left unconnected. The common terminal of every circuit in the equipment has been tied together and electrically connected to the conducting equipment chassis;

Chassis Ground

This terminal is often denoted using the chassis ground symbol of Fig(a). The electrical connection to earth, however, has been made (inadvertently) a nonzero resistance; it may be the table for all we know. In any event, there is no reason to expect this chassis ground to be the earth ground.



A pseudo-schematic (some liberty was taken with the person's equivalent resistance symbol) of the situation is shown in Fig. If the equivalent resistance of the person is significantly less than the resistance of all other possible paths to ground... well, let's just say not all stories have happy endings.



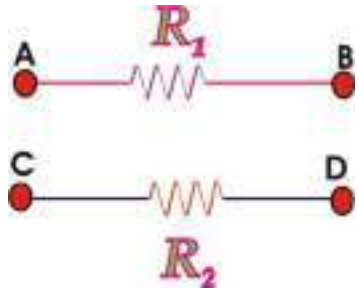
The fact that "ground" is not always "earth ground" can cause a wide range of safety and electrical noise problems.

LECTURE 3

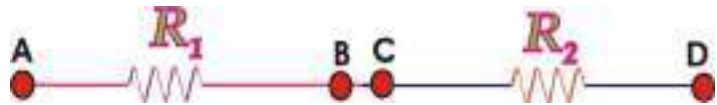
(Resistance in Series)

Resistance in Series

- If we connect resistors across a source such that the ending point of one resistor is joined with starting point of the other resistor then they are said to be connected in series. The combined effect of all the resistors will be equal to the sum of individual resistances.
Consider two resistances R_1 and R_2 with terminals A, B and C, D as shown in the figure.



They will be in series if we connect B with C as shown in the figure



Combined effect of these two resistances will be

$$R_{eq} = R_1 + R_2$$

Therefore, if we connect N resistances in series then

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N$$

To illustrate this effect we take some examples.

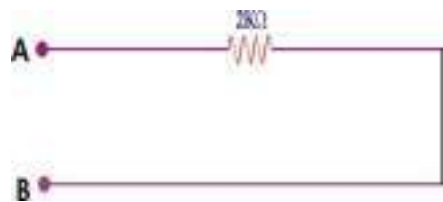
Example 1

Simplify the given circuit.



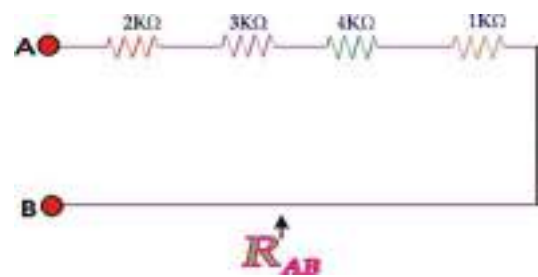
Solution:

$$\begin{aligned} \text{So } R_{AB} &= R_1 + R_2 \\ &= 1\text{ k} + 1\text{ k} = 2\text{ k}\Omega \end{aligned}$$



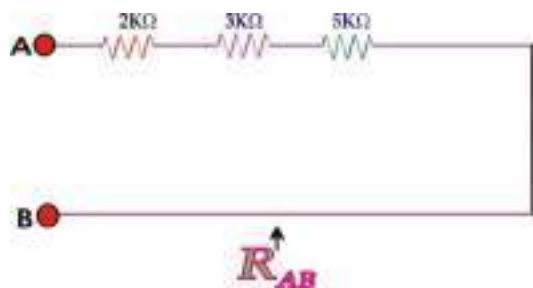
Example 2

Simplify the given circuit.



Solution:

1 k Ω and 4 k Ω are in series so they will be combined as



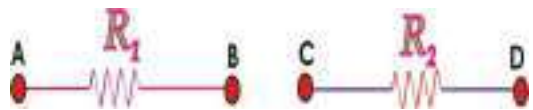
Now 5k Ω and 3k Ω are also in series so



2 k Ω and 8k Ω are also in series
So $R_{AB} = 2k + 8k = 10k \Omega$

Resistance in Parallel

Consider two resistances with terminals A, B and C, D as shown in the fig



If we connect A with C and B with D they are said to be connected in parallel.



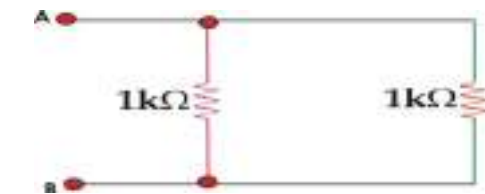
The equivalent of these two resistances will be

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{(R_1 R_2)}{(R_1 + R_2)}$$

If we connect N number of resistances in parallel their equivalent will be

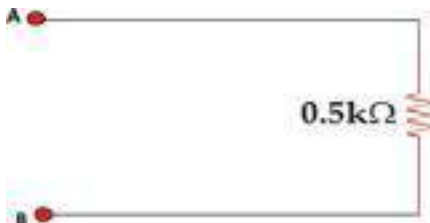
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

To illustrate this effect let us take some examples.
Example 3 Simplify the given circuit.

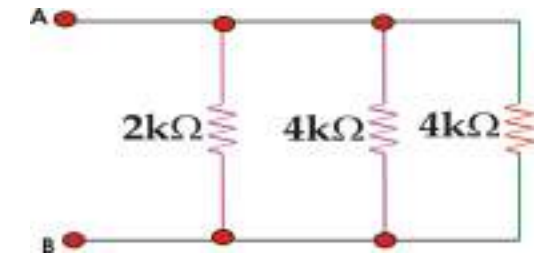


1k is in parallel with 1k Ω so

$$R_{eq} = (R_1 R_2) / (R_1 + R_2)$$
$$= 1/2$$
$$= 0.5 \text{ k } \Omega$$

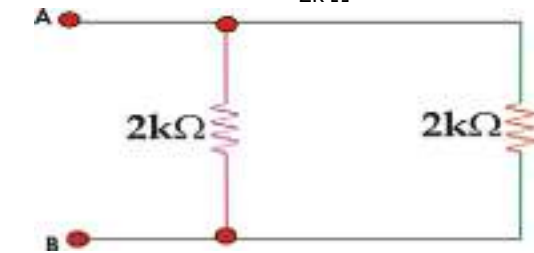


Example 4 Simplify the given circuit.



4k Ω is parallel with 4k Ω so $4k || 4k = (4 \times 4) / (4 + 4)$

$$= 16/8$$
$$= 2 \text{ k } \Omega$$



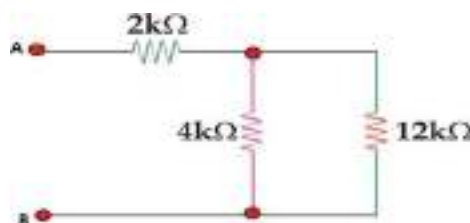
2k Ω is parallel with 2k Ω.

So $R_{eq} = (2 \times 2) / (2 + 2) = 4/4$

$$= 1 \text{ K } \Omega$$



Example 5 Simplify the given circuit.



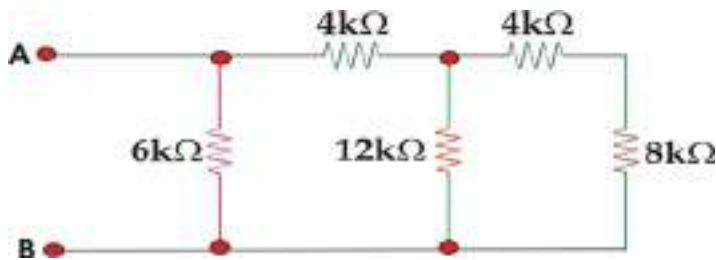
12k Ω is parallel with 4k Ω so
 $12k||4k = (12 \times 4)/(4 + 12)$
 $= 48/16$
 $= 3k \Omega$



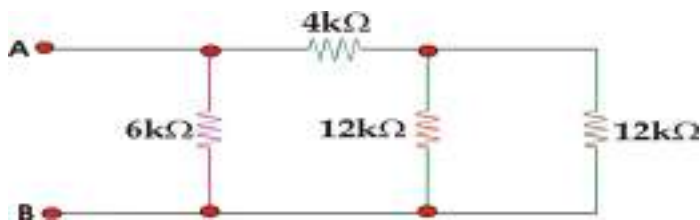
Now 2k Ω is in series with 3k Ω so
 $R_{AB} = 2k + 3k = 5k \Omega$



Example 6 Simplify the given circuit.

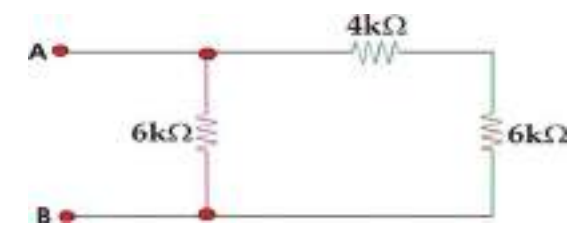


4k Ω is in series with 8k Ω so the combined effect=12k Ω

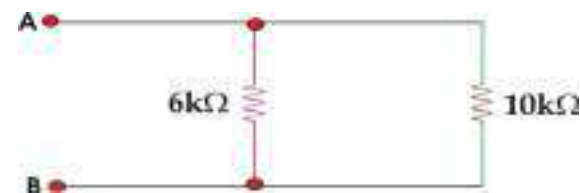


12k Ω is in parallel with 12k Ω so
 $12k||12k = (12 \times 12)/(12 + 12)$

$=144/24=6k\ \Omega$



4k Ω is in series with 6k Ω so their combined effect = 4k+6k
=10k Ω

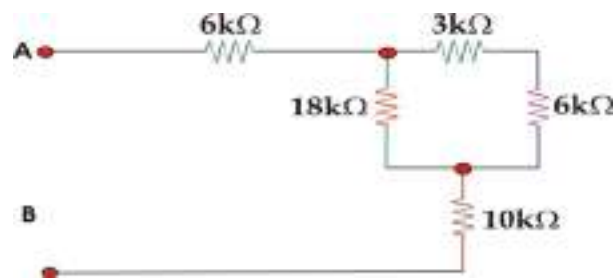


6k Ω is in parallel with 10k Ω so
 $R_{AB} = (6 \times 10)/(6+10)=3.75k\ \Omega$

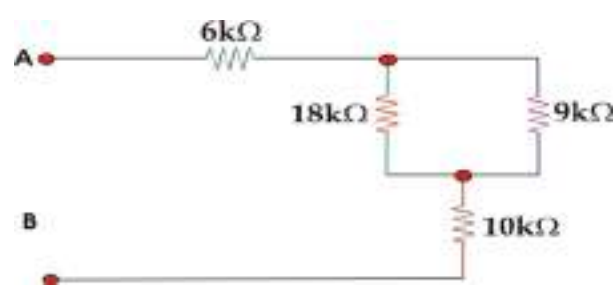


Example 7

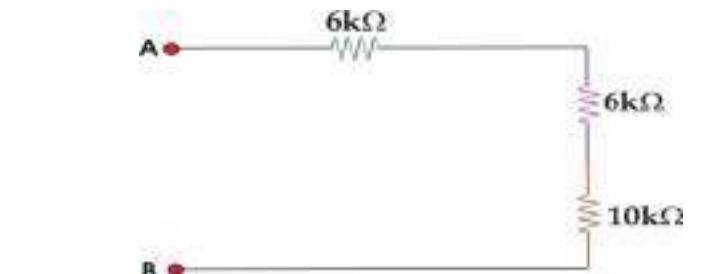
Simplify the given circuit.



3k Ω is in series with 6k Ω , therefore, their combined effect=3k +6k= 9k Ω



9k Ω is in parallel in 18k Ω so
 $9k||18k= (9 \times 18)/(9+18)$
 $=162/27=6k\ \Omega$



$6\text{k}\Omega$ is in series with $10\text{k}\Omega$
So their combined effect = $6\text{k} + 10\text{k}$
= $16\text{k}\Omega$

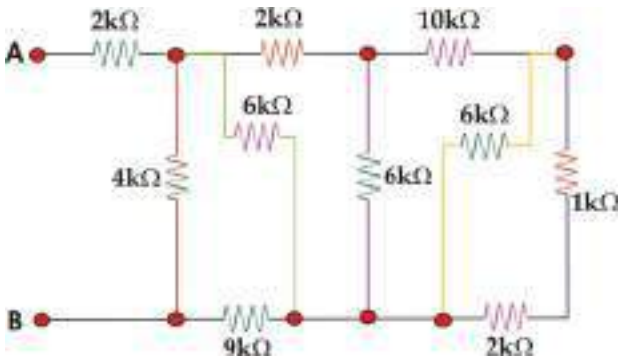


$6\text{k}\Omega$ is in series with $16\text{k}\Omega$ so
 $R_{AB} = 6 + 16 = 22\text{k}\Omega$

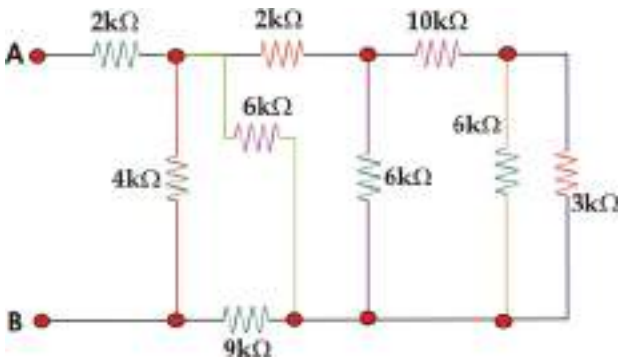


Example 8

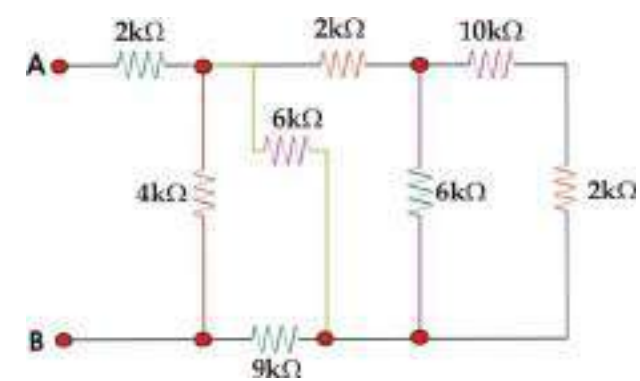
Simplify the given circuit.



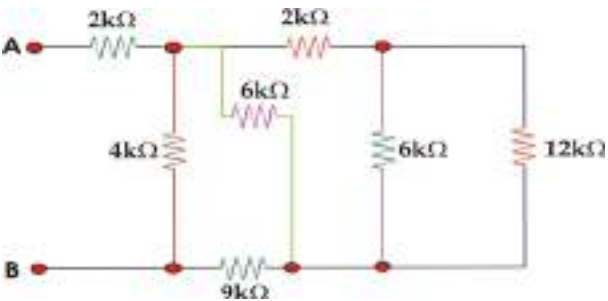
$1\text{k}\Omega$ is in series with $2\text{k}\Omega$ so
Their combined effect = $1 + 2 = 3\text{k}\Omega$



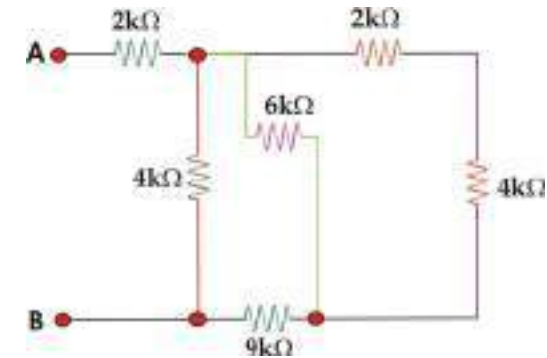
3k Ω is in parallel with 6k Ω
 $3k || 6k = (3 \times 6)/(6+3) = 18/9 = 2k \Omega$



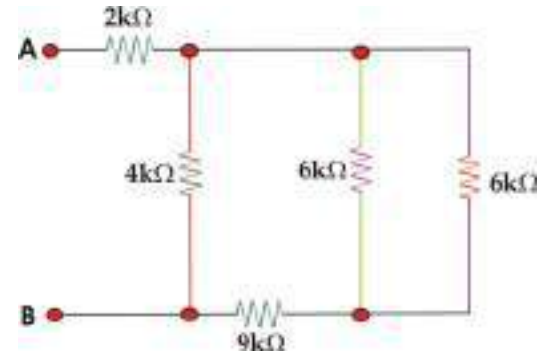
10k Ω is in series with 2k Ω, therefore, their combined effect
 $= 10k + 2k = 12k \Omega$



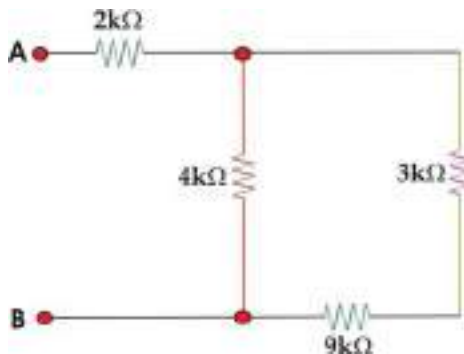
12k Ω is in parallel with 6k Ω, hence
 $12k || 6k = (12 \times 6)/(12+6) = 4k \Omega$



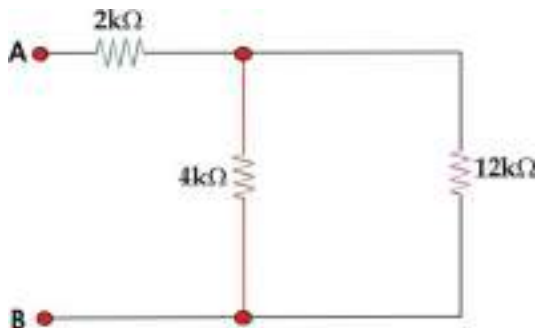
2k Ω is in series with 4k Ω
combined effect= 2+4=6k Ω



6k Ω is in parallel with 6k Ω, therefore,
 $6k||6k=(6 \times 6)/(6+6)=36/12=3k \Omega$



3k Ω is in series with 9k Ω, therefore,
combined effect=3k +9k=12k Ω



12k Ω is in parallel with 4k Ω so
 $12k||4k = (12 \times 4)/(12+4)=48/16=3k \Omega$



2k Ω is in series with 3k Ω so
 $R_{AB}=2k+3k=5k \Omega$



LECTURE 4
(Series parallel combination)

Inductance

- Resistance offered by an inductor in an circuit is called inductance.
- The unit of inductance is Henry.
- It is denoted by L.

Inductance in Series

- If we connect n inductances in series the combined effect of all these inductances is equal to the sum of individual inductance.

$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_n$$

Inductance in Parallel

- we connect n inductances in parallel, the reciprocal of combined effect of all these inductances is equal to the sum of reciprocals of individual inductances.

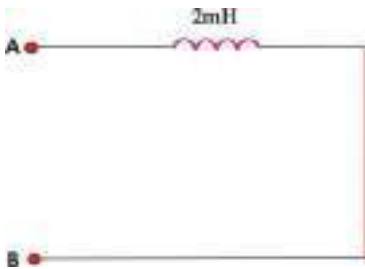
$$1/L_{eq} = 1/L_1 + 1/L_2 + 1/L_3 + \dots + 1/L_n$$

Example: Simplify the given inductance circuit.

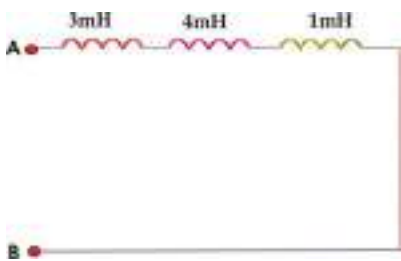


Solution:

1mH and 1mH are in series
so their combined effect = 1 + 1 = 2mH

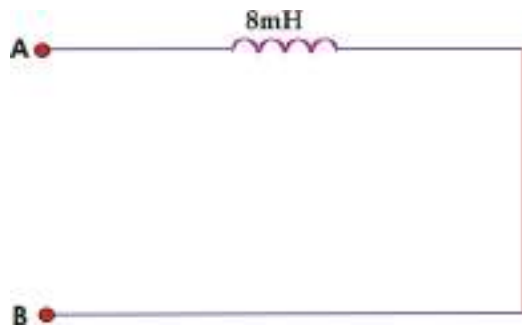


Example: Simplify the given inductance circuit.

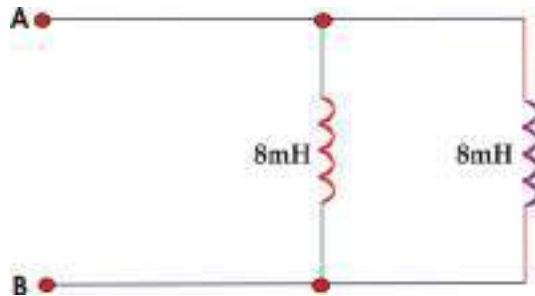


Solution:

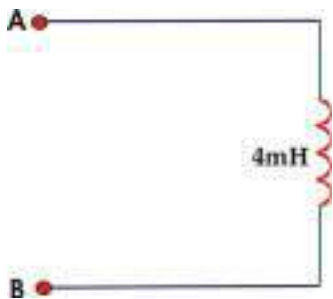
1mH is in series with 4 mH and with 3mH, therefore, their effect=1 + 4+ 3=8mH



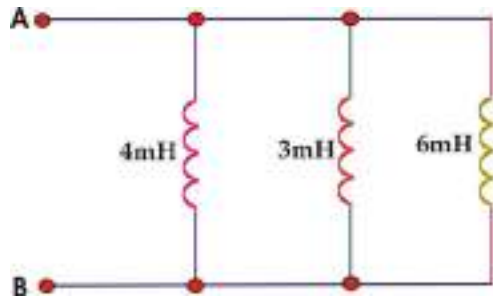
Example: Simplify the given inductance circuit.



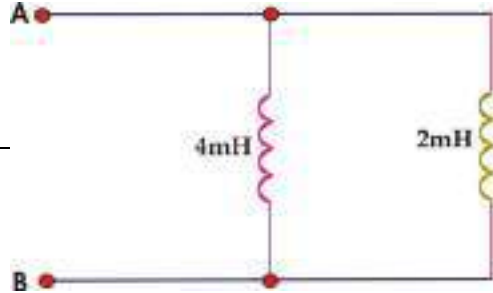
Solution: 8mH is in parallel with 8mH so
 $L_{AB} = (8 \times 8)/(8+8)$
 $= 64/16 = 4\text{mH}$



Example: Simplify the given inductance circuit.



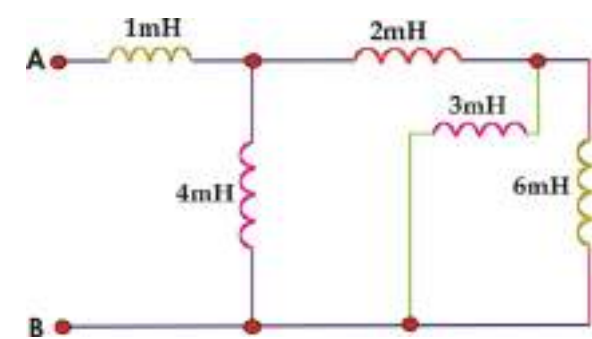
Solution: 6mH is in parallel with 3mH so
 $6\text{mH}||3\text{mH}=(6 \times 3)/(6+3)$
 $=18/9$
 $= 2\text{mH}$



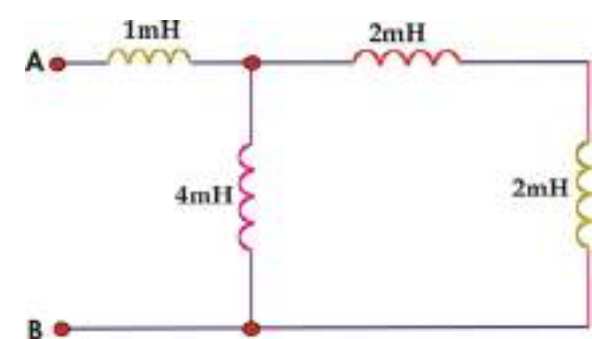
2mH is in parallel with 4mH, therefore,

$$2\text{mH}||4\text{mH} = (2 \times 4)/(2+4)$$
$$= 8/6=1.33\text{mH}$$

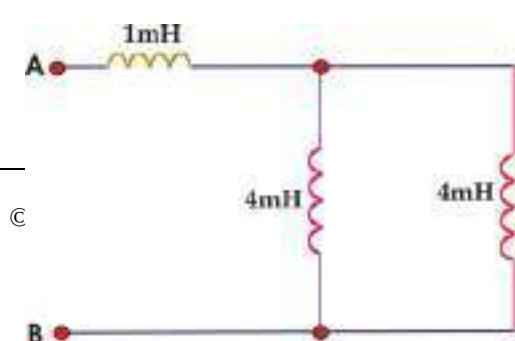

Example: Simplify the given inductance circuit.



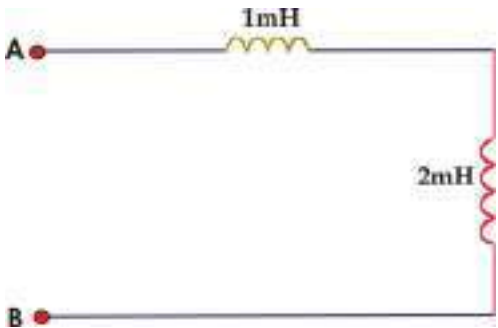
Solution: 3mH is in parallel with 6mH so

$$3\text{mH}||6\text{mH}=(3 \times 6)/(3+6)$$
$$= 18/9$$
$$= 2\text{mH}$$


2mH is in series with 2mH, therefore,
The combined effect of these two = 2+2
= 4mH



4mH is in parallel with 4mH So
 $4\text{mH}||4\text{mH} = (4 \times 4)/(4 + 4)$
 $= 16/8$
 $= 2 \text{ mH}$



1mH is in series with 2mH so
 $L_{AB} = 1 + 2 = 3\text{mH}$

Capacitance:

- Resistance offered by a
- The unit of capacitance i
- It is denoted by C.

Capacitance in parallel:

- If we connect n capacita of individual capacitance

$C_{eq} = C_1 + C_2 + C_3 + + C_n$

Capacitance in Series:

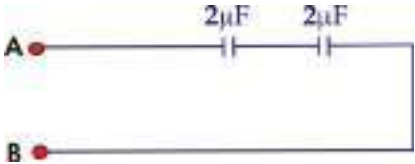
- If we connect n capacitances in series, the reciprocal of combined effect of all these capacitances is equal to the sum of reciprocals of individual capacitances.

$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + + 1/C_n$

The diagram shows a circuit with two terminals, A and B. A horizontal wire connects A to a vertical wire. On this vertical wire, there is a red inductor labeled '3mH'.

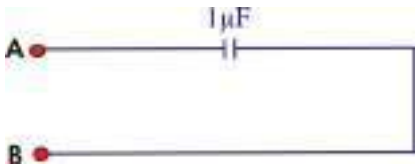
the capacitance is equal to the sum

Example: Simplify the given circuit.

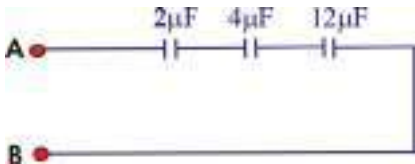


Solution: 2μF capacitor is in series with other 2μF capacitor their combined

effect will be= $(2 \times 2)/(2+2)=4/4=1\mu\text{F}$

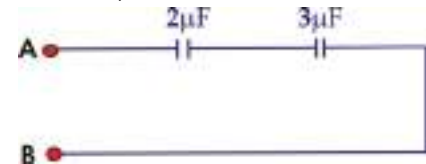


Example: Simplify the given circuit.

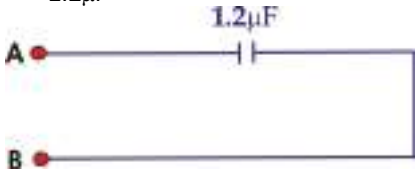


Solution:

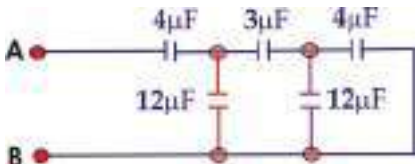
12 μF is in series with 4 μF.
So their combined effect will be = $48/16$
=3 μF



2 μF is in series with 3 μF so
 $C_{AB} = (2 \times 3)/(2+3)$
=1.2 μF

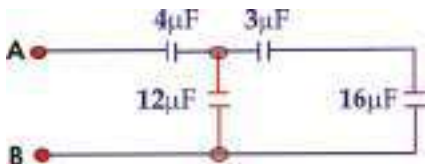


Example: Simplify the given circuit.

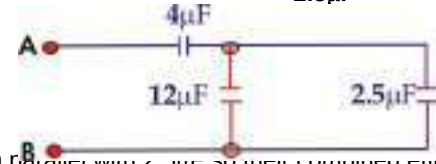


Solution:

4 μF is in parallel with 12 μF so
 $12 || 4 = 12 + 4 = 16 \mu\text{F}$



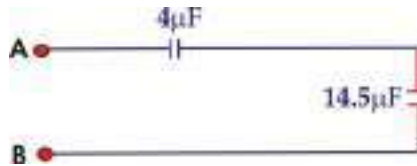
And 16 μF is in series with 3 μF so
 $= (3 \times 16)/(3+16) = 48/19$
= 2.5 μF



12 μF is in parallel with 2.5 μF so their combined effect will be

$$= 12 + 2.5$$

$$= 14.5 \mu\text{F}$$



The capacitors are in series so
 $C_{AB} = (4 \times 14.5) / 18.5$
 $= 3.13 \mu\text{F}$



OHM'S LAW:

- If a voltage across a conductor is applied, the 'current' passing through the conductor is directly proportional to the 'voltage provided temperature remains constant.' i.e.
 $V \propto I$
 $V = IR$
- Where 'R' is the resistance of the conductor.
- Resistance ('R') depends upon the material of the conductor.

The Current "I=V/R"

- If we keep the same resistance in a circuit but vary the voltage, the current will vary.
- For general case, for any values of 'V' & 'R' OHM'S Law is "I=V/R".
- Where 'I' is the amount of current through resistance 'R', which is connected across a Potential difference 'V'.
- Volt (V) is the practical unit of potential difference & Ohm (Ω) for Resistance, therefore,
- **Ampere=Volts/Ohms**
- This Formula tells us, to calculate the Amperes of Current through 'R', simply divide the voltage across 'R' by the Ohms of Resistance.

High Voltage but Low Current:

- It is important to realize that with high voltage the Current can have a Low value when there is a very high Resistance in the circuit.
- For example 1000 (1k) volts applied across 1000000 (1M) Ω results in a Current of only 0.001 (1m)A.
- The practical fact is that high voltage circuits usually do have a small value of current in electronic equipment otherwise tremendous amount of power would be necessary for operation.

Low Voltage but High Current:

- At the opposite extreme a low value of voltage in a very low resistance circuit can cause a very high current to flow.
- For example a 6 volt battery connected across a Resistance of .001 Ω causes 600 A of Current to flow.

$$I = V/R$$

$$= 6\text{v} / .01\Omega$$

$$= 600 \text{ A}$$

Similarly more 'R' will result in less 'I'.

Example: A heater with a Resistance of 8 Ω is connected across the 220 volt power line. How much is the current 'I' flowing through the heater coil?

Solution:

$$I = V/R$$

$$= 220/8$$

$$= 27.5 \text{ A}$$

Example: A small light bulb with a resistance of 2400Ω is connected across the same 220 volt power line. How much is the current 'I' through the bulb filament?

Solution:

$$\begin{aligned} I &= V/R \\ &= 220/2400 \\ &= .09 \text{ A} \\ &= 90 \text{ m A} \end{aligned}$$

The Voltage $V=IR$

- It is the other Form of the same formula.
- Besides the numerical calculations possible with the 'IR' formula, it is useful to consider that the 'IR' product means voltage.
- Whenever, there is current through a resistance, it must have a potential difference across its two terminals equal to the product 'IR'.
- As we studied in the last lecture if there was no potential difference, no electrons could flow to produce the current.

The Resistance $R=V/I$

- As a third and final version of Ohm's Law, three factors V, I, and R are related by the formula $R=V/I$.
- As we studied in the last lecture physically, a resistance can be considered as some material with elements having an atomic structure that allows free electrons to drift through it.
- Electrically, a more practical & general way of considering Resistance is simply as a V/I ratio.

Example: A conductor allows 1 A of current with 10 volts applied at its ends. How much will be the Resistance of the conductor?

Solution:

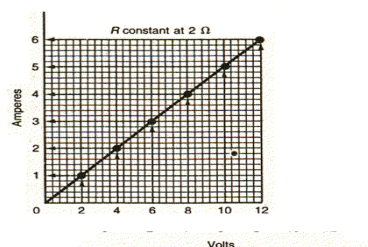
$$\begin{aligned} R &= V/I \\ &= 10/1 \\ &= 10 \text{ ohm} \end{aligned}$$

The Linear Proportion Between V & I

- The Ohm's law formula $V=IR$ states that V and I are directly proportional for any one value of R. This relation is true for constant values of R.

Volt-Ampere Characteristics

- The graph in figure is called the volt-ampere characteristic of R. It shows how much current the resistor allows for different voltages.



Power Dissipation in Resistance

- When current flows through a resistance, heat is produced due to friction between the moving free electrons and the atoms which obstruct the path of electron flow.
- The heat is evidence that power is used in producing current.
- The electric energy converted to heat is considered to be dissipated or used up because the calories of work can not be returned to the circuit as electric energy.
- Since power is dissipated in the resistance of a circuit, it is convenient to express the power in terms of resistance R.
- The formula $P=V \times I$ can be arranged as follows.
- Substituting IR for V,

$$\begin{aligned}P &= V \times I \\&= IR \times I \\&= I^2 R\end{aligned}$$

- This is the common form of the formula because of heat produced in a resistance due to current R.
For another form, substitute, V/R for I

$$\begin{aligned}P &= V \times I \\&= V \times V/R \\&= V^2/R\end{aligned}$$

In all the formulae, V is the voltage across R in ohms producing the current in amperes for power in Watts.

LECTURE 5
(AC, DC Current, voltages)

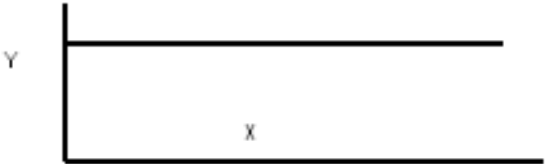
TYPES OF SOURCES:

IDEAL VOLTAGE SOURCE

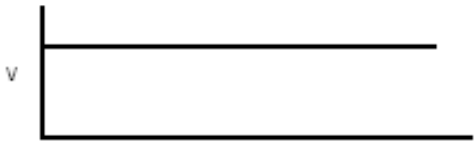
A source in which terminal voltage remains same independent of the amount of current drawn is called ideal voltage source.

GRAPHICAL REPRESENTATION

We can define this graph as y is independent of x, whatever the value of x, the value of y will remain same.

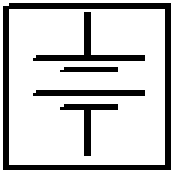


So, applying this circuit theory, the graph in figure can be defined as V is independent of I.



SYMBOLIC REPRESENTATION of DC VOLTAGE SOURCE

The dc voltage source can be symbolically represented as

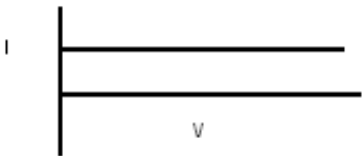


IDEAL CURRENT SOURCE

A source which gives constant current independent of the terminal voltage is called independent current source.

GRAPHICAL REPRESENTATION

This graph can be defined as I is independent of V.



DIRECT VOLTAGE

The voltage which is independent of time and its magnitude and direction do not change with time is called direct voltage.

DIRECT QUANTITIES

The quantities whose magnitude and direction do not change with time are called direct quantities for example may be V or I.

ALTERNATING QUANTITIES

The quantities whose magnitude and direction changes with respect to time are called alternating quantities. For example V or I.

DEPENDENT OR CONTROLLED SOURCES

- VOLTAGE**
- (1)Voltage controlled voltage source.
 - (2)Current controlled voltage source.

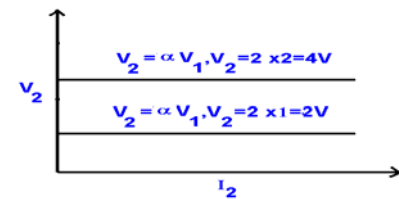
VOLTAGE CONTROLLED VOLTAGE SOURCE

The source whose magnitude is controlled by voltage is called voltage controlled voltage source.

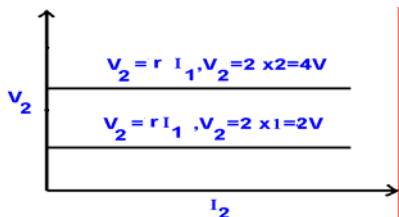
CURRENT CONTROLLED VOLTAGE SOURCE

Current controlled Voltage source can be defined as the source whose voltage is controlled by current is called current controlled voltage source.

VOLTAGE CONTROLLED VOLTAGE SOURCE

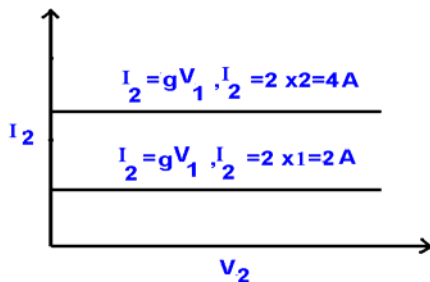


CURRENT CONTROLLED VOLTAGE SOURCE



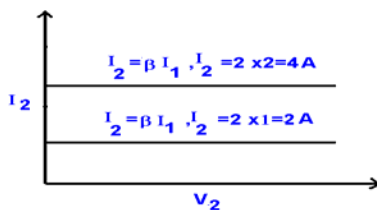
VOLTAGE CONTROLLED CURRENT SOURCE

If magnitude of current is controlled by input voltage then the source is called voltage controlled current source.



CURRENT CONTROLLED CURRENT SOURCE

If magnitude of the current is controlled by input current the source is called current controlled current source.



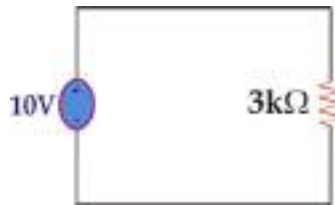
Example**Solution:**

Figure shows a battery of 10 V has been applied across a resistor of 3k, so by Ohm's law the current flowing through it, can be given as

$$\begin{aligned} I &= V/R \\ &= 10/3k \\ &= 3.33 \text{ mA} \end{aligned}$$

Example

Calculate the Current I through the circuit.

**Solution:**

2kΩ and 3kΩ are in series so, the combined effect will be $2k + 3k = 5k\Omega$
We want to calculate current through 5k Ω resistor.

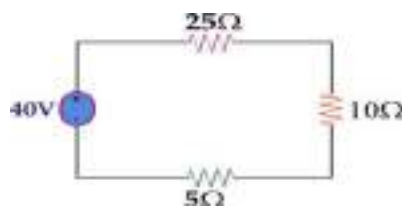


The current flowing through 5kΩ resistor, by Ohm's law

$$\begin{aligned} I &= V/R \\ &= 10/5k \\ &= 2 \text{ mA} \end{aligned}$$

Example

Calculate the Current I through the circuit.

**Solution:**

The resistors of 25,10, and 5 ohm are in series, so their combined effect = $25+10+5 = 40\Omega$



The current flowing through 40 ohm resistor by Ohm's law is

$$I = V/R = 40/40 = 1 \text{ A}$$

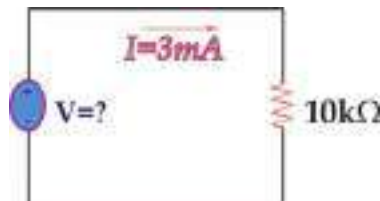
Example: Find R₁.



Solution:

The current flowing through the resistor R₁ is 5mA so by Ohm's law the value of R₁ is
 $R = V/I$
 $R_1 = 10V/5mA$
 $= 2k\Omega$

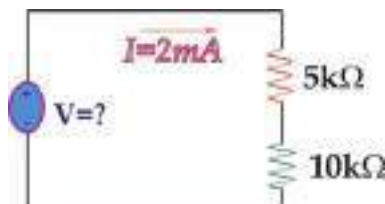
Example: Find V.



Solution:

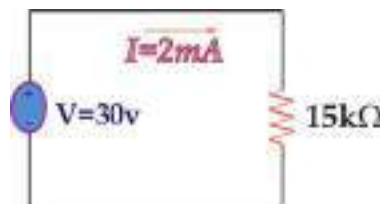
The current flowing through 10k resistor is 3mA so the voltage, by Ohm's law
 $V = IR$
 $= 10k \times 3m = 30V$

Example: Find V.



Solution:

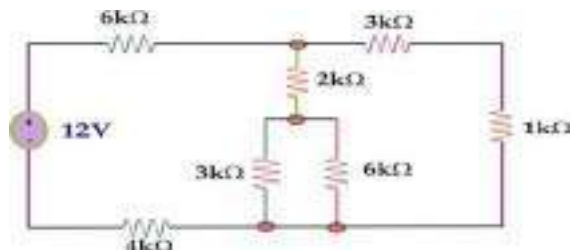
10k and 5k are in series so their combined effect will be $= 5k + 10k = 15k$
The current flowing through the circuit is 2mA.



So by Ohm's law, the voltage is

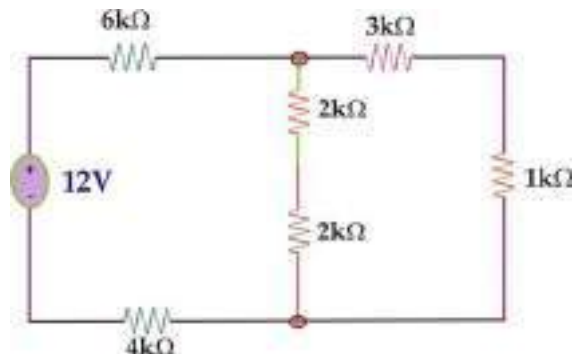
$$\begin{aligned} V &= IR \\ &= 2m \times 15k \\ &= 30V \end{aligned}$$

Example: Calculate the current through all the resistors in the circuit.

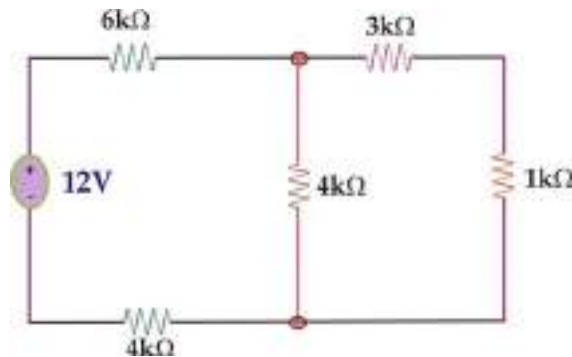


Solution:

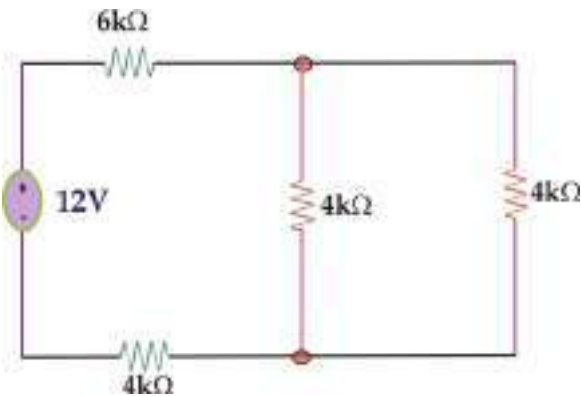
we need to simplify the circuit first.
 $3k \parallel 6k = (3 \times 6)/(3+6) = 18/9 = 2k \Omega$



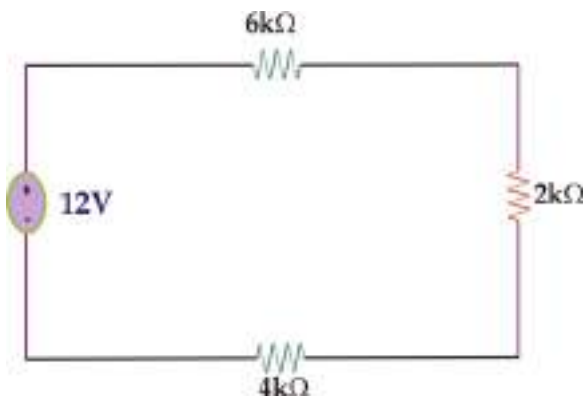
2k Ω is in series with 2k Ω so their combined effect = 2k+2k=4k Ω



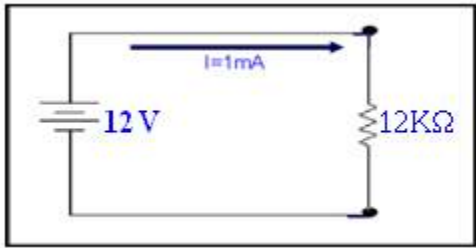
3k is in series with 1k so their combined effect= 3k+1k=4k



4k is in parallel with 4k, so
 $4k \parallel 4k = 16/8 = 2k$



The resistors 6kΩ, 2kΩ and 4kΩ are in series so they will be combined as=6k+2k+4k=12k Ω.

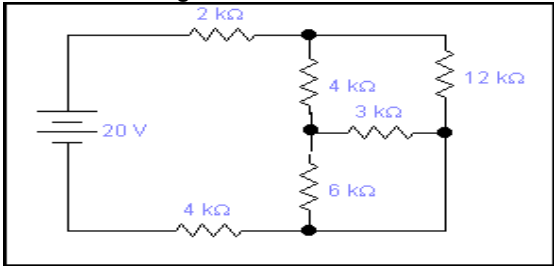


So the current flowing through all resistors by Ohm's law

$$=12/12k$$
$$= 1k^{-1} A$$
$$= 1mA$$

Example:

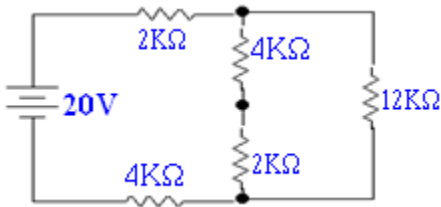
Find current through circuit.



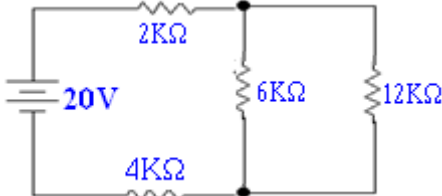
Solution:

3k Ω is in parallel with 6k Ω, therefore,

$$3k||6k = (3 \times 6)/(3 + 6)$$
$$= 18/9$$
$$=2k \Omega$$



4kΩ is series with 2kΩ so their combined effect = 2k + 4k = 6kΩ.



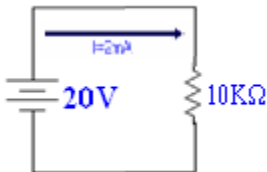
6k Ω is in parallel with 12k Ω so

$$6k||12k =(6 \times 12)/(12+6)$$
$$= 72/18$$
$$= 4k \Omega$$



The resistors 2kΩ,4kΩ, 4kΩ are in series so the combined effect will be
=2k+4k+4k
=10kΩ

So the current flowing through all the Resistors by Ohm's law will be
 $I=V/R$
 $=20/10k$
 $=2mA$



VOLTAGE DIVIDERS AND CURRENT DIVIDERS:

- Any series circuit is a voltage divider. The IR drops are proportional parts of the applied voltage.

VOLTAGE DIVIDERS AND CURRENT DIVIDERS:

- Special formulae can be used for voltage and current division as short cuts in calculations.
- The voltage division formula gives the series voltage even when the current is not known.

SERIES VOLTAGE DIVIDERS:

- The current is same in all resistances in a series circuit.
- Also, the voltage drop is equal to the product IR.
- Therefore, IR voltages are proportional to series resistances.
- A higher resistance has a greater IR voltage than a lower resistance in same series circuit; equal resistances have the same amount of IR drop across each resistance.
- If R_1 is double than R_2 then V_1 will be double than V_2 .
- The series string can be considered as a voltage divider.
- Each resistance provides an IR drop V equal to its proportional part of the applied voltage. Stated by formula,

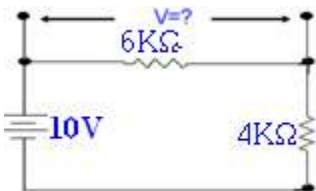
$$V=R/R_T \times V_T$$

Example: Calculate the voltage drop across 4kΩ resistor



Solution:

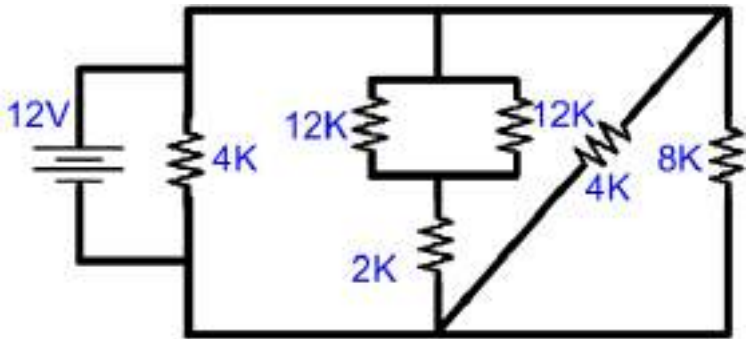
We want to calculate the voltage drop across 4kΩ resistor. So by voltage division rule $V = (4/10) \times 10$
=4 volts



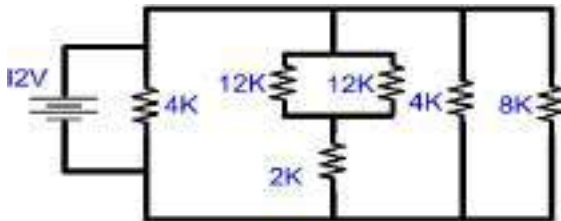
Now we want to calculate the voltage drop across 6k resistor so by formula

$$\begin{aligned} V &= (R/R_t) \times V_t \\ &= (6/10) \times 10 \\ &= 6 \text{ volts} \end{aligned}$$

Example: Calculate the power dissipated in the Circuit.

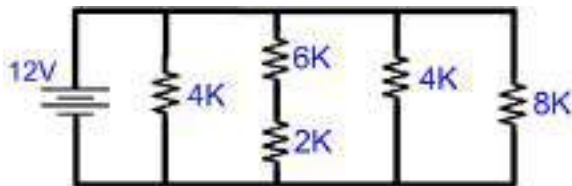


Solution: The circuit can be redrawn as:

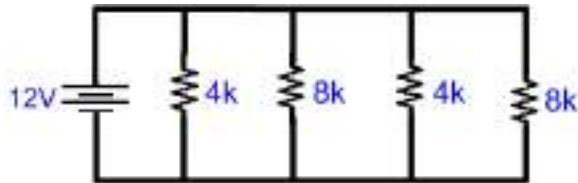


12KΩ resistor is in parallel with 12K. So their parallel combination will be $(12K \times 12K)/(12K + 12K) = 6K$

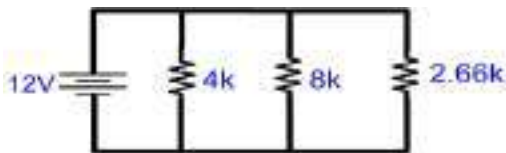
The circuit will become



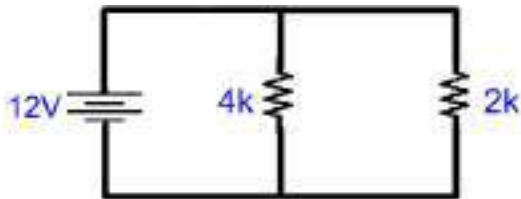
6k is in series with 2k. So there



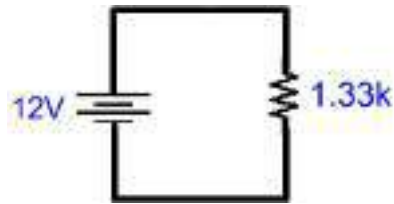
$8K || 4K = 2.66K$



$2.66K || 8K = 2K$



$$2K || 4K = 1.33K$$



So the current flowing through it

$$I = 12V / 1.33K$$

$$= 9.022 \text{ mA}$$

So the power dissipation in the circuit

$$P = VI$$

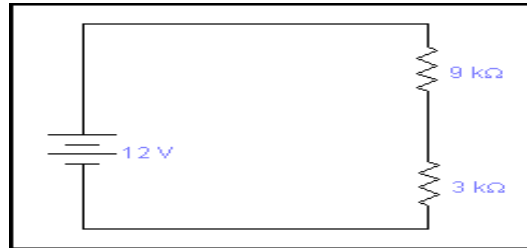
$$= 12 \times 9.022 \text{ m A}$$

$$= 108.26 \text{ m}$$

LECTURE 6

(Voltage divider, Current divider)

EXAMPLE: Find the voltage drop across each resistance.



Solution:

The voltage drop across 9k resistor
 $V_1 = (9/9+3) \times 12$
 $= 9/12 \times 12$
 $= 9 \text{ volts}$

The voltage drop across 3k resistor
 $V_2 = (3/12) \times 12$
 $= 3 \text{ volts.}$

EXAMPLE: Find the voltage drop across each resistance.



Solution:

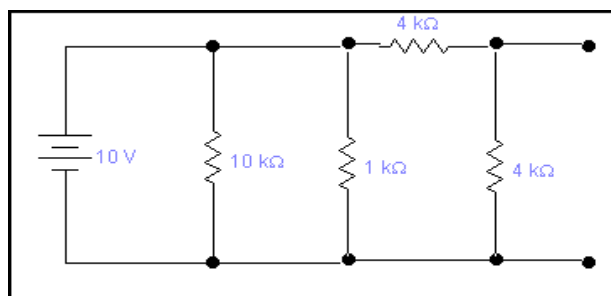
Three resistors are in series and we want to calculate the voltage drop across each resistor it will be calculated as

The voltage drop across 50 k resistor
 $V = R/R_t \times V_t$
 $= 50k/100k \times 200 = 100V$

The voltage drop across 30k resistor
 $V = 30/100 \times 200$
 $= 60 V$

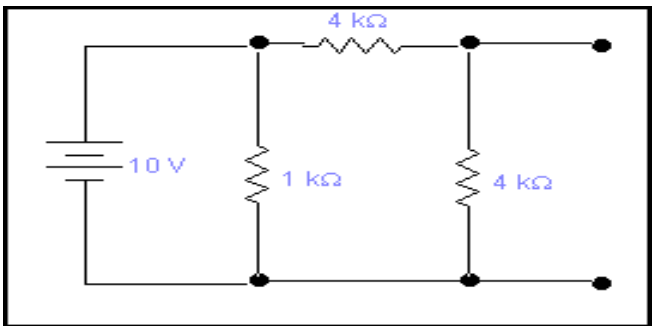
The voltage across 20k resistor
 $V = 20/100 \times 200$
 $= 40 \text{ volts}$

EXAMPLE: Find the voltage across each resistance.

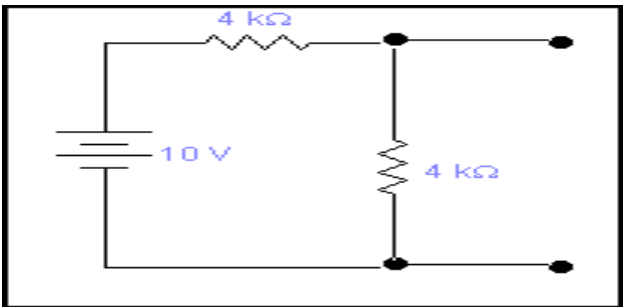


Solution:

As we studied in the last lecture, the same voltage appears across the parallel resistances, so the same 10V source voltage will appear across the 10k resistor.



Now the voltage source is in parallel with the 1 k resistor so the voltage will be same.



Now 4k is in series with 4k so we can
Apply voltage division rule
 $V = \frac{4}{8} \times 10 = 5$ volts

TWO VOLTAGE DROPS IN SERIES

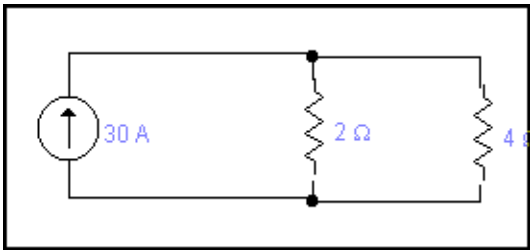
- For this case, it is not necessary to calculate both voltages. After finding one we can subtract it from V_t to find the other.
- As an example, assume V_t is 48V across two series resistances R_1 and R_2 . if V_1 is 18 volts then V_2 must be $48 - 18 = 30$ volts

CURRENT DIVIDER WITH TWO PRALLEL RESISTANCES

- It is often necessary to find individual branch currents in a circuit but without knowing the value of branch voltage.
- This problem can be solved by using the fact that currents divide inversely as branch resistance. The formula is

$$I_1 = I_s \times \frac{R_2}{(R_1 + R_2)}$$

EXAMPLE: Find the current through each resistance.

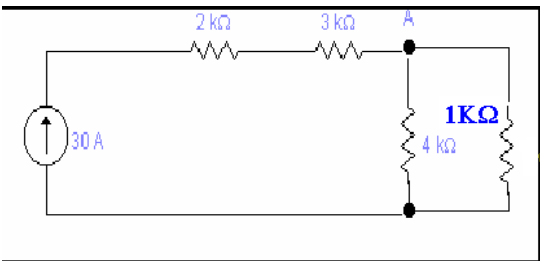


Solution:

The current flowing through 4 ohm Resistor will be
 $I = \frac{2}{2+4} \times 30$
 $= \frac{2}{6} \times 30$
 $= 10A$

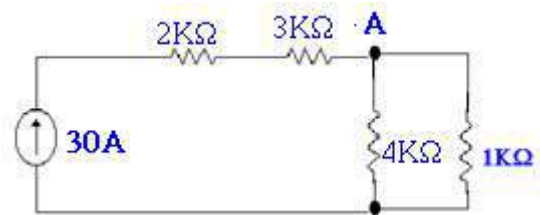
The current flowing through 2 ohm resistor
 $I = \frac{4}{2+4} \times 30$
 $= \frac{4}{6} \times 30$
 $= 20 A$

EXAMPLE: Find the current through each resistance.



Solution:

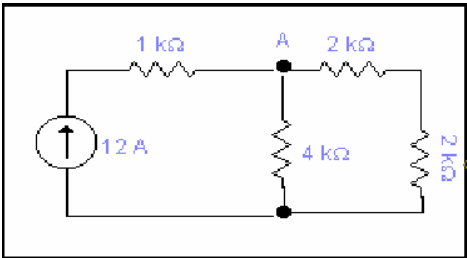
The same 30 A current is flowing through series combination 2k and 3k so they are leaving no effect on the value of current. Current divides at node A into two parts.



Now by current division rule the current flowing through 1k resistor

$$\begin{aligned} I &= \frac{4}{4+1} \times 30 \\ &= \frac{4}{5} \times 30 \\ I &= 24A \end{aligned}$$

EXAMPLE: Calculate the current through 4k Ω resistance.

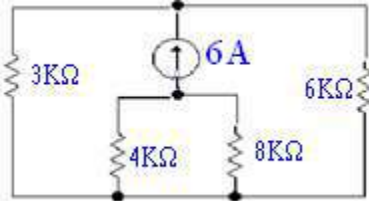


Solution:

We want to calculate the current through 4k resistor. Current will divide in two parts at Node A. 2k Ω is in series with 2k Ω so the current flowing through 4k resistor is

$$\begin{aligned} I &= \frac{4}{4+4} \times 12 \\ &= \frac{4}{8} \times 12 \\ I &= 6A \end{aligned}$$

EXAMPLE: Calculate the current through 8k Ω resistance.

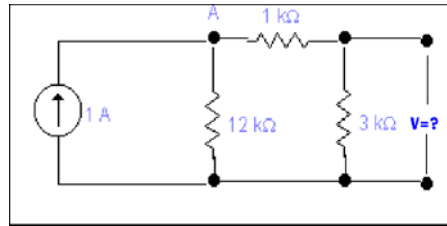


Solution:

We want to calculate the current through 8k resistor. Now if we take the direction of current source downward the value of it will become -6A. Now the current flowing through 8k resistor can be calculated as

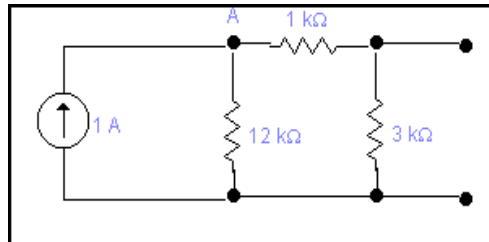
$$I = \frac{4}{12} \times -6 = -2A$$

EXAMPLE: Calculate the voltage across $3\text{ k}\Omega$ resistance.



Solution:

We want to calculate the voltage across $3\text{ k}\Omega$ resistor. At point A the current divides into two parts, one through $12\text{ k}\Omega$ resistor and other through series combination of $3\text{ k}\Omega$ and $1\text{ k}\Omega$ resistors.



So the current flowing through series combination of $1\text{ k}\Omega$ and $3\text{ k}\Omega$ can be calculated as

$$I = \frac{12}{4+12} \times 1$$

$$= \frac{12}{16} \times 1$$

$$I = 0.75 \text{ A}$$

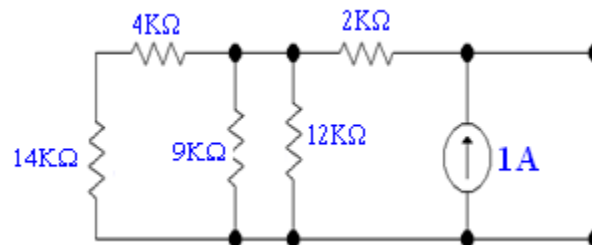
Same current is flowing through the series combination of two resistors so the voltage across $3\text{ k}\Omega$ resistor will be

$$V = IR$$

$$= 0.75 \times 3\text{ k}$$

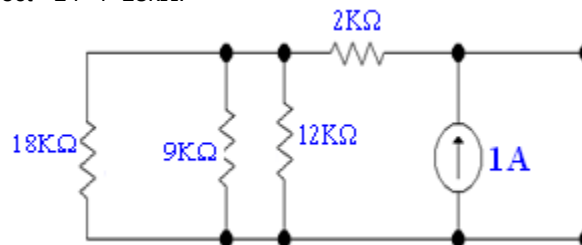
$$V = 2250 \text{ volts}$$

Example: Find V.

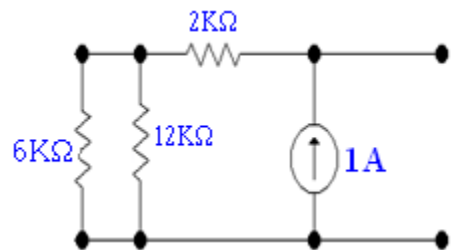


Solution:

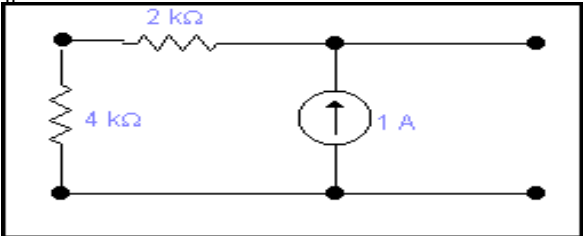
We want to calculate the voltage across entire circuit. $4\text{ k}\Omega$ resistor and $14\text{ k}\Omega$ resistor are in series so their combined effect $= 14 + 4 = 18\text{ k}\Omega$.



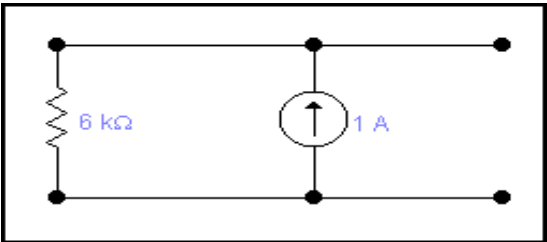
$18\text{ k}\Omega$ resistor and $9\text{ k}\Omega$ resistor are parallel
 $18\text{ k} || 9\text{ k} = 18 \times 9 / 27 = 6\text{ k}\Omega$



6k resistor is in parallel with 12kΩ resistor
 $6k \parallel 12k = \frac{6 \times 12}{18} = 4k\Omega$

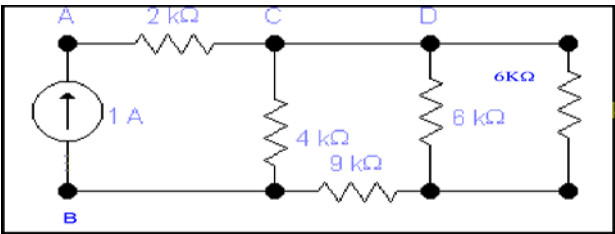


2kΩ resistor is in series with 4kΩ resistor so the total resistance will be
 $= 2k + 4k = 6k\Omega$



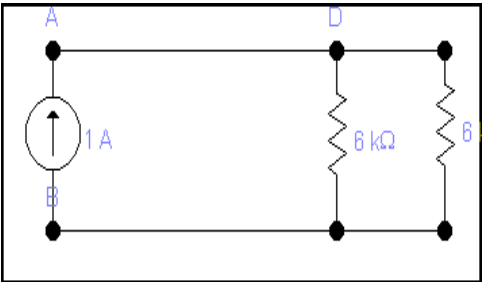
So the voltage drop across the equivalent 6kΩ resistor
 $V = IR$
 $= 6k \times 1$
 $= 6000\text{volts.}$

Example: Find current through 6k ohm resistance.



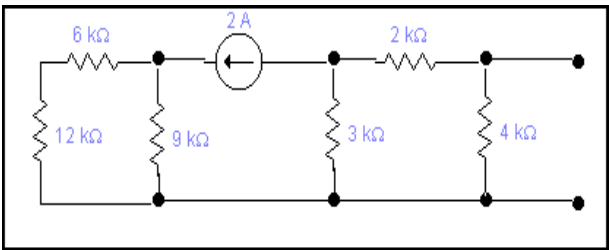
Solution:

At point c the current should divide into two parts but due to short circuit between c and D whole current will come at point D.

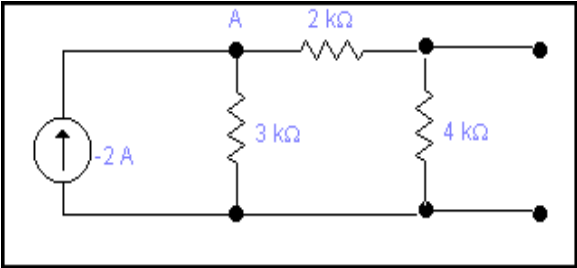


Now by current division rule the required current will be
 $I = (6k/12k) \times 1$
 $= 0.5A$

Example Calculate the voltage across 4kΩ resistor.

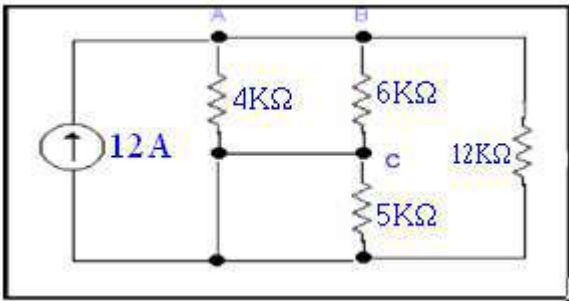


Solution: We want to calculate the voltage across 4kΩ resistor if we take the direction of current source negative our circuit will become



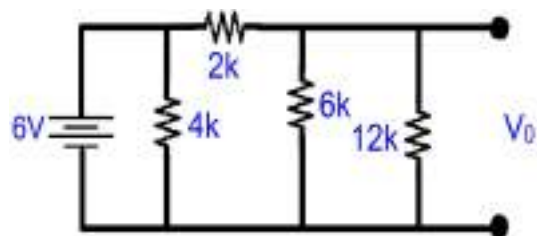
The current is dividing between 3kΩ and series combination of 2kΩ and 4kΩ so by current division rule
 $I = (3k/(3k + 6k)) \times (-2)$
 $= (3k/9k) \times (-2)$
 $= -0.66 A$
so the voltage across the 4kΩ resistor
 $V = IR$
 $= 0.66 \times 4k$
 $= 2640 \text{ volts}$

EXAMPLE: Calculate the value of current through 5k Ω resistance.

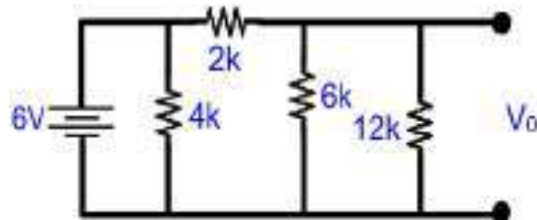


Solution: We want to calculate the value of current though 5kΩ resistor.
Look closely at the circuit. Resistor 5KΩ has zero resistance in parallel, so current arriving at node C, will take the zero resistance path and no current will pass through 5KΩ,

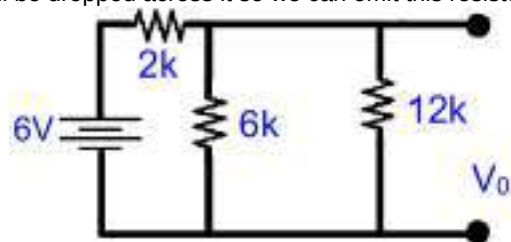
EXAMPLE: Calculate the voltage across 12k resistor.



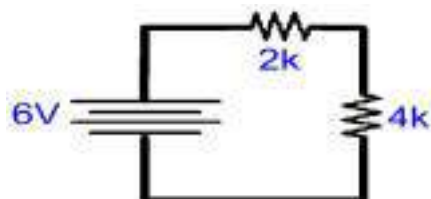
Solution:



We want to calculate voltage across 12k resistor using voltage division rule. 4k resistor is in parallel with 6V so same voltage will be dropped across it so we can omit this resistor from this circuit.



$$12\text{ k} \parallel 6\text{ k} = \frac{12\text{ k} \times 6\text{ k}}{12\text{ k} + 6\text{ k}} = 4\text{ k}$$

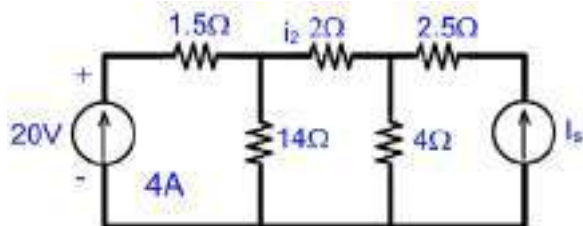


Now applying Voltage division rule

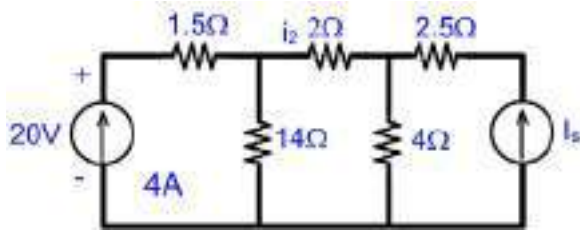
$$V_{4\text{k}} = \frac{4\cancel{\text{k}}}{\cancel{6\text{k}}} \times \cancel{6\text{V}} = 4\text{V}$$

Since 4k is the parallel combination of 6k and 12k. So the same voltage will be across 12k.
So $V_0 = 4\text{volts}$.

EXAMPLE: Find power absorbed by each element.



Solution: Beginning from the left we find



$$P_{20V} = -(20)(4) = -80$$

$$V_{1.5} = 4(1.5) = 6V$$

Therefore,

$$p_{1.5} = (v_{1.5})^2 / 1.5 = 24W$$

$$V_{14} = 20 - V_{1.5} = 20 - 6 = 14V$$

Therefore,

$$p_{14} = 14^2 / 14 = 14W$$

$$i_2 = V_2 / 2 = V_{1.5} / 1.5 - V_{14} / 14$$

$$= 6 / 1.5 - 14 / 14$$

$$i_2 = 3A$$

Therefore,

$$V_2 = 2(3) = 6V$$

and

$$p_2 = 6^2 / 2 = 18W$$

$$V_4 = V_{14} - V_2$$

$$= 14 - 6$$

$$V_4 = 8V$$

Therefore

$$p_4 = 8^2 / 4 = 16W$$

$$i_{2.5} = V_{2.5} / 2.5$$

$$= V_2 / 2 - V_4 / 4$$

$$= 3 - 2$$

$$i_{2.5} = 1A$$

$$V_{2.5} = (2.5)(1)$$

$$= 2.5V$$

and so

$$p_{2.5} = (2.5)^2 / 2.5$$

$$p_{2.5} = 2.5W$$

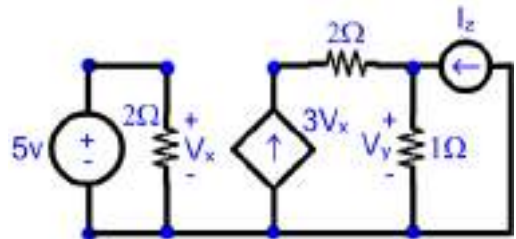
$$I_{2.5} = -I_s$$

Therefore $I_s = -1A$

LECTURE 7
(Kirchhoff's Laws Kirchhoff's Current Law (KCL))

Example:

- (a) Calculate v_y if $i_z = -3A$.
(b) What voltage would need to replace the 5v source to obtain $v_y = -6v$ if $i_z = 0.5A$

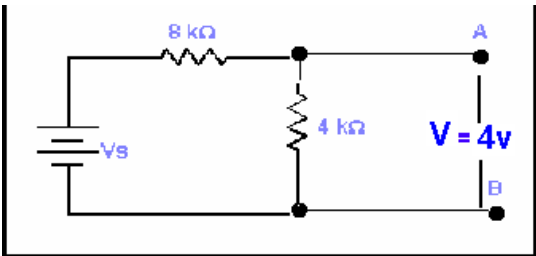


Solution:

- (a) $V_y = 1(3V_x + i_z)$
 $v_x = 5V$ and given that $i_z = -3A$,
We find that
 $v_y = 3(5) - 3 = 12 V$
(b) $V_y = 1(3V_x + i_z) = -6 = 3v_x + 0.5$
Solving, we find that
 $V_x = (-6 - 0.5) / 3 = -2.167V$

EXAMPLE:

Calculate the source voltage V_s , while the voltage between node A and B is 4V.

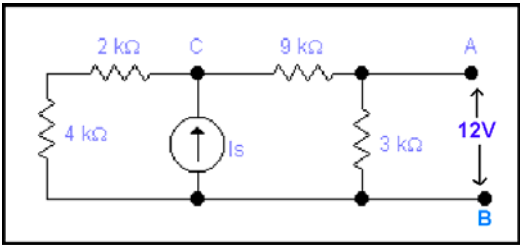


Solution:

The voltage between node A and B is 4V the source voltage can be calculated as
 $V = (R/R_t) V_s$
 $4 = (4/12) V_s$
 $V_s = (12 \times 4)/4$
 $= 12V$

EXAMPLE:

Calculate the source current I_s .

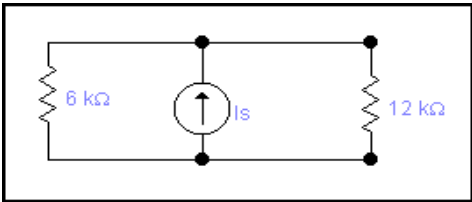


Solution:

We want to calculate the source current. The voltage across 3kΩ resistor is 12V. So the current (I) flowing through it, will be

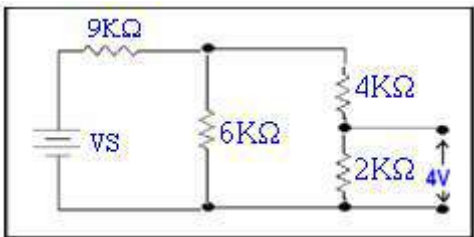
$$\begin{aligned} I &= V/R \\ &= 12/3k \\ &= 4mA \end{aligned}$$

The same current is following through the series combination of 3kΩ resistor and 9kΩ resistor. 3kΩ is in series with 9kΩ and 2kΩ is in series with 4kΩ, so



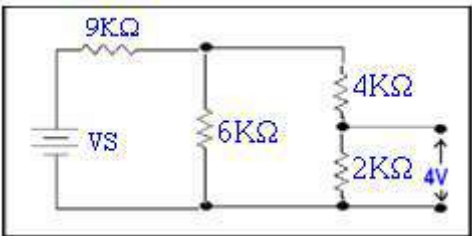
Now by current division rule the source current will be
 $I = (6k/(12k + 6k)) I_s$
 $I_s = (18k/6k) 4mA$
 $= 12mA$

EXAMPLE: Calculate the source voltage Vs.



Solution: We want to calculate the source voltage. The voltage across 2kΩ resistor is 4V, so the current flowing through it will be

$$I = V/R$$
$$= (4/2k)$$
$$= 2mA$$

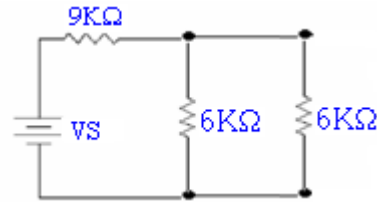


The same current is following through the series combination of 2kΩ and 4kΩ. So the voltage across 4kΩ resistor will be

$$V = IR$$
$$= (2m) \times (4k)$$
$$= 8 \text{ Volts}$$

So the total voltage across 2kΩ and 4kΩ resistor will be
 $V = 4V + 8V$
 $= 12V$

4kΩ is in series with 2kΩ these may be combined as
 $= 4k + 2k = 6kΩ$

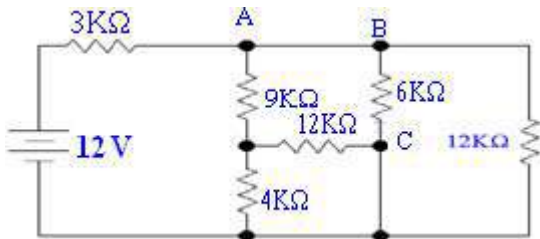


6kΩ is parallel with 6kΩ resistor so
 $6k || 6k = (6 \times 6)/(6 + 6) = 3kΩ$



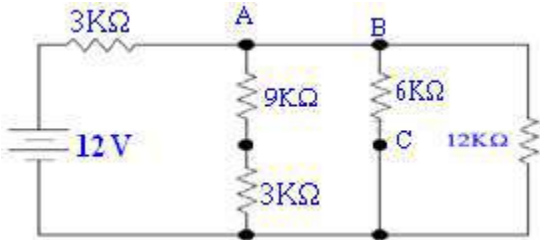
The voltage across 3kΩ resistor is 12V so the source voltage is
 $V = (3/(3+9)) V_s$
 $V_s = 12 (12/3)$
 $= 48V$

EXAMPLE: Calculate the voltage across 4kΩ resistor.

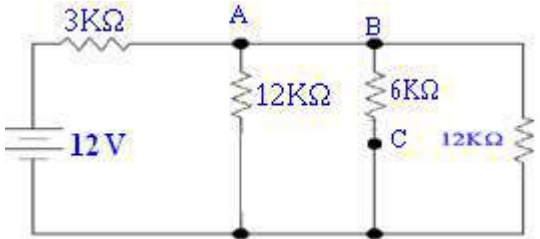


Solution:

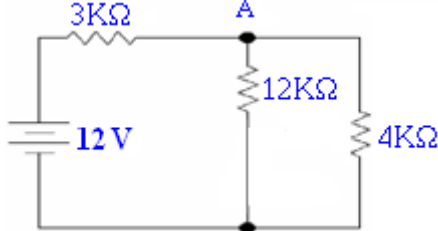
We want to calculate the voltage across 4kΩ resistor.
 $12k||4k = (12 \times 4)/(12+4)$
 $= 3k\Omega$



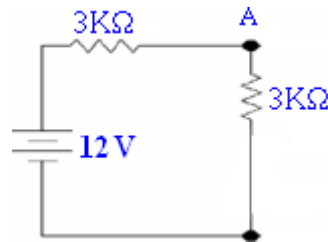
9kΩ is in series with 3kΩ their combined effect will be = 9k + 3k
 $= 12k\Omega$



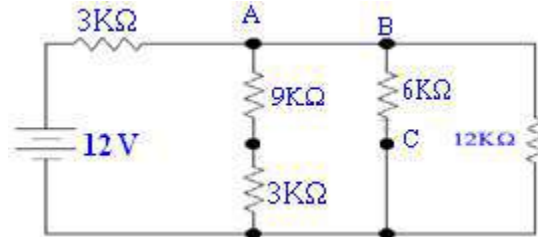
12kΩ is parallel with 6kΩ resistor so
 $12k||6k = (12 \times 6)/18$
 $= 72/18$
 $= 4k\Omega$



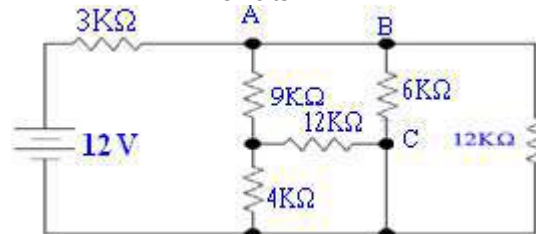
12kΩ is parallel with 4kΩ resistor so
 $12k||4k = (12 \times 4)/(12+4)$
 $= 48/16$
 $= 3k\Omega$



So by voltage division rule, the voltage across the equivalent $3k\Omega$ resistor is
 $V = (3/6) \times 12$
 $= 6$



So the voltage across $3k\Omega$ resistor is
 $V = (3/12) \times 6$
 $= 1.5 \text{ volts}$



The same voltage will be drop across $4k\Omega$ resistor.

KIRCHHOF'S LAW

KIRCHHOF'S CURRENT LAW

Sum of all the currents entering in the node is equal to sum of currents leaving the node.
 It can also be defined as

$$\text{Sum of entering currents} + \text{sum of leaving currents} = 0$$

ASSUMPTIONS

All the entering currents are taken as negative.
 All the leaving currents are taken as positive.

NODE

It is the junction of two or more than two elements
 OR
 It is simply a point of connection between circuit elements.

BRANCH

It is the distance or link between two nodes.

LOOP

It is the closed path for the flow of current in which no node is encountered more than once.
 Let's take some examples of node analysis or Kirchhof's current law.

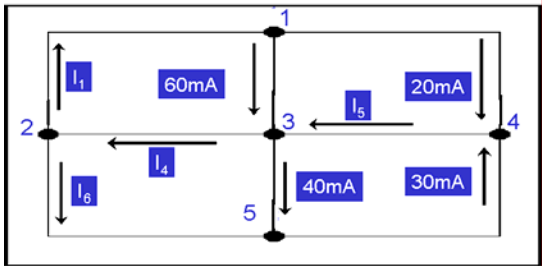
Formula for Writing Equation

Number of equations in node analysis is one minus than the number of total nodes.
 Number of equations = $N - 1$
 Where N is the number of nodes.

Ground

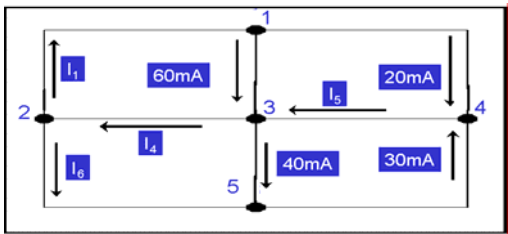
This is a common or reference point among all the nodes without insertion of any component between.

Example: Find the value of I_1, I_2, I_3, I_4 .



Solution:

Assuming the currents leaving the node are positive, the KCL equations for node 1 through 4 are

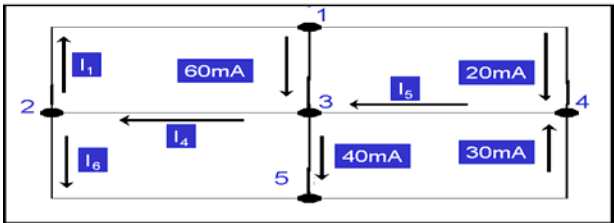


For node 1

$$-I_1 + 0.06 + 0.02 = 0$$

$$-I_1 + 0.08 = 0$$

$$I_1 = 0.08 \text{ A}$$

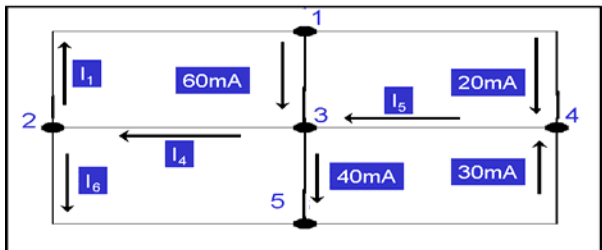


For node 2

$$I_1 - I_4 + I_6 = 0$$

$$0.08 - I_4 + I_6 = 0$$

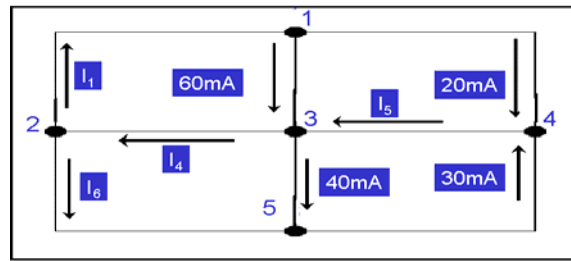
$$-I_4 + I_6 = -0.08 \text{ A}$$



For node 3

$$-0.06 + I_4 - I_5 + 0.04 = 0$$

$$I_4 - I_5 = 0.02 \text{ A}$$



For node 4

$$-0.02 + \frac{1}{5}(-0.03) = 0$$

$$I_5 = 0.05 \text{ A}$$

Now putting the value of I_5 in equation of node 3

$$I_4 - 0.05 = 0.02$$

$$I_4 = 0.07 \text{ A}$$

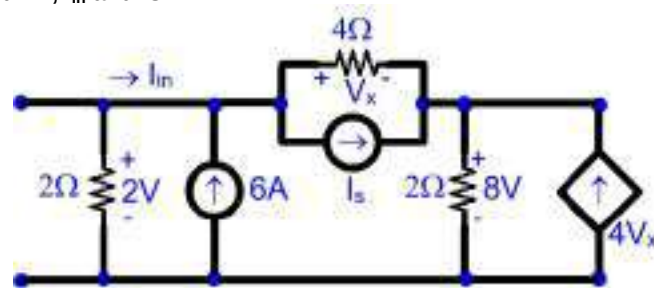
Putting the value of I_6 in the equation for node 2

$$-0.07 + I_6 = -0.08$$

$$I_6 = -0.01 \text{ A}$$

EXAMPLE:

Find V_x , I_{in} and I_s .



Solution:

(a) By KVL $-2 + V_x + 8 = 0$

So that $V_X = -6V$

(b) By KCL at the top right node

$$I_s + 4V_x = 4 - V_x / 4$$

So

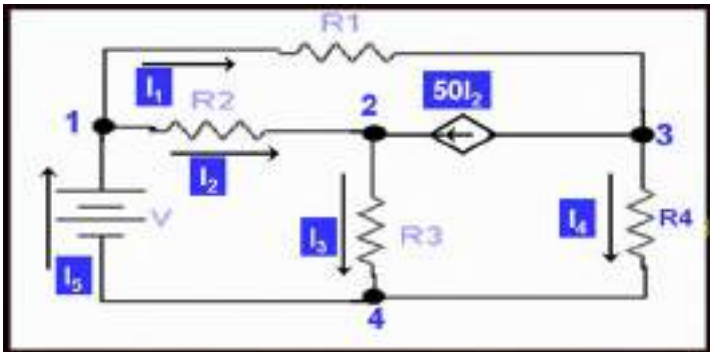
$I_s = 29.5A$

$$I_{lin} = 1 + I_s + V_x / 4 - 6$$

or $I_{in} = 23A$

LECTURE 8
(Application of Nodal Analysis)

Example: Write KCL equations for all nodes.



Solution:

The KCL equations for nodes 1 through 4 follow

For node 1

$$I_1 + I_2 - I_5 = 0$$

For node 2

$$-I_2 + I_3 - 50I_2 = 0$$

For node 3

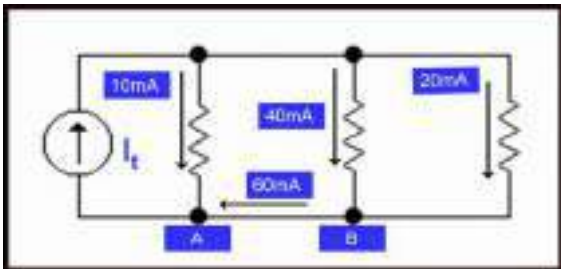
$$-I_1 + 50I_2 + I_4 = 0$$

For node 4

$$I_5 - I_3 - I_4 = 0$$

Example

Write KCL equations for all nodes.



Solution:

For node A 10mA is entering the node and the source current I_t is leaving and 60mA is also entering
so,

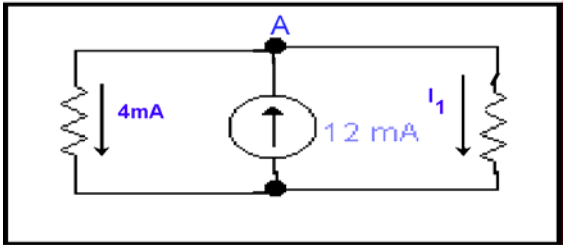
For node A

$$I_t - 60\text{mA} - 10\text{mA} = 0$$

For node B

$$60\text{mA} - 40\text{mA} - 20\text{mA} = 0$$

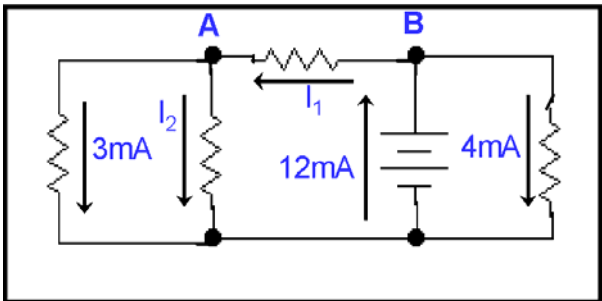
Example Write KCL equation for node A.



Solution:

By KCL the equation for **node A**
 $-12\text{mA} + 4\text{mA} + I = 0$

Example Write KCL equation for node A and node B.



Solution:

The equation of currents at node A and B by KCL

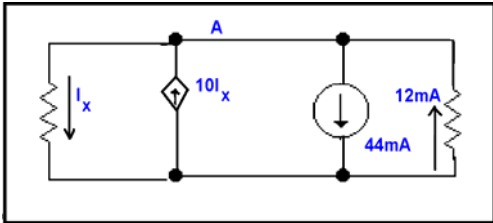
For node A

$$-I_1 + I_2 + 3\text{mA} = 0$$

For node B

$$-12\text{mA} + 4\text{mA} + I_1 = 0$$

Example Write KCL equation for node A .

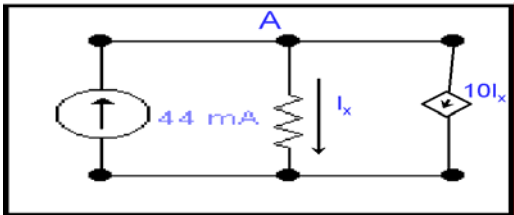


Writing equation for node A .

For node A

$$-10 I_x + I_x + 44 \text{ mA} - 12 \text{ mA} = 0$$

Example Write KCL equation for node A .

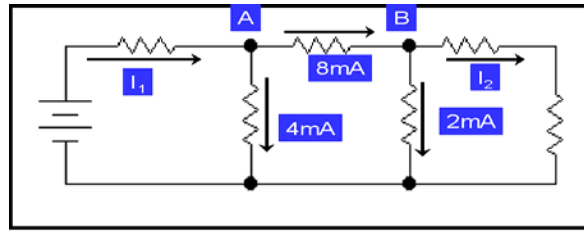


Solution:

For node A equation will be by KCL

$$I_x + 10I_x - 44\text{mA} = 0$$

Example Calculate the values of I_1 and I_2 .



Solution:

We want to calculate the values of I_1 and I_2

For node A

$$4\text{mA} + 8\text{mA} - I_1 = 0$$

$$I_1 = 12\text{mA}$$

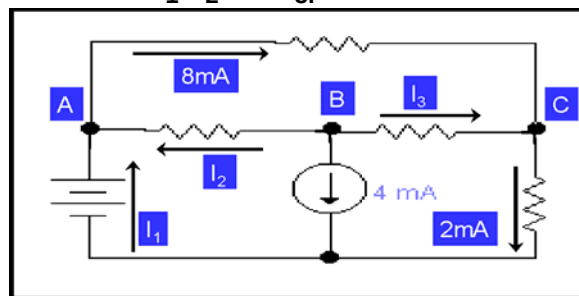
Now for I_2

For node B

$$-8\text{mA} + 2\text{mA} + I_2 = 0$$

$$I_2 = 6\text{mA}$$

Example Calculate the values of I_1 , I_2 , and I_3 .



Solution:

We want to calculate the values of I_1 , I_2 , and I_3 , so we will use node analysis.

For node A

$$-I_1 - I_2 + 8\text{mA} = 0$$

$$-I_1 - I_2 = -8\text{mA}$$

For node B

$$I_2 + I_3 + 4\text{mA} = 0$$

$$I_2 + I_3 = -4\text{mA}$$

For node C

$$-I_3 + 2\text{mA} - 8\text{mA} = 0$$

$$I_3 = -6\text{mA}$$

Putting the value of I_3 in equation of **node B**

$$I_2 - 6\text{mA} = -4\text{mA}$$

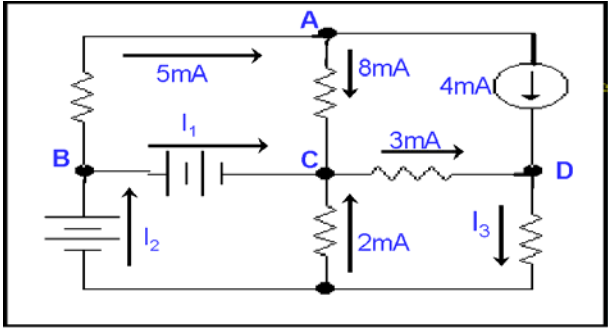
$$I_2 = 2\text{mA}$$

Putting the value of I_2 in equation of **node A**

$$-I_1 - 2\text{mA} = -8\text{mA}$$

$$I_1 = 6\text{mA}$$

Example: Find the KCL equations for node A, node B, node C and node D.



Solution:

We want to write the equations for nodes A, B, C and D.

For node A

$$-5\text{mA} + 8\text{mA} + 4\text{mA} = 0$$

For node B

$$I_1 - I_2 + 5\text{mA} = 0$$

For node C

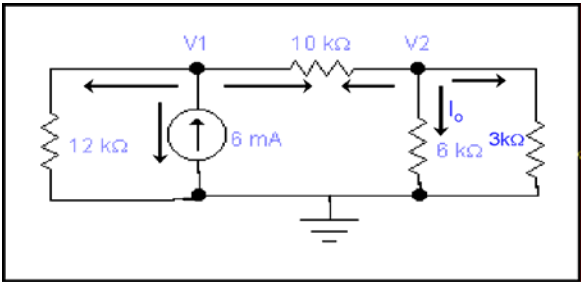
$$-I_1 - 2\text{mA} + 3\text{mA} - 8\text{mA} = 0$$

For node D

$$-4\text{mA} - 3\text{mA} + I_3 = 0$$

Example

Calculate the current I_o .



Solution:

At node 1

$$\begin{aligned} (V_1/12\text{k}) + ((V_1 - V_2)/10\text{k}) &= 6\text{mA} \\ 5V_1 + 6V_1 - 6V_2 &= (60\text{k})(6\text{mA}) \\ 11V_1 - 6V_2 &= 360 \end{aligned}$$

At node 2

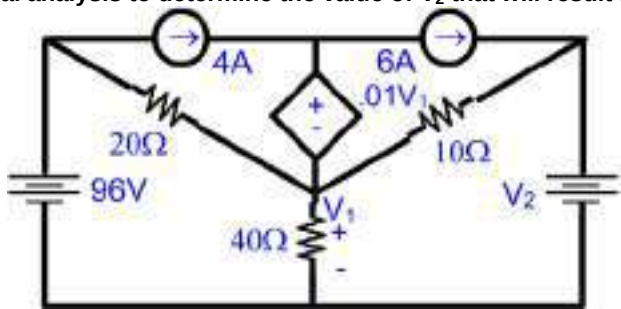
$$\begin{aligned} (V_2/3\text{k}) + (V_2/6\text{k}) + ((V_2 - V_1)/10\text{k}) &= 0 \\ 10V_2 + 5V_2 + 3V_2 - 3V_1 &= 0 \\ 18V_2 - 3V_1 &= 0 \end{aligned}$$

Equating equation of **node1** and **node 2**

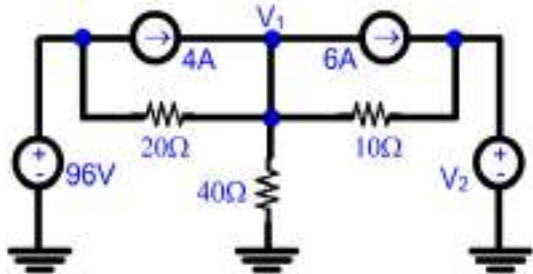
$$\begin{aligned} 33V_1 - 18V_2 &= 1080 \\ -3V_1 + 18V_2 &= 0 \\ \hline 30V_1 &= 1080 \\ V_1 &= 36 \text{ volts} \\ 18V_2 - 3V_1 &= 0 \\ 6V_2 - V_1 &= 0 \\ 6V_2 - 36 &= 0 \end{aligned}$$

$$\begin{aligned} V_2 &= 36/6 \\ V_2 &= 6 \text{ volts} \\ I_o &= V_2/6\text{k} \\ &= 6/6\text{k} \\ &= 1\text{mA} \end{aligned}$$

Example: Use nodal analysis to determine the value of V_2 that will result in $V_1 = 0$

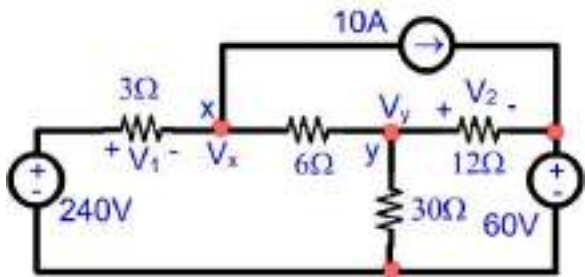


Solution: If $V_1 = 0$, the dependent source is a short circuit and we may redraw the circuit as :



At NODE 1 : $4 - 6 = V_1/40 + (V_1 - 96)/20 + (V_1 - V_2)/10$
Since $V_1 = 0$, this simplifies to
 $-2 = -96/20 - V_2/10$
So that $V_2 = -28V$

Example Use nodal analysis to find V_1 and V_2 , compute the power absorbed by 6Ω resistance.

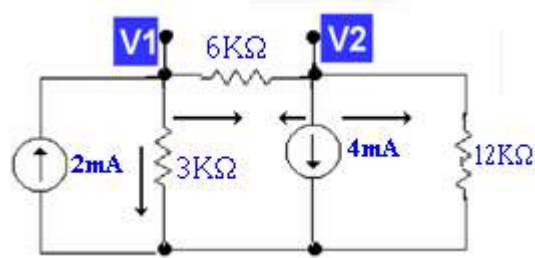


Solution:
Designate the node between the 3Ω and 6Ω resistors as node X, and the right hand node of the 6Ω resistor node Y. The bottom node is chosen as the reference node.
(a) Writing the two nodal equation, then

NODE X :
 $-10 = (V_x - 240)/3 + (V_x - V_y)/6$ [1]
NODE Y :
 $0 = (V_y - V_x)/6 + V_y/30 + (V_y - 60)/12$ [2]
Simplifying, $-180 + 1440 = 9V_x - 3V_y$ [1]
 $10800 = -360V_x + 612 V_y$ [2]
Solving $V_x = 181.5V$ and $V_y = 124.4V$
Thus, $V_1 = 240 - V_x = 58.50V$
and $V_2 = V_y - 60 = 64.40V$
(b) The power absorbed by the 6Ω resistor is
 $(V_x - V_y)^2 / 6 = 543.4 W$

LECTURE 9
(Application of Nodal Analysis)

Example Find current flowing through 6k ohm resistor.



Solution:

At node 1

$$\begin{aligned} ((V_1 - V_2)/6k) + (V_1/3k) &= 2mA \\ V_1 - V_2 + 2V_1 &= 6k \times 2mA \\ -V_2 + 3V_1 &= 12 \\ 9V_1 - 3V_2 &= 36 \quad \dots\dots\dots(A) \end{aligned}$$

At node 2

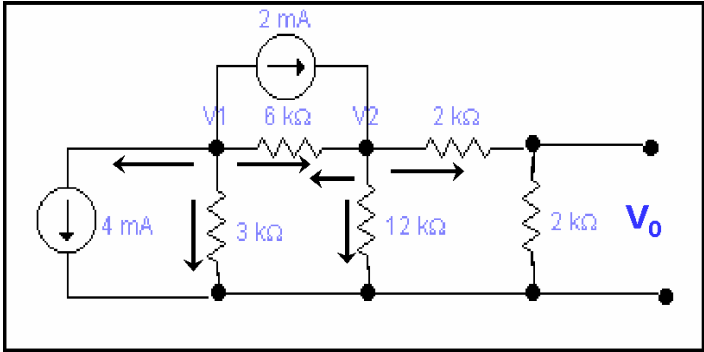
$$\begin{aligned} V_2/12k + 4mA + (V_2 - V_1)/6k &= 0 \\ V_2 + 48 + 2V_2 - 2V_1 &= 0 \\ 3V_2 - 2V_1 &= -48 \quad \dots\dots\dots(B) \end{aligned}$$

Adding equation (A) & (B)

$$\begin{array}{r} 3V_2 - 2V_1 = -48 \\ -3V_2 + 9V_1 = 36 \\ \hline \end{array}$$

$$\begin{aligned} V_1 &= -12/7 \\ \text{Now putting the value of } V_1 \text{ in (A)} \\ V_2 &= 3V_1 - 12 \\ &= 3 \times (-12/7) - 12 \\ &= (-36/7) - 12 \\ &= (-36 - 84)/7 \\ V_2 &= -120/7 \\ V_0 &= V_2 - V_1 \\ &= (-120/7) + 12/7 \\ &= -108/7 \\ I_0 &= -108/7 \times (1/6k) \\ &= -108/42 \\ &= 2.57 \text{ mA} \end{aligned}$$

Example Find voltage across 2k ohm resistor.



Solution:

Combine 2k and 2k and then

At node (1)

$$\begin{aligned} V_1/3k + 4mA + 2mA + (V_1-V_2)/6k &= 0 \\ 2V_1 + 24 + 12 + V_1 - V_2 &= 0 \\ 3V_1 - V_2 &= -36 \end{aligned}$$

At node (2)

$$\begin{aligned} V_2/4k + V_2/12k - 2mA + (V_2-V_1)/6k &= 0 \\ 3V_2 + V_2 - 24 + 2V_2 - 2V_1 &= 0 \\ 6V_2 - 2V_1 &= 24 \end{aligned}$$

$$\begin{aligned} 9V_2 - 3V_1 &= 36 \\ -V_2 + 3V_1 &= -36 \end{aligned}$$

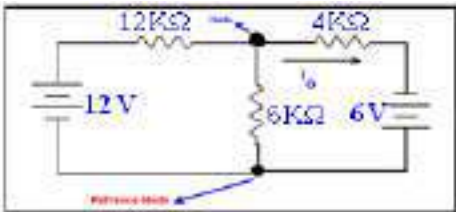
$$8V_2 = 0$$

$$V_2 = 0$$

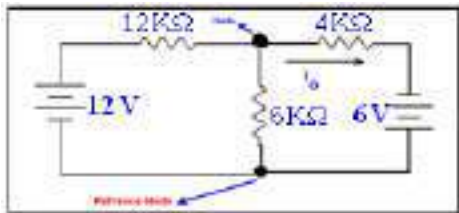
you can see from the circuit on the right side two resistance are in series i.e 2k and 2k ohm so we will apply voltage divider rule here

$$\begin{aligned} V_0 &= 2/4 \times V_2 \\ &= 2/4 \times 0 \\ &= 0 \end{aligned}$$

Example : Find current flowing through 4k ohm resistor.



Solution:



At node 1

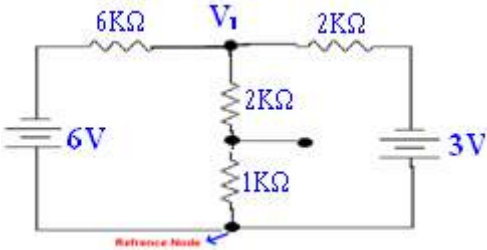
$$\begin{aligned} (V_1-12)/12k + V_1/6k + (V_1+6)/4k &= 0 \\ V_1 - 12 + 3V_1 + 3(6) + 2V_1 &= 0 \\ 6V_1 &= -6 \\ V_1 &= -6/6 \\ &= -1 \text{ volts} \end{aligned}$$

$$I_0 = (V_1 - (-6))/4k$$

Here negative sign with -6 is due to the negative reference of the battery ,so

$$I_o = (-1+6)/4k$$
$$= 5/4$$
$$I_o = 1.2mA$$

Example : Find voltage across 1k ohm resistor.

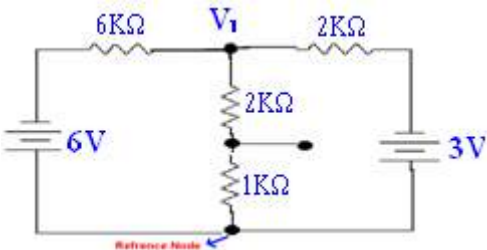


Solution:

As we see in the circuit 2k and 1k ohm resistors are in series and the same current will pass through them so we will combine them and they will become 3k ohm
So,

At node 1

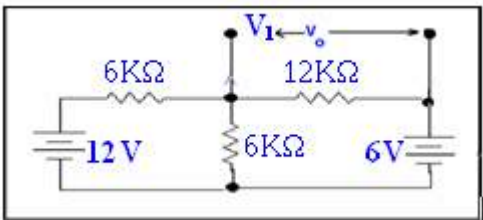
$$(V_1 + 6)/6k + (V_1+3)/2k + V_1/3k = 0$$
$$6 + V_1+9 +2V_1+3V_1 = 0$$
$$6V_1+15 = 0$$
$$V_1 = - 15/6 \text{ volts}$$



By Voltage divider rule at V₀

$$V_0 = 1/3 \times -15/6$$
$$= -5/6 \text{ volts}$$

Example : Find the voltage across a 12K ohm resistance. When we don't know the value of the current in the circuit.



Solution:

In this circuit we have three nodes one is reference node while the other node is between the 6v battery and the 12K resistance here we will neglect this node and calculate the voltage at point A so the node equation for the node V₁ is

At node (1)

$$(V_1 - 12)/6k + (V_1 -6)/12k + V_1/6k = 0$$
$$5V_1 = 30$$
$$V_1 = 6\text{volts}$$

As we know that we want to calculate the voltage across 12K resistor and it is between 6V battery and V₁

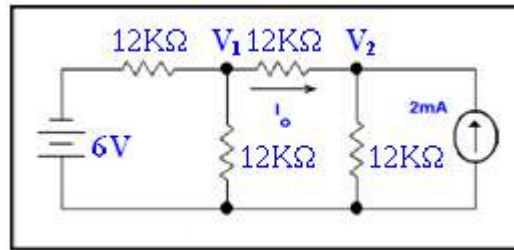
So,

$$V_0 = V_1 - V_s$$

In the above equation V_s =6V

$$V_0 = V_1 -6$$
$$= 0 \text{ volts}$$

Example : Find the current I_o through the 12K ohm resistor.



Solution:

At node 1

$$(V_1 + 6)/12 + V_1/12 + (V_1 - V_2)/12 = 0$$

$$V_1 + 6 + V_1 + V_1 - V_2 = 0$$

$$3V_1 - V_2 = -6 \text{ ----- (A)}$$

Multiplying both sides by 2

$$6V_1 - 2V_2 = -12 \text{ ----- (A1)}$$

At node 2

$$(V_2 - V_1)/12 + V_2/12 = 2\text{mA}$$

$$V_2 - V_1 + V_2 = 24$$

$$2V_2 - V_1 = 24 \text{ ----- (B)}$$

adding equation A1 & B we have

$$2V_2 - V_1 = 24$$

$$-2V_2 + 6V_1 = -12$$

$$5V_1 = 12$$

$$V_1 = 12/5$$

Put the value of V_1 in A we have

$$V_2 = 3V_1 + 6$$

$$= 3 \times 12/5 + 6$$

$$= (36 + 30)/5$$

$$V_2 = 66/5$$

Now we know the voltage at both nodes so now we will calculate the value of current through 12K resistance. So for output current I_o

$$I_o = (V_1 - V_2)/12$$

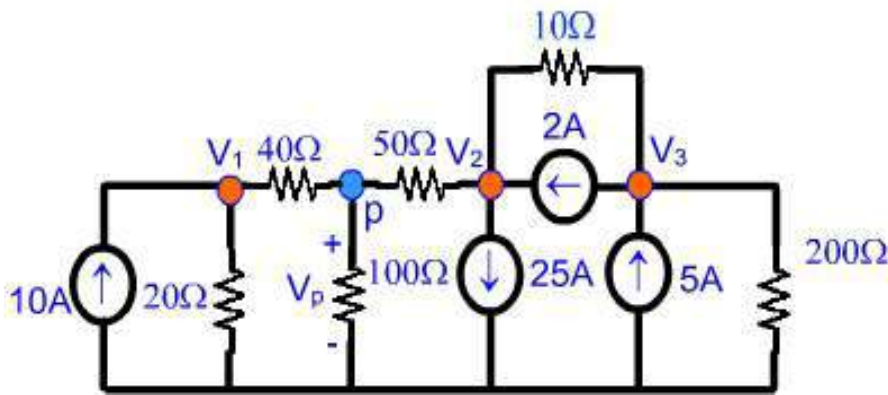
$$= (12/5 - 66/5)/12$$

$$= -54/5 \times 1/12$$

$$= -54/60$$

$$= -0.9\text{mA}$$

Example: Use nodal analysis to find V_P .



Solution:

The bottom node has the largest number of branch connections, so we choose that our reference node.

This also makes V_P easier to find, as it will be a nodal voltage. Working from left to right, we name

our nodes 1, P, 2, 3.

At node 1

$$10 = V_1/20 + (V_1 - V_P)/40 \quad [1]$$

At node p

$$0 = (V_P - V_1)/40 + V_P/100 + (V_P - V_2)/50 \quad [2]$$

At node 2

$$-2.5 + 2 = (V_2 - V_P)/50 + (V_2 - V_3)/10 \quad [3]$$

At node 3

$$5 - 2 = V_3/200 + (V_3 - V_2)/10 \quad [4]$$

Simplifying

$$60 V_1 - 20 V_P = 8000 \quad [1]$$

$$-50 V_1 + 110 V_P - 40 V_2 = 0 \quad [2]$$

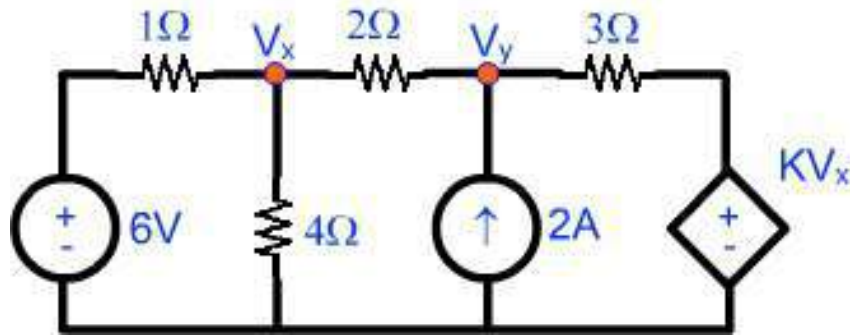
$$- V_P + 6 V_2 - 5 V_3 = -25 \quad [3]$$

$$- 200 V_2 + 210 V_3 = 6000 \quad [4]$$

Solving,

$$V_P = 171.6V$$

Example: Use nodal analysis to find the value of K that will cause V_Y to be zero.



Solution:

At node x :

$$V_x/4 + (V_x - V_y)/2 + (V_x - 6)/1 = 0 \quad [1]$$

At node y :

$$(V_y - K V_x)/3 + (V_y - V_x)/2 = 2 \quad [2]$$

Our additional constraints is that $V_Y = 0$, so we may simplify Equations [1] and [2].

$$14 V_x = 48 \quad [1]$$

$$-2K V_x - 3 V_x = 12 \quad [2]$$

Since Eq. [1] yields $V_x = 48/14 = 3.429 \text{ V}$

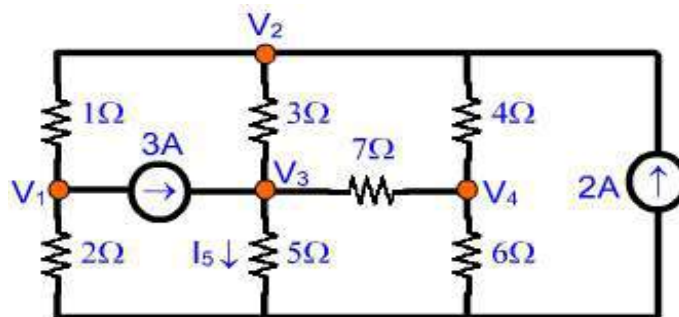
From equation 2 we have

$$K = (12 + 3 V_x)/(-2 V_x)$$

After putting the values of V_x

$$K = -3.250$$

Example: Use nodal Analysis to find I_5



Solution:

We choose the bottom node as ground to make calculation of i_5 easier. The left-most node is named "1", the top node is named "2", the central node is named "3", and the node between the 4W and 6W resistors is named "4".

NODE 1 :

$$-3 = v_1/2 + (v_1 - v_2)/1 \quad [1]$$

NODE 2 :

$$2 = (v_2 - v_1)/1 + (v_2 - v_3)/3 + (v_2 - v_4)/4 \quad [2]$$

NODE 3 :

$$3 = v_3/5 + (v_3 - v_4)/7 + (v_3 - v_2)/3 \quad [3]$$

NODE 4 :

$$0 = v_4/6 + (v_4 - v_3)/7 + (v_4 - v_2)/4 \quad [4]$$

Rearranging and grouping terms,

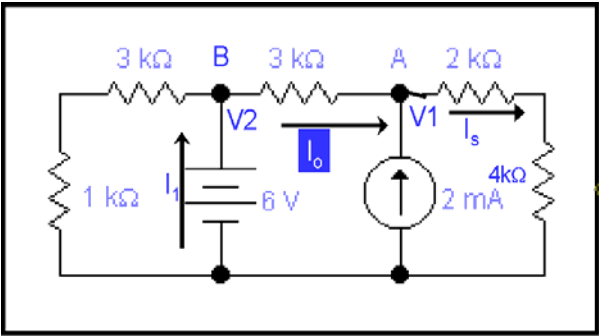
$$\begin{aligned} 3v_1 - 2v_2 &= -6 & [1] \\ -12v_1 + 19v_2 - 4v_3 - 3v_4 &= 24 & [2] \\ -35v_2 + 71v_3 - 15v_4 &= 315 & [3] \\ -42v_2 - 24v_3 + 94v_4 &= 0 & [4] \end{aligned}$$

Solving, we find that $v_3 = 6.76\text{V}$ and so $i_5 = v_3/5 = 1.352\text{ A}$

LECTURE 10

(Super Node - Constraint or Coupling Equation)

Example Calculate the current I_o through the 3k ohm resistor.



Solution:

At node A

$$((V_1 - V_2)/3k) + (V_1/6k) = 2mA$$

where $V_2 = 6V$

so by putting its value in the equation

$$((V_1 - 6)/3k) + (V_1/6k) = 2mA$$

$$2V_1 - 12 + V_1 = 12$$

$$3V_1 = 24$$

$$V_1 = 8 \text{ volts} \text{ ----- (A)}$$

by ohm's law

$$I_o = (v_1 - v_2)/3k$$

by putting the value of V_1 and V_2 we have

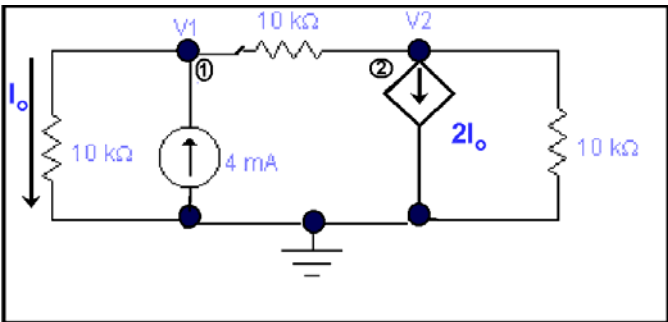
$$I_o = (8 - 6)/3k$$

$$= 0.667 \text{ mA}$$

$$I_1 = 6/4k$$

$$= 1.5 \text{ mA}$$

Example: Calculate the current I_o through the 10k ohm resistor also find V_1 and V_2 .



Solution:

$$I_o = V_1/10k \text{ -----(A)}$$

where V_1 is the voltage for node 1 We want to calculate the node voltages.

So, **For node 1**

$$(V_1/10k) + ((V_1 - V_2)/10k) = 4mA$$

$$(V_1/10k) + (V_1/10k) - (V_2/10k) = 4mA$$

by putting the value of I_o

$$I_o + I_o - (V_2/10k) = 4mA$$

$$2 I_o - (V_2/10k) = 4mA$$

$$20 I_o - V_2 = 40mA \text{ -----(B)}$$

For node 2

$$(V_2/10k) + ((V_2 - V_1)/10k) = -2I_o$$

$$(V_2/10k) + (V_2/10k) - (V_1/10k) = -2I_o$$

$$(V_2/10k) + (V_2/10k) - I_o = -2I_o$$

$2V_2 + 10I_0 = 0$ -----(C)

Equating equations of node 1 and 2
 $40 I_0 - 2V_2 = 80\text{mA}$
 $10 I_0 + 2V_2 = 0$

Hence $50 I_0 = 80\text{mA}$
 $I_0 = 8/5 \text{ mA}$

So, put the value of I_0 in equation (A)
 $I_0 = (V_1/10\text{k})$
 $8/5 \text{ mA} = (V_1/10\text{k})$
 $V_1 = 16 \text{ volts}$
by putting the value of I_0 from (A) into (C) we have
 $2V_2 + V_1 = 0$
 $2V_2 = -V_1$
 $V_2 = -16/2 = -8\text{volts}$

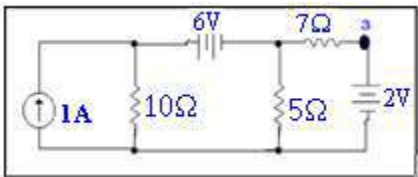
SUPER NODE:

A node which emerges as a result of combination of two ordinary nodes around a voltage source.

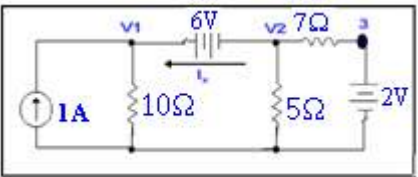
CONSTRAINT OR COUPLING EQUATION:

This is an equation which describes a super node mathematically, instead of writing equations individual ordinary nodes of the super node.

Example Calculate the power supplied by current source.

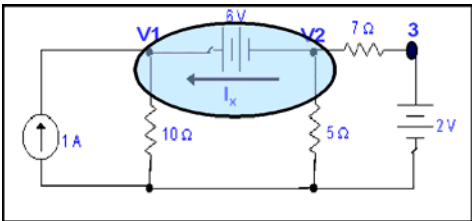


Solution:



Power = 1A(V₁)
Applying KCL to **node 1**
 $(V_1/10) - I_x = 1$
Apply KCL to **node 2**
 $0 = I_x + (V_2/5) + ((V_2-2)/7)$

Now, 3 unknown quantities can not be calculated from 2 equations, so try super node technique,



Equation for super node
 $1 = (V_1/10) + (V_2/5) + ((V_2-2)/7)$
Simplifying it, we can write
 $35 V_1 + 120 V_2 = 450$

Also the constraint or coupling equation is

$$V_1 - V_2 = 6$$

Solving the two equations simultaneously,

$$35V_1 + 120V_2 = 450 \text{ ----- (A)}$$

$$V_2 = V_1 - 6$$

put the value of V_2 in A we have

$$35V_1 + 120(V_1 - 6) = 450$$

$$35V_1 + 120V_1 - 720 = 450$$

$$155V_1 = 1170$$

$$V_1 = 7.55V$$

Therefore,

$$V_1 = 7.55V$$

$$\& V_2 = 1.55V$$

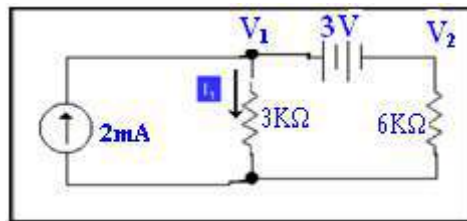
Hence, power

$$P = I(A)(V_1)$$

$$= 7.55W$$

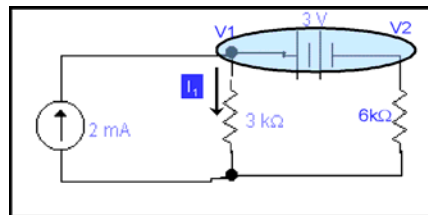
Example:

Calculate I_1 from the given circuit.



Solution:

We can redraw the circuit as



Constraint equation for super node

$$V_1 - V_2 = 3$$

KCL equation for super node

$$(V_1/3k) + (V_2/6k) = 2 \times 10^{-3}$$

$$2V_1 + V_2 = 12$$

Now from constraint equation

$$V_2 = V_1 - 3$$

Putting this value in the KCL equation of super node

$$2V_1 + V_1 - 3 = 12$$

$$3V_1 = 15$$

$$V_1 = 5 \text{ volts}$$

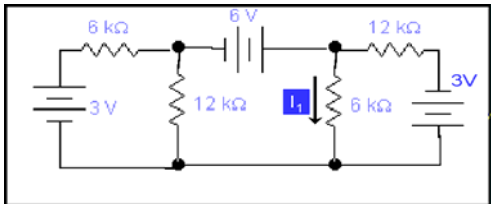
Now

$$I_1 = \frac{V_1}{3k}$$

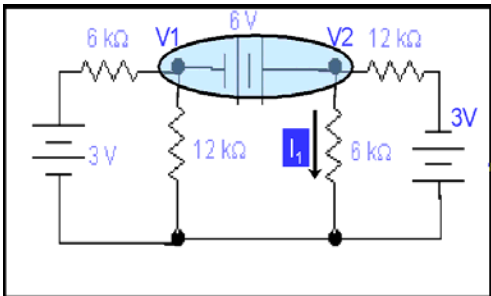
$$= \frac{5}{3k}$$

$$= 1.6 \text{ mA}$$

Example: Find I_1 from the circuit.



Solution: We will redraw this circuit as



Constraint equation for the super node
 $V_2 - V_1 = 6V$

KCL equation for the super node

$$\begin{aligned} ((V_1 - 3)/6k) + (V_1/12k) + ((V_2 + 3)/12k) + (V_2/12k) &= 0 \\ 2V_1 - 6 + V_1 + V_2 + 3 + 2V_2 &= 0 \\ 3V_1 + 3V_2 &= 3 \\ V_2 + V_1 &= 1 \end{aligned}$$

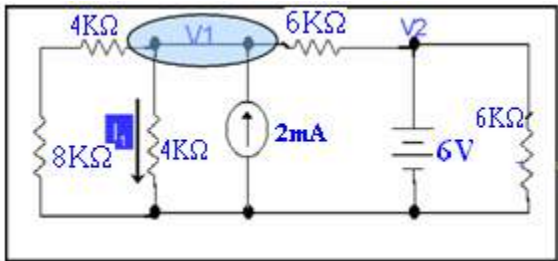
Adding constraint equation and KCL equation

$$\begin{array}{r} V_2 - V_1 = 6 \\ V_2 + V_1 = 1 \\ \hline \end{array}$$

or

$$\begin{aligned} 2V_2 &= 7 \\ V_2 &= 3.5V \\ I_1 &= 3.5/6k \\ &= 35/60k \\ &= 7/12 \text{ mA} \end{aligned}$$

Example: Calculate the value I_1 .



Solution:

We want to calculate the value of current I_1

For node 1

$$\begin{aligned} (V_1/12k) + (V_1/4k) + (V_1/6k) - (6/6k) &= 2 \text{ mA} \\ ((1/12k) + (1/4k) + (1/6k))V_1 - (6/6k) &= 2 \text{ mA} \\ (V_1/2k) - (1/1k) &= 2 \times 10^{-3} \\ V_1 - 2 &= 4 \\ V_1 &= 6V \end{aligned}$$

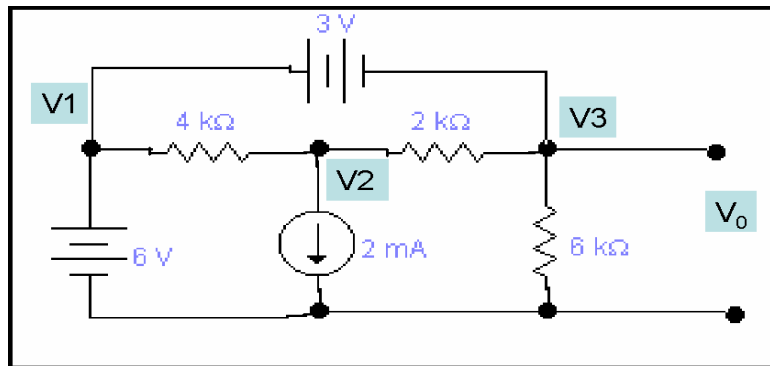
Now

$$\begin{aligned} I_1 &= V_1/4k \\ &= 6/4k \\ &= 1.5 \text{ m} \end{aligned}$$

LECTURE 11

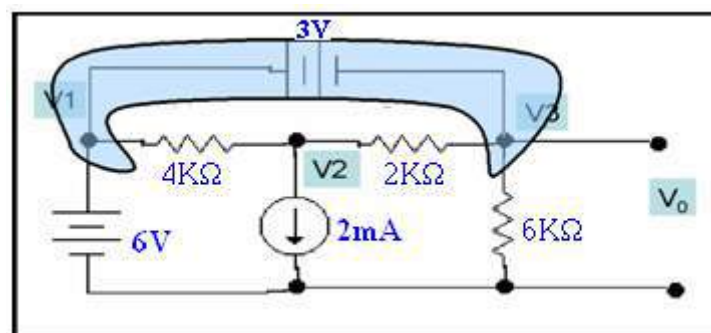
(Examples of Nodal Analysis - Super Node technique)

Example: Find the out put voltage V_0 .



Solution:

we can redraw this circuit as



Here $V_1 = 6$ and the constraint equation is

$$V_1 - V_3 = 3V$$

$$6 - V_3 = 3V$$

$$V_3 = 6 - 3 \text{ volts}$$

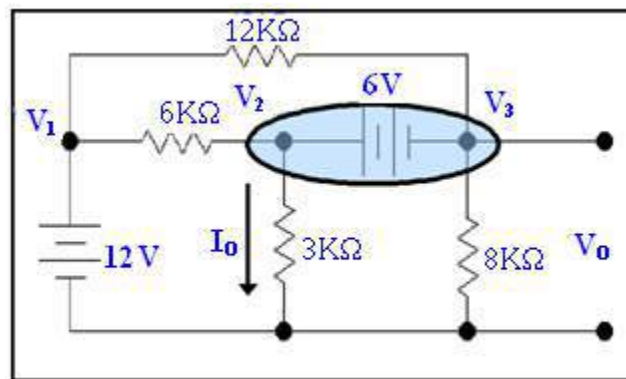
$$V_3 = 3 \text{ volts}$$

Also,

$$V_3 = V_0 = 3 \text{ volts}$$

So

$$V_0 = 3 \text{ Volts}$$

Example:Calculate V_0 and I_0 .**Solution:**

Node 2 and 3 constitute a super node Constraint equation will be

$$V_2 - V_3 = 6 \text{ -----(A)}$$

KCL equation at super node is

$$((V_2 - 12)/6k) + (V_2/3k) + (V_3/6k) + ((V_3 - 12)/12k) = 0$$

$$(V_2/6k) + (V_2/3k) + (V_3/6k) + (V_3/12k) - (12/12k) - (12/6k) = 0$$

$$(V_2/2k) + (V_3/4k) - (12/4k) = 0$$

$$2V_2 + V_3 - 12 = 0$$

$$2V_2 + V_3 = 12 \text{(B)}$$

Now from constraint equation (A), value of V_3 is

$$V_3 = V_2 - 6$$

put this in equation (B)

Also, the super node equation becomes

$$2V_2 + V_2 - 6 = 12$$

$$3V_2 = 18$$

$$V_2 = 6 \text{ Volts}$$

Putting this value in super node equation

$$V_3 = 0$$

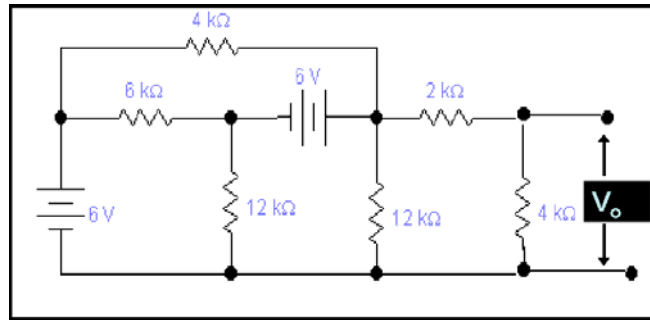
Therefore,

$$V_0 = 0 \text{ Volt}$$

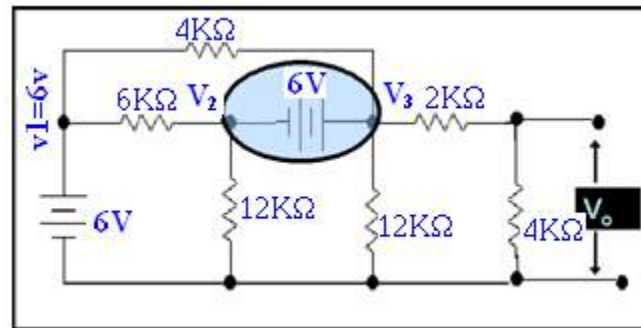
$$I_0 = V_2/3k$$

$$= 6/3k$$

$$= 2\text{mA}$$

Example:**Calculate the value of V_0 .****Solution:**

The circuit can be redrawn as



Node 2 and 3 constitute a super node, having constraint equation

$$V_3 - V_2 = 6 \text{ volts}$$

KCL equation at super node

$$\begin{aligned} ((V_2 - 6)/6k) + (V_2/12k) + ((V_3 - 6)/4k) + (V_3/12k) + (V_3/6k) &= 0 \\ (V_2/6k) + (V_2/12k) + (V_3/6k) + (V_3/12k) + (V_3/4k) - (6/4k) - (6/6k) &= 0 \\ (V_2/4k) + (V_3/2k) - (30/12k) &= 0 \end{aligned}$$

$$3V_2 + 6V_3 - 30 = 0$$

$$V_2 + 2V_3 = 10 \text{ -----(A)}$$

From the constraint equation of super node

$$V_2 = V_3 - 6$$

KCL equation (A) of super node will become

$$3V_3 - 6 = 10$$

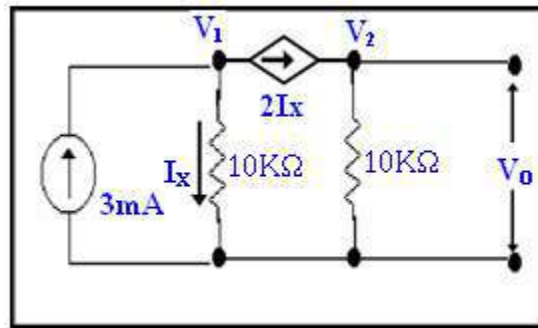
$$V_3 = 16/3 \text{ volts}$$

$$\begin{aligned} V_2 &= (16/3) - 6 \\ &= -2/3 \text{ V} \end{aligned}$$

Now by voltage division rule

$$\begin{aligned} V_0 &= (4k \times V_3)/(2k + 4k) \\ &= 64/18 \\ &= 3.55 \text{ Volts} \end{aligned}$$

Example: In the given circuit, find V_o .



Solution:

Applying KCL at node 1

$$(V_1/10k) = 3mA - 2I_x$$

$$\text{Now } I_x = (V_1/10k)$$

Putting this value in the equation of node 1

$$V_1/10k = I_x = 3mA - 2I_x$$

$$I_x = 1mA$$

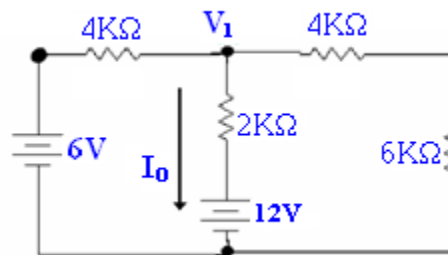
Now KCL at node 2

$$(V_2/10k) = 2I_x$$

$$(V_2/10k) = 2(1mA)$$

$$V_2 = 20 \text{ Volts} = V_o$$

Example: Find I_o in the given circuit.



Solution:

KCL equation for node 1

$$((V_1 - 6)/4k) + (V_1/2k) + ((V_1 - 12)/4k) = 0$$

$$5V_1 - 30 + 2V_1 + 10V_1 - 120 = 0$$

$$17V_1 = 150$$

$$V_1 = 8.82 \text{ Volts}$$

$$I_o = (V_1 - 12)/2k$$

$$= (8.82 - 12)/2k$$

$$I_o = -1.58 \text{ mA}$$

So the current value is in negative sign, so we conclude that as it is given in the circuit the direction of I_o is opposite.

LECTURE 12

(Kirchhoff's Voltage Law (KVL) - Loop - Examples)

KIRCHHOFF'S VOLTAGE LAW:

This law states that the algebraic sum of the voltages around any loop is zero.

OR

Sum of voltages rises and voltage drops around any closed path or loop is equal to zero.

LOOP:

It is the closed path for the flow of current in which no node is encountered more than one. Loop can be considered as closed path in which work done in moving a unit charge is equal to zero.

ASSUMPTIONS:

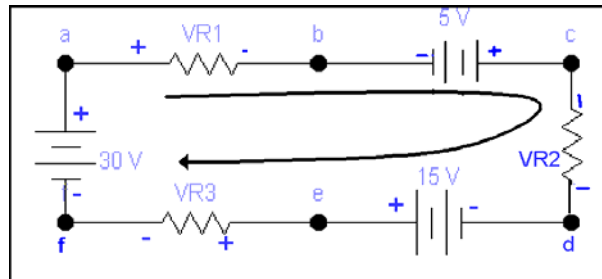
Any increase in energy level is taken as positive and any decrease in energy level is taken as negative.

Current leaving the source is taken as positive and current entering the voltage source is taken as negative.

NO. OF EQUATIONS:

No. of equations to be written are equal to the no. of loops or closed paths.

Example:



We want to calculate the value of VR_3 where values of VR_1 and VR_2 are known by using KVL.

Solution:

Now we will take increase in energy level positive and decrease in energy level negative. By using this assumption and KVL the equation of this loop will be

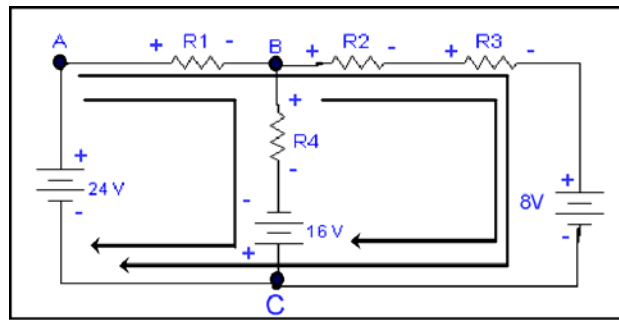
$$+VR_1 - 5 + VR_2 - 15 + VR_3 - 30 = 0$$

which can be written as

$$VR_1 + VR_2 + VR_3 = 5 + 15 + 30 \\ = 50$$

Now suppose that VR_1 and VR_2 are known to be 18V and 12V respectively then

$$VR_3 = 20V$$

Example:**Solution:**

Note that this network has three closed paths the left loop, the right loop and outer loop.

Applying KVL to left loop

$$VR_1 + VR_4 - 16 - 24 = 0$$

KVL equation for right loop starting at point B.

$$VR_2 + VR_3 + 8 + 16 - VR_4 = 0$$

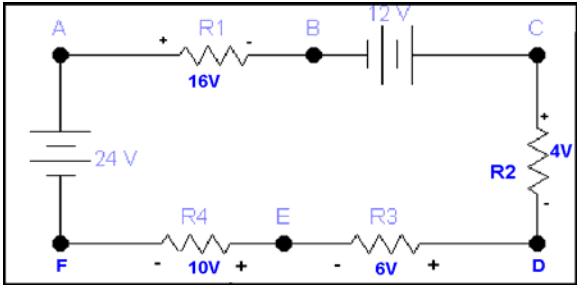
The equation for outer loop starting at point A

$$VR_1 + VR_2 + VR_3 + 8 - 24 = 0$$

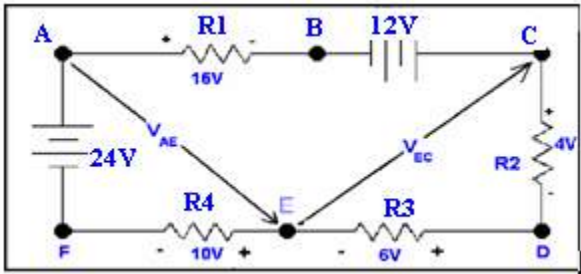
Note that if we add first two equations, we obtain the third equation. So these three equations are not linearly independent. We will discuss this issue in next lectures that we will use only linearly independent equations to solve for all voltages.

LECTURE 13
(Applications of Loop Analysis)

Example: Calculate the voltages V_{AE} and V_{EC} .



Solution: We will draw some imaginary arrows across point A E and C. The circuit can be redrawn as



Since points A and E as well as E and C are not physically close, we use the arrow notation. Our approach to determining the unknown voltage is to apply KVL with the unknown voltage in the closed path, therefore to V_{AE} we will use the path **AEFA** or **ABCDEA**. The equation for two paths in which V_{AE} is the only known are

For path **AEFA**

$$V_{AE} + 10 - 24 = 0 \text{ ----- (A)}$$
$$V_{AE} = 14 \text{ volt}$$

For path **ABCDEA**

$$16 - 12 + 4 + 6 - V_{AE} = 0 \text{ ----- (B)}$$
$$V_{AE} = 14 \text{ volt}$$

Solving both loops i.e. **AEFA** and **ABCDEA** we will get $V_{AE} = 14$ volt. We can calculate V_{EC} using paths **CDEC** or **CEFABC**. The value of V_{AE} is also known so we can also use path **CEABC**.

KVL for each of the paths is

For the loop or path **CDEC**

$$4 + 6 + V_{EC} = 0 \text{ ----- (1)}$$
$$V_{EC} = -10 \text{ volts}$$

For the loop or path **CEABC**

$$-V_{EC} + V_{EA} + 16 - 12 = 0 \text{ ----- (A)}$$

in above equation (A) $V_{EA} = -V_{AE}$

$$+V_{EC} - V_{AE} + 16 - 12 = 0 \text{ ----- (2)}$$

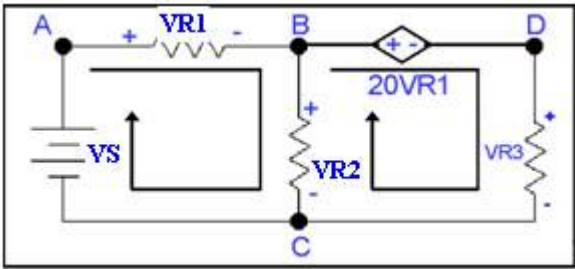
For the loop or path **CEFABC**

$$-V_{EC} + 10 - 24 + 16 - 12 = 0 \text{ ----- (3)}$$

Solving each of three equations we have

$$V_{EC} = -10 \text{ volts}$$

Example: Write KVL equations for the given circuit.



Solution: We have two loops in this circuit one is on the left hand side i.e. **ABCA** and other is on the right hand side i.e. **BDCB**.

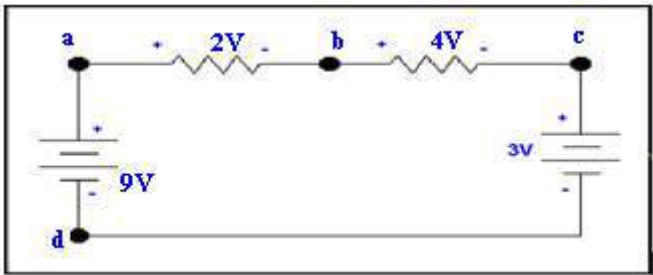
KVL equation for left hand loop **ABCA**

$$VR_1 + VR_2 - V_s = 0$$

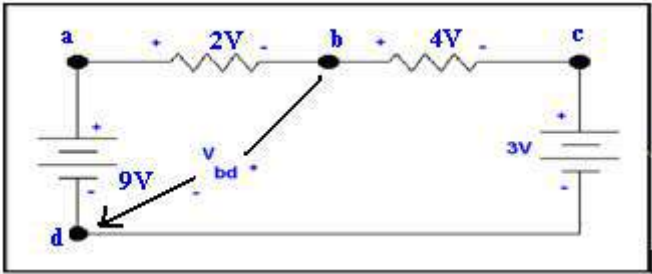
KVL equation for right hand loop **BDCB**

$$20VR_1 + VR_3 - VR_2 = 0$$

Example: Calculate the voltage V_{bd}



Solution: We want to calculate the voltage V_{bd} . As we can see there is no closed path between b and d, so the circuit can be redrawn as



We can consider two closed paths **abda** and **bcd b**. The **KVL equation** for these two paths will be written as

The KVL equation for the path **abda**

$$2 + V_{bd} - 9 = 0$$

$$V_{bd} = 7 \text{ volts}$$

KVL equation for the **bcd b**

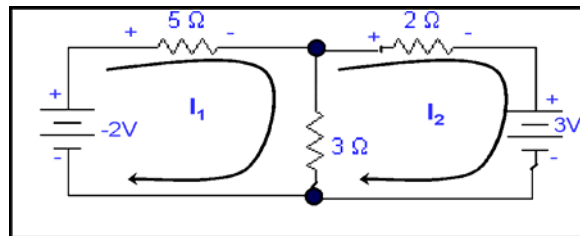
$$4 + 3 - V_{bd} = 0$$

$$V_{bd} = 7 \text{ volts}$$

LECTURE 14

(Applications of Loop Analysis)

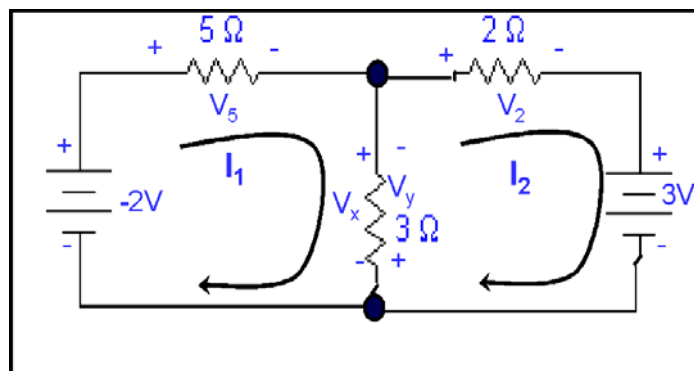
Example: Find the power dissipated by 3Ω resistance.



Solution:

We can write $I_3 = I_1 - I_2$, I_1 and I_2 are opposite to each other so they cancel each other up to some extent to give I_3 . It is given in the circuit that direction of I_1 is in clock wise direction while I_2 is also in clockwise direction.

We suppose that voltage V_x for mesh 1 and V_y for mesh 2 across the 3 ohm resistance.



For mesh 1

$$-(-2) + V_5 + V_x = 0$$

Also from **Ohm's law**

$$V_5 = 5 I_1 \text{ and } V_x = 3 (I_1 - I_2)$$

$$\therefore 2 + 5I_1 + 3(I_1 - I_2) = 0$$

$$\text{Or } 8I_1 - 3I_2 = -2 \text{ ----- (A)}$$

For mesh 2

$$3(I_2 - I_1) + 2I_2 + 3 = 0$$

$$\text{Or } -3I_1 + 5I_2 = -3 \text{ ----- (B)}$$

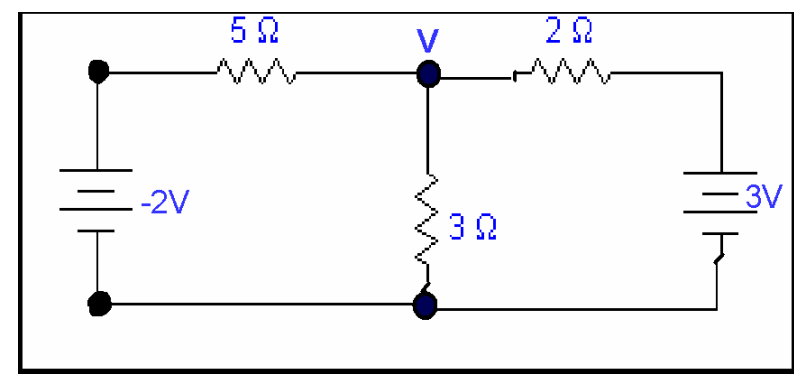
Solving two mesh equations (A) and (B), we get

$$I_1 = -612.9 \text{ mA}$$

$$I_2 = -967.7 \text{ mA}$$

$$\therefore I_3 = 354.8 \text{ mA}$$

Hence Power = $3 (354.8)^2$
=377.7 mW
It does not matter whether we write I_1-I_2 or I_2-I_1 due to square term.
Now using nodal analysis

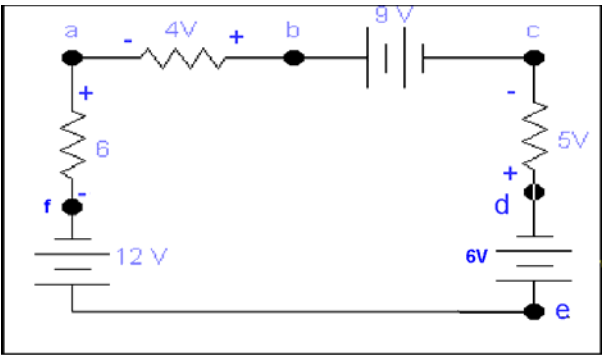


$$0 = (V-(-2)/5) + (V/3) + (V-3)/2$$
$$V = 1.065V$$

∴ $Power_{3\Omega} = | (V^2/R)=378.1 \text{ mW}$

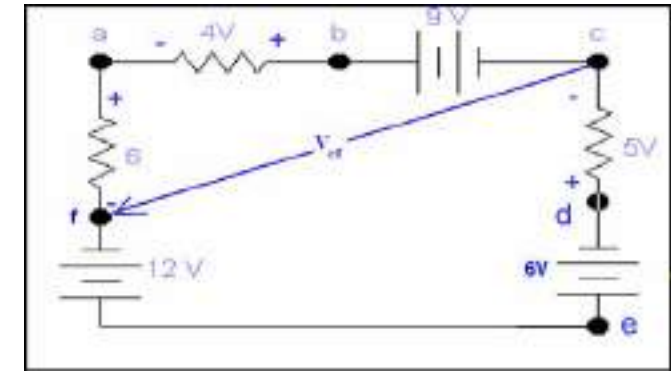
Example:

Calculate the voltage V_{cf} in the network.



Solution:

We want to calculate the voltage V_{cf} in the network. The network can be redrawn as



We can consider two paths for finding voltage V_{cf} that are **abcfa** and **cdefc**.
KVL equation for the path **abcfa**

$$-4 + 9 + V_{cf} - 6 = 0$$

$$V_{cf} = 1 \text{ volts} \text{ ----- (A)}$$

KVL equation **cdefc** will be
 $-5 - 6 - 12 + V_{fc} = 0$

$$V_{fc} = -1 \text{Volts} \text{ ----- (B)}$$

In the above equation we have V_{fc} instead of V_{cf} . But we want to calculate V_{cf} .

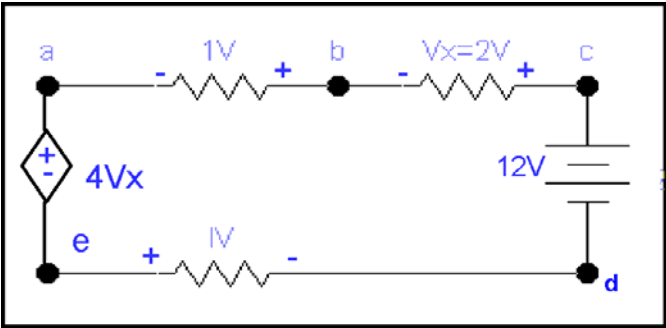
Where $V_{cf} = -V_{fc}$

so from equation B we have

$$V_{cf} = 1 \text{Volts}$$

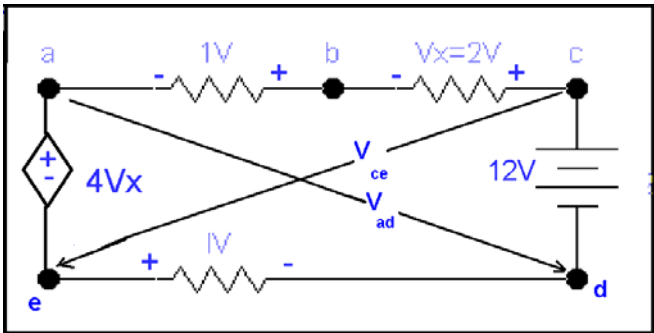
Example

Calculate the voltage V_{ad} V_{ce} in the network.



Solution:

We want to calculate V_{ad} V_{ce} . As there is no physical closed path in between these points the circuit can be redrawn as



For V_{ad}

There are two paths in which V_{ad} is the only unknown quantity they are **adea** and **adcba**.

For path **adea** the KVL equation will be

$$V_{ad} - 1V - 4Vx = 0$$

where $Vx=2V$

$$V_{ad} - 1 - 8 = 0$$

$$V_{ad} = 9 \text{ volts}$$

For the path **adcba**

$$V_{ad} - 12 + 2 + 1 = 0$$

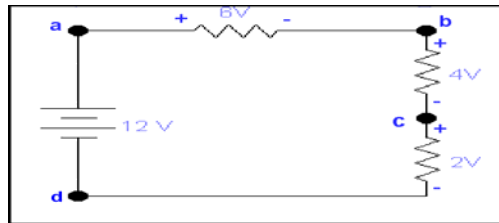
$$V_{ad} = 9 \text{ volts}$$

Note: Calculate V_{ce} yourself.

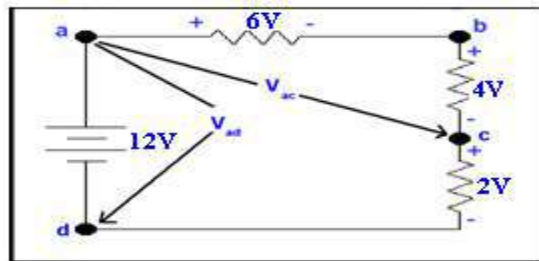
LECTURE 15

(Applications of Loop Analysis)

Example: Calculate the voltages V_{ad} , V_{ac} and V_{bd} .



Solution: We want to calculate V_{ad} and V_{ac} . The circuit can be redrawn as



As seen from the figure there is only a voltage source between points a and d so the voltage V_{ad} will be **12 volts**. There is no physical closed path between point a and c we can consider two closed paths **acba** and **acda**.

KVL equation for the path **acba**

$$V_{ac} - 4 - 6 = 0$$

$$\mathbf{V_{ac} = 10 \text{ Volts}}$$

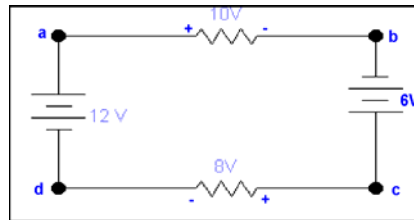
Second path in which only unknown value is V_{ac} is **acda**. KVL equation for this will be

$$V_{ac} + 2 - 12 = 0$$

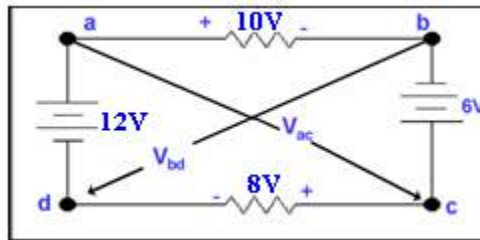
$$\mathbf{V_{ac} = 10 \text{ Volts}}$$

Note: Calculate V_{bd} yourself.

Example: Calculate the voltages V_{ac} and V_{db} .



Solution: We want to calculate V_{ac} and V_{db} . The circuit can be redrawn as



To calculate V_{ac} , the two paths will be **acda** and **acba**.

KVL equation for the path **acda**

$$V_{ac} + 8 - 12 = 0$$

$$\mathbf{V_{ac} = 4 \text{ volts}}$$

KVL equation for the path **acba** will be

$$V_{ac} + 6 - 10 = 0$$

$$\mathbf{V_{ac} = 4 \text{ volts}}$$

Two paths for V_{bd} will be **bdcb** and **bdab**.

KVL equation for **bdcb**

$$V_{bd} - 8 + 6 = 0$$

$$V_{bd} = 2 \text{ volts}$$

$$\mathbf{V_{db} = - 2 \text{ volts}}$$

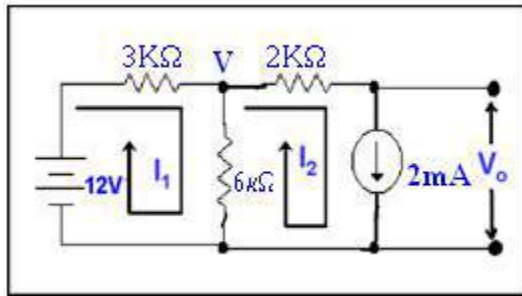
KVL equation for the path **bdab**

$$V_{bd} - 12 + 10 = 0$$

$$V_{bd} = 2 \text{ volts}$$

$$\mathbf{V_{db} = - 2 \text{ volts}}$$

Example: Calculate the voltage V_0 .



Solution: We want to calculate V_0 .

Here

$$I_2 = 2 \text{ mA}$$

KVL for **mesh 1**

$$3kI_1 + 6k(I_1 - I_2) - 12 = 0$$

$$9kI_1 - 6kI_2 = 12$$

$$9kI_1 - 12 = 12$$

$$I_1 = 8/3 \text{ mA}$$

Voltage across **6k** resistor

$$V = 6k(I_1 - I_2)$$

$$= 6k(8/3 - 2)$$

$$= 4 \text{ volts}$$

As current through 2k resistor is

$$I_2 = 2 \text{ mA}$$

$$\text{Voltage across } 2k = 2k \times 2 \text{ mA}$$

$$= 4 \text{ volts}$$

$$V_0 = V - V_{2k}$$

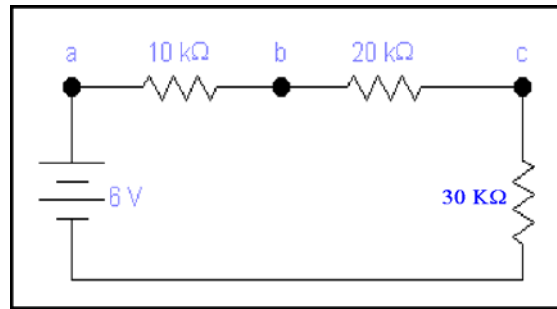
$$= 4 - 4$$

$$V_0 = 0 \text{ volts}$$

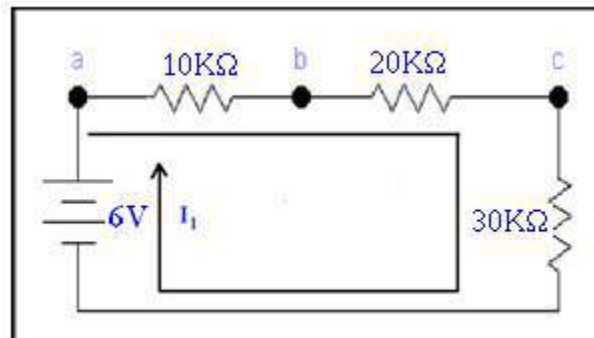
LECTURE 16

(Applications of Loop Analysis)

Example: Calculate the voltage V_{ac} .



Solution: We want to calculate the voltage V_{ac} . To solve this problem the circuit can be redrawn as



Let the current I_1 is flowing through the circuit. The **KVL equation** will be

$$10kI_1 + 20kI_1 + 30kI_1 - 6 = 0$$

$$10kI_1 + 20kI_1 + 30kI_1 = 6$$

$$60kI_1 = 6$$

$$I_1 = 6/60k$$

$$I_1 = 0.1mA$$

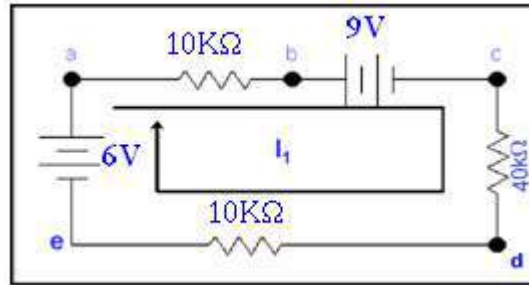
Now V_{ac} will be equal to

$$V_{ac} = (10k + 20k)I_1$$

$$= 30k(0.1mA)$$

$$V_{ac} = 3 \text{ Volts}$$

Example: Calculate the voltage V_{bd} .



Solution: We want to calculate the voltage V_{bd} . First we will have to calculate the voltage across 40k resistor. Let the current I_1 be flowing through the loop.

Applying KVL

$$10kI_1 + 9 + 40kI_1 + 10kI_1 - 6 = 0$$

$$60kI_1 = -3$$

$$I_1 = -3/60k$$

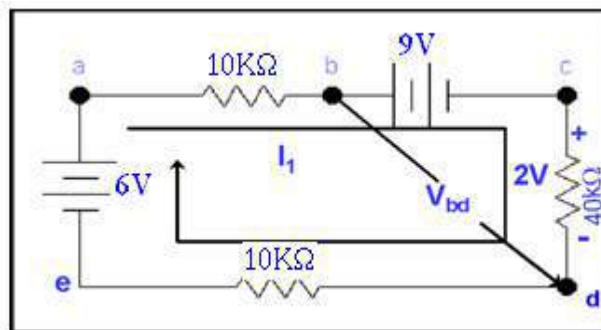
$$= -0.05 \text{ mA}$$

So, voltage across 40kΩ resistor

$$V_{40k} = (-0.05\text{m})(40k)$$

$$= -2 \text{ volts} \text{ ----- (A)}$$

We want to calculate V_{bd} we will redraw the circuit as



Now to calculate V_{bd} we take the path **bdcb**

Applying KVL

$$V_{bd} - V_{40k} - 9 = 0$$

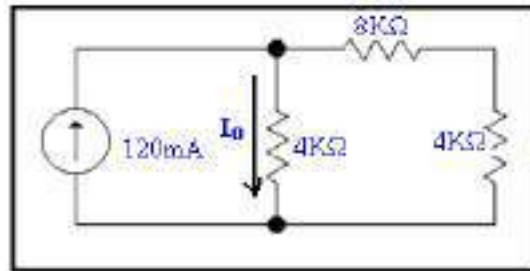
by putting the value of V_{40k} from equation (A) we have

$$V_{bd} - (-2) - 9 = 0$$

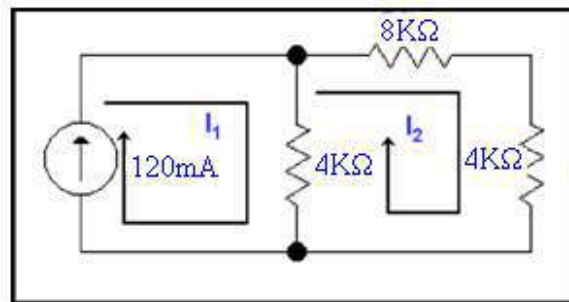
$$V_{bd} + 2 - 9 = 0$$

$$V_{bd} = 7 \text{ Volts}$$

Example: Calculate the current I_0 .



Solution: We want to calculate the current I_0 . The circuit can be redrawn as



KVL for mesh 1

Here

$$I_1 = 120 \text{ mA}$$

Applying KVL to mesh 2

$$8kI_2 + 4kI_2 + 4k(I_2 - I_1) = 0$$

$$16kI_2 - 4kI_1 = 0$$

$$16kI_2 = 480$$

$$I_2 = 480/16k$$

$$I_2 = 30 \text{ mA}$$

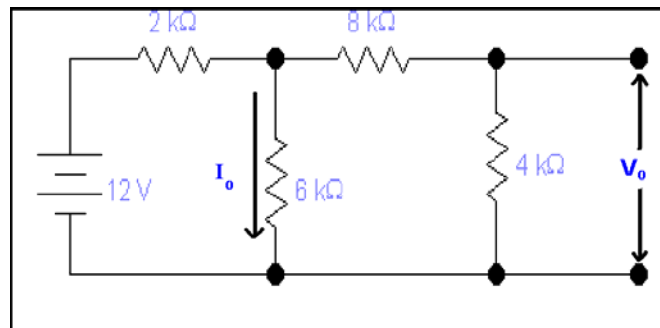
So I_0 will be

$$I_0 = I_1 - I_2$$

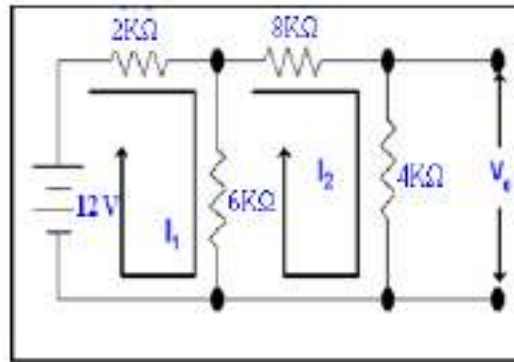
$$I_0 = 120 - 30$$

$$I_0 = 90 \text{ mA}$$

Example: Calculate the current I_0 and the voltage V_0 .



Solution: We want to calculate the current I_0 and the voltage V_0 . The circuit can be redrawn as



Applying KVL to **mesh 1**

$$2kI_1 + 6k(I_1 - I_2) = 12$$

$$8kI_1 - 6kI_2 = 12$$

Applying KVL at **mesh 2**

$$8kI_2 + 4kI_2 + 6k(I_2 - I_1) = 0$$

$$18kI_2 - 6kI_1 = 0$$

Solving equations of mesh 1 and mesh 2

$$24kI_1 - 18kI_2 = 36$$

$$-24kI_1 + 72kI_2 = 0$$

$$54kI_2 = 36$$

$$I_2 = 36/54k$$

$$I_2 = 0.67mA \text{ -----(A)}$$

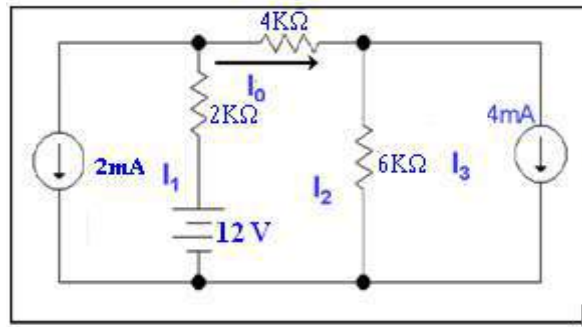
Now V_0 will be equal to

$$V_0 = 4k I_2$$

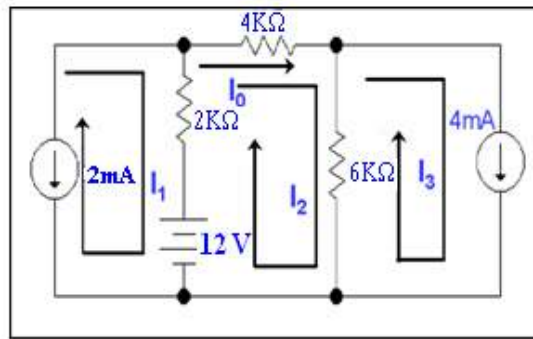
by putting the value of I_2 from equation (A) we have

$$V_0 = 2.66 \text{ Volts}$$

Example: Calculate the current I_0 .



Solution: We want to calculate the current I_0 . To apply KVL the circuit can be redrawn as



Here

$$I_1 = -2\text{mA}$$

Negative value of I_1 is due to this reason because the direction of I_1 is going opposite to the Independent current source.

$$I_3 = 4\text{mA}$$

KVL for **mesh 2**

$$4kI_2 + 6k(I_2 - I_3) + 2k(I_2 - I_1) = 12$$

$$12kI_2 - 24 + 4 = 12$$

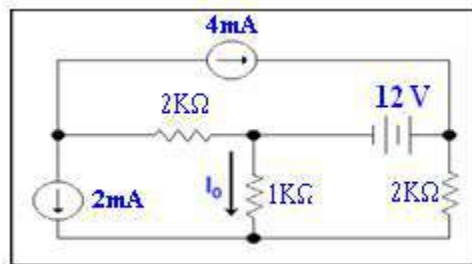
$$I_2 = 32/12k$$

$$= 8/3 \text{ mA}$$

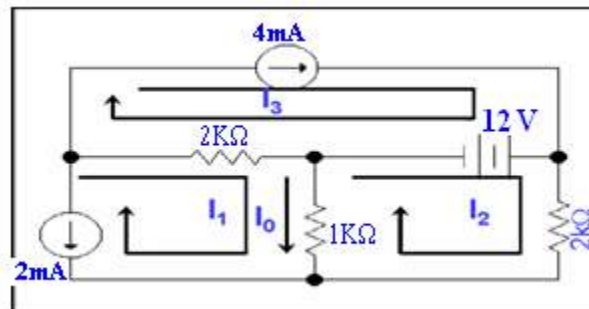
$$= 2.66 \text{ mA}$$

$$I_0 = I_2 = 2.66\text{mA}$$

Example: Calculate the current I_0 .



Solution: We want to calculate the current I_0 . The circuit can be redrawn as



Here

$$I_1 = -2\text{mA}$$

$$I_3 = 4\text{mA}$$

Apply KVL on the **mesh 2**

$$2kI_2 + 1k(I_2 - I_1) = 12$$

$$3kI_2 + 2 = 12$$

$$I_2 = 10/3 \text{ mA}$$

$$I_2 = 3.33\text{mA}$$

Now

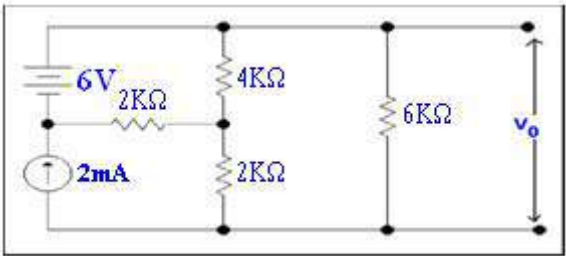
$$I_0 = I_1 - I_2$$

$$= -2 - 3.33$$

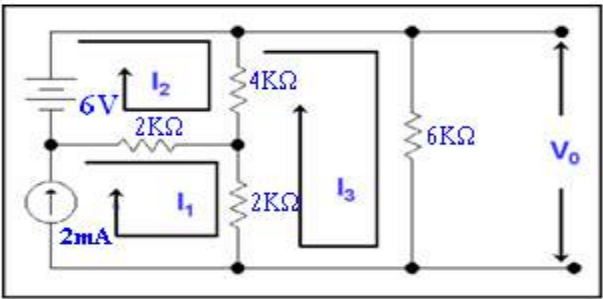
$$I_0 = -5.33 \text{ mA}$$

LECTURE 17
(Applications of Loop Analysis)

Example: Calculate the voltage V_0 .



Solution: We want to calculate the voltage V_0 . The circuit can be redrawn as



Here

$$I_1 = 2\text{mA}$$

For mesh 2

$$\begin{aligned} 4k(I_2 - I_3) + 2k(I_2 - I_1) &= 6 \\ 4kI_2 - 4kI_3 + 2kI_2 - 2kI_1 &= 6 \\ 6kI_2 - 4kI_3 - 4 &= 6 \\ 6kI_2 - 4kI_3 &= 10 \end{aligned}$$

For mesh 3

$$\begin{aligned} 6kI_3 + 2k(I_3 - I_1) + 4k(I_3 - I_2) &= 0 \\ 6kI_3 + 2kI_3 - 2kI_1 + 4kI_3 - 4kI_2 &= 0 \\ 12kI_3 - 4kI_2 - 4 &= 0 \\ 12kI_3 - 4kI_2 &= 4 \end{aligned}$$

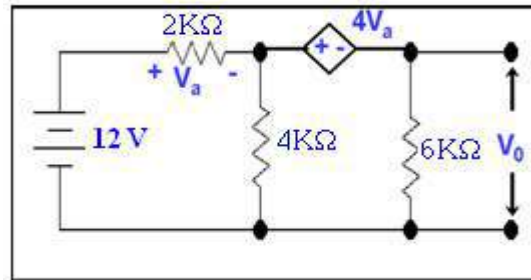
Solving equations of mesh2 and mesh3

$$\begin{array}{rcl} 24kI_2 - 16kI_3 &= & 40 \\ -24kI_2 + 72kI_3 &= & 24 \\ \hline 56kI_3 &= & 64 \\ I_3 &= & 1.142\text{mA} \end{array}$$

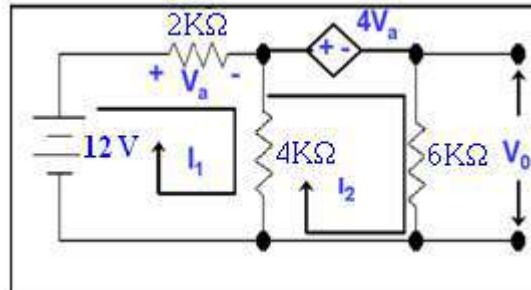
Now

$$\begin{aligned} V_0 &= 6kI_3 \\ &= 6k \times 1.142\text{mA} \\ V_0 &= 6.85 \text{ volts} \end{aligned}$$

Example: Calculate the voltage V_0 .



Solution: We want to calculate the **voltage V_0** . The circuit can be redrawn as



KVL equation for **mesh 1**

$$V_a + 4k(I_1 - I_2) = 12$$

$$V_a + 4kI_1 - 4kI_2 = 12$$

KVL equation for **mesh 2**

$$4V_a + 6kI_2 + 4k(I_2 - I_1) = 0$$

$$4V_a + 10kI_2 - 4kI_1 = 0$$

Here

$$V_a = 2kI_1$$

Putting this value in the equation of **mesh 1** we will get

$$2kI_1 + 4kI_1 - 4kI_2 = 12$$

$$6kI_1 - 4kI_2 = 12$$

Now put the value of **V_a** in the equation of **mesh 2**

$$8kI_1 + 10kI_2 - 4kI_1 = 0$$

$$4kI_1 + 10kI_2 = 0$$

Now multiplying modified equation of **mesh 1** by **1.5** and subtracting modified equation of **mesh 2** from modified equation of **mesh 1**

$$6kI_1 - 4kI_2 = 12$$

$$-6kI_1 - 15kI_2 = 0$$

$$-19kI_2 = 12$$

$$I_2 = -12/19 \text{ mA}$$

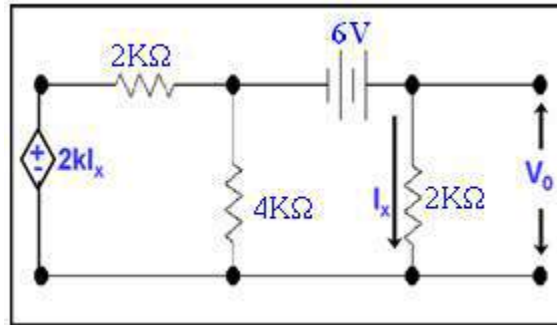
$$I_2 = 0.6315 \text{ mA}$$

Now

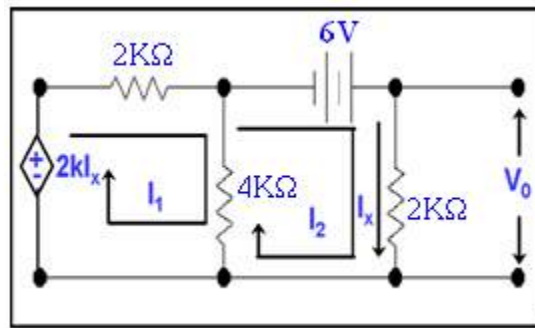
$$V_0 = 6k \times 0.6315$$

$$V_0 = 3.78 \text{ volts}$$

Example: Calculate the voltage V_0 .



Solution: We want to calculate the **voltage V_0** . The circuit can be redrawn as



KVL equation for **mesh 1**

$$2kI_1 + 4k(I_1 - I_x) - 2kI_x = 0$$

$$6kI_1 - 4kI_x - 2kI_x = 0$$

$$I_1 = I_x$$

KVL equation for **mesh 2**

$$2kI_x + 4k(I_x - I_1) - 6 = 0$$

$$\text{As } I_1 = I_x$$

$$2kI_x - 6 = 0$$

$$2kI_x = 6$$

$$I_x = 3\text{mA}$$

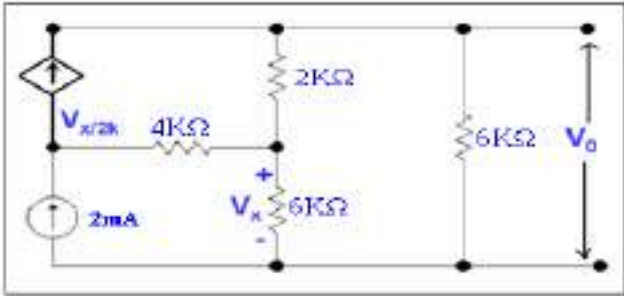
$$V_0 = 2kI_x$$

$$= 2k \times 3\text{mA}$$

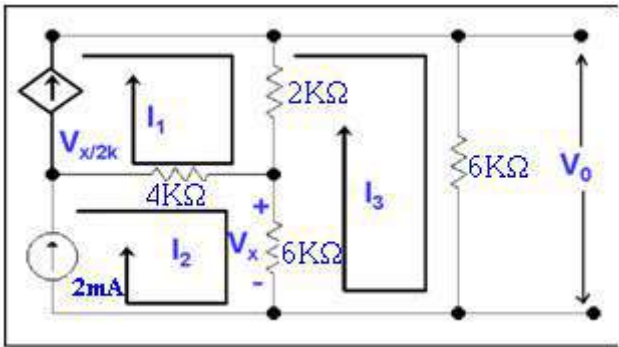
$$V_0 = 6 \text{ Volts}$$

LECTURE 18
(Applications of Loop Analysis-super mesh technique)

Example: Calculate the voltage V_o .



Solution: We want to calculate the voltage V_o . The circuit can be redrawn as



Now

$$V_x = (I_2 - I_3) 6k \text{ -----(A)}$$

$$I_1 = V_x / 2k \text{ -----(B)}$$

Put V_x from A into B we have

$$\begin{aligned} I_1 &= (I_2 - I_3) 6k / 2k \\ &= 3(I_2 - I_3) \end{aligned}$$

Now

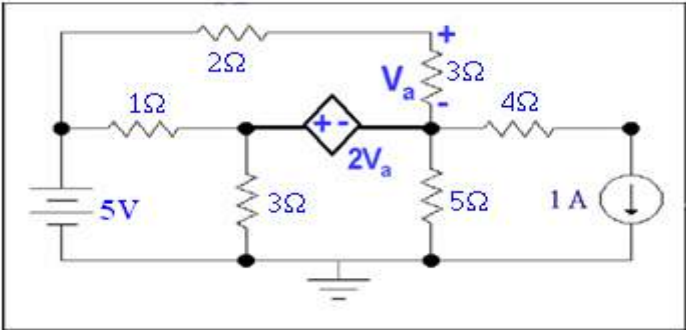
$$I_2 = 2mA$$

$$I_1 = 6mA - 3I_3$$

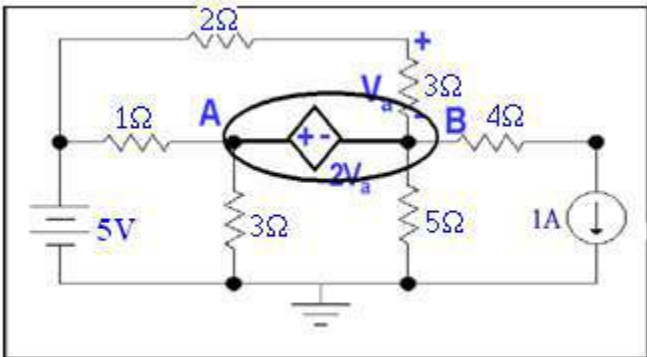
Now KVL equation for mesh 3

$$\begin{aligned} 6kI_3 + 6k(I_3 - I_2) + 2k(I_3 - I_1) &= 0 \\ 6kI_3 + 6kI_3 - 12 + 2kI_3 - 2k(6mA - 3I_3) &= 0 \\ 14kI_3 - 12 - 12 + 6kI_3 &= 0 \\ 20kI_3 &= 24 \\ I_3 &= 1.2mA \\ V_o &= 6kI_3 \\ &= 6k(1.2mA) \\ &= 7.2 \text{ volts} \end{aligned}$$

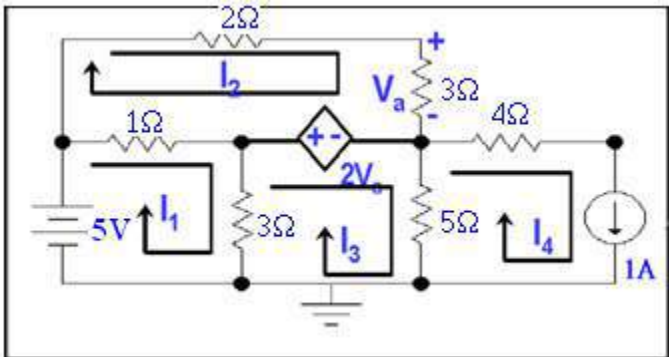
Example: Calculate the voltage V_a .



Solution: We want to calculate the voltage V_a . The circuit can be redrawn as



V_a can also be found by super node technique but it will take a lengthy calculation lets see how it becomes very easy with loop analysis.



Here

$$I_4 = 1A$$

KVL for mesh 1

$$1(I_1 - I_2) + 3(I_1 - I_3) - 5 = 0$$

$$I_1 - I_2 + 3I_1 - 3I_3 - 5 = 0$$

$$4I_1 - 3I_3 - I_2 = 5 \text{ -----(A)}$$

KVL for mesh 2

$$2I_2 + 3I_2 - 2V_a + I_2 - I_1 = 0$$

$$6I_2 - I_1 = 2V_a \text{ -----(B)}$$

$$V_a = 3I_2$$

Put this value in (b)

$$6I_2 - I_1 = 2(3I_2)$$

$$I_1 = 0$$

KVL for **mesh 3**

$$2V_a + 5(I_3 - I_4) + 3(I_3 - I_1) = 0$$

Putting the values of I_1, I_4 and V_a

$$6I_2 + 3I_3 - 0 + 5I_3 - 5 = 0$$

$$8I_3 + 6I_2 = 5$$

Solving equation for mesh 1 and mesh 3

$$-18I_3 - 6I_2 = 30$$

$$8I_3 + 6I_2 = 5$$

$$-10I_3 = 35$$

$$I_3 = -3.5A$$

Putting values of I_3 and I_1 in eq of mesh 1 i.e in (A) we have

$$10.5 - I_2 = 5$$

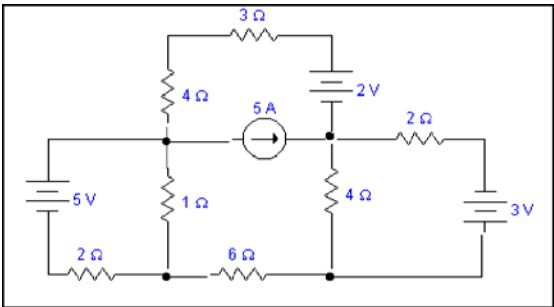
$$I_2 = 5.5 A$$

As $V_a = 3I_2$

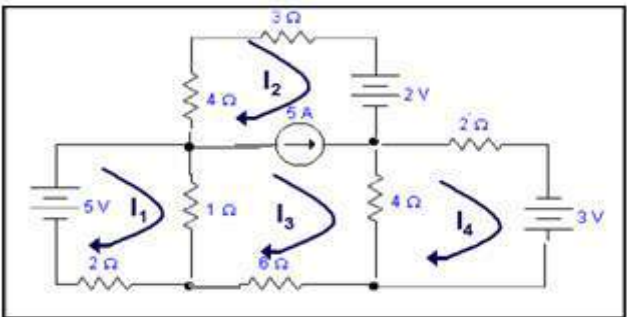
Therefore $V_a = 3 \times 5.5$

$$V_a = 16.5 V$$

Example: Find Current through all meshes.



Solution:



Incorrect approach

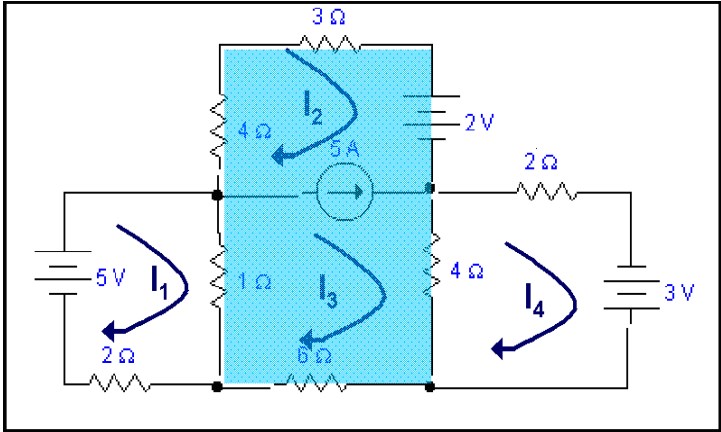
$$I_2 = -5A$$

$$I_3 = +5A$$

We derive all these

KVL equation for mesh 1

$$-5 + I_1 - I_3 + 2I_1 = 0 \text{ ----- (1)}$$



KVL equation for **super mesh**

$$1(I_3-I_1)+4I_2+3I_2+2+4I_3-4I_4+6I_3=0 \text{ ----- (2)}$$

KVL equation for **Mesh 4**

$$4I_4-4I_3+2I_4-3=0 \text{ ----- (3)}$$

Simplifying

Equation (1)	$3I_1-I_3=5$
Equation (2)	$-I_1+7I_2+11I_3-4I_4=-2$
Equation (3)	$-4I_3+6I_4=3$

Also $-I_2+I_3=5$

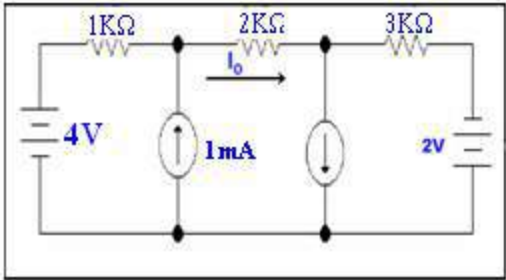
Therefore $I_1=2.481\text{A}$

$I_2=-2.556\text{A}$

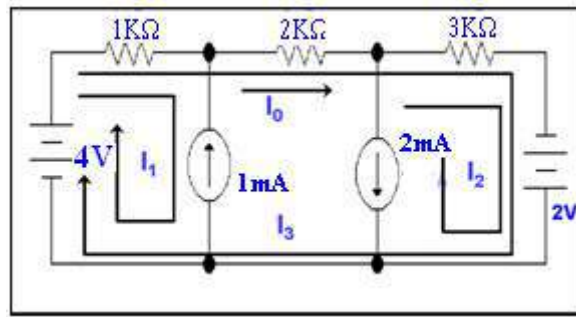
$I_3=2.444\text{A}$

$I_4=2.130\text{A}$

Example: Calculate current I_0 .



Solution: We want to calculate the **current I_0** . The circuit can be redrawn as



Here

$$I_1 = -1\text{mA}$$

$$I_2 = -2\text{mA}$$

KVL equation for loop 3

$$1\text{k}(I_3 + I_1) + 2\text{k}I_3 + 1\text{k}(I_3 + I_2) = 2 + 4$$

$$1\text{k}I_3 + 1\text{k}I_1 + 2\text{k}I_3 + 1\text{k}I_3 + 1\text{k}I_2 = 6$$

$$4\text{k}I_3 - 1 - 2 = 6$$

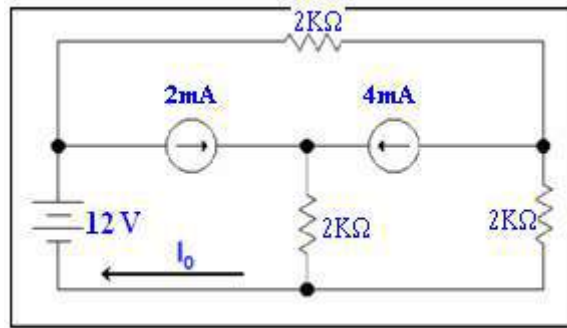
$$4\text{k}I_3 - 3 = 6$$

$$I_3 = 9/4 \text{ mA}$$

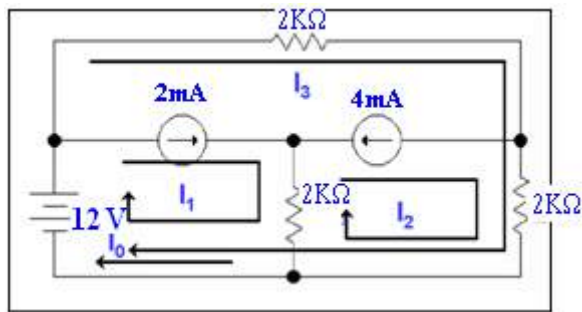
$$I_3 = 2.25 \text{ mA}$$

$$I_3 = I_0 = 2.25 \text{ mA}$$

Example: Calculate current I_0 .



Solution: We want to calculate the **current I_0** . The circuit can be redrawn as



Here

$$I_1 = 2\text{mA}$$

$$I_2 = -4\text{mA}$$

KVL for loop 3

$$2kI_3 + 2k(I_3 + I_2) = 12$$

$$2kI_3 + 2kI_3 + 2kI_2 = 12$$

$$4kI_3 - 8 = 12$$

$$4kI_3 = 20$$

$$I_3 = 5\text{mA}$$

Now

$$I_0 = I_3 + I_1$$

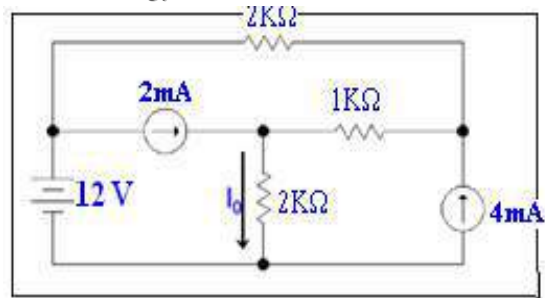
$$I_0 = 5\text{mA} + 2\text{mA}$$

$$= 7\text{mA}$$

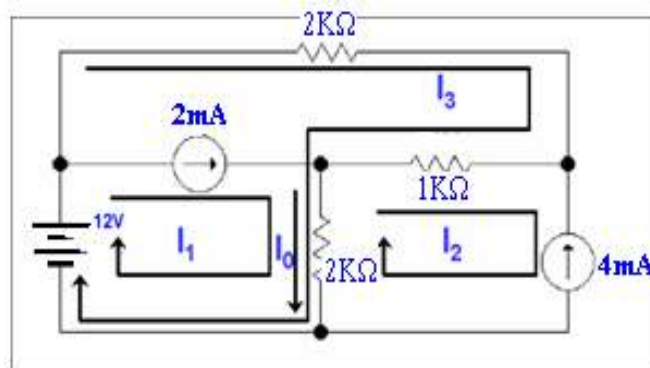
LECTURE 19

(Examples of Loop Analysis)

Example: Calculate the current I_0 .



Solution: we want to calculate the **current I_0** . The circuit can be redrawn as



Here

$$I_1 = 2\text{mA}$$

$$I_2 = -4\text{mA}$$

KVL for loop 3

$$2kI_3 + 1k(I_3 - I_2) + 2k(I_3 + I_1 - I_2) = 12$$

$$2kI_3 + 1kI_3 - 1kI_2 + 2kI_3 + 2kI_1 - 2kI_2 = 12$$

$$5kI_3 - 3kI_2 + 2kI_1 = 12$$

$$5kI_3 + 12 + 4 = 12$$

$$5kI_3 = -4$$

$$I_3 = -4/5 \text{ mA}$$

$$I_3 = .8\text{mA}$$

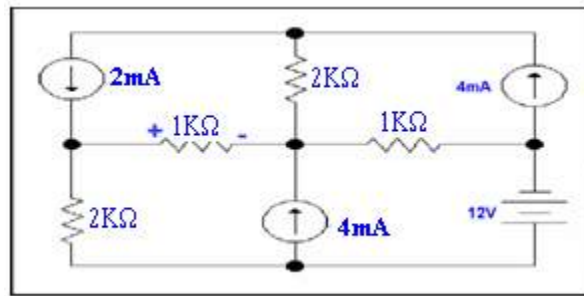
Now

$$I_0 = I_3 + I_1 - I_2$$

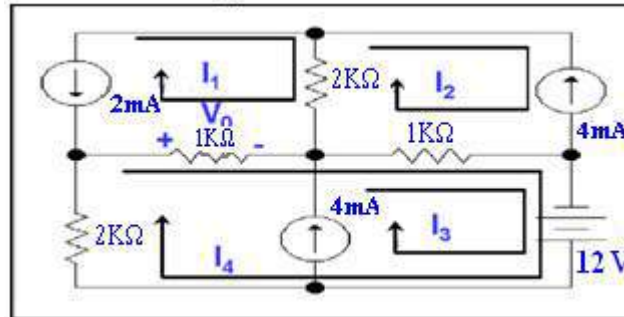
$$= -.8 + 2 + 4$$

$$I_0 = 5.2 \text{ mA}$$

Example: Calculate the voltage V_0 .



Solution: we want to calculate the voltage V_0 . The circuit can be redrawn as



Here

$$I_1 = -2\text{mA}$$

$$I_2 = -4\text{mA}$$

$$I_3 = 4\text{mA}$$

KVL for loop 4

$$1\text{k}(I_4 - I_1) + 1\text{k}(I_4 - I_2) + 1\text{k}I_3 + 2\text{k}I_4 = 12$$

$$1\text{k}I_4 - 1\text{k}I_1 + 1\text{k}I_4 - 1\text{k}I_2 + 1\text{k}I_3 + 2\text{k}I_4 = 12$$

$$4\text{k}I_4 + 2 + 4 + 4 = 12$$

$$I_4 = 2/4\text{k}$$

$$I_4 = 0.5\text{ mA}$$

Now current through 1k resistor

$$I_{1\text{k}} = I_4 - I_1$$

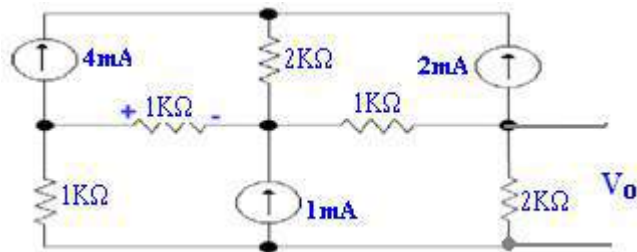
$$= 0.5 - (-2)$$

$$I_{1\text{k}} = 2.5\text{mA}$$

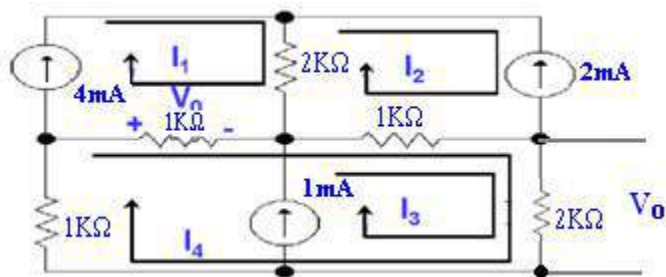
$$V_0 = 1\text{k} \times 2.5\text{mA}$$

$$V_0 = 2.5\text{ volts}$$

Example: Calculate the voltage V_0 .



Solution: we want to calculate the **voltage V_0** . The circuit can be redrawn as



Here

$$\begin{aligned} I_1 &= 4\text{mA} \\ I_2 &= -2\text{mA} \\ I_3 &= 1\text{mA} \end{aligned}$$

KVL for loop 4

$$\begin{aligned} 1k(I_4 - I_1) + 1k(I_4 - I_2 + I_3) + 2k(I_4 + I_3) + 1kI_4 &= 0 \\ 1kI_4 - 1kI_1 + 1kI_4 - 1kI_2 + 1kI_3 + 2kI_3 + 2kI_4 + 1kI_4 &= 0 \\ 5kI_4 - 4 + 1 + 2 + 2 &= 0 \\ 5kI_4 &= -1 \\ I_4 &= -0.2 \text{ mA} \end{aligned}$$

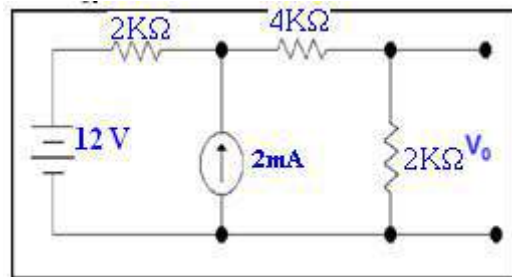
Now

$$\begin{aligned} V_0 &= 2k(I_4 + I_3) \\ &= 2k(-0.2 + 1) \\ \mathbf{V_0} &= \mathbf{1.6 \text{ volts}} \end{aligned}$$

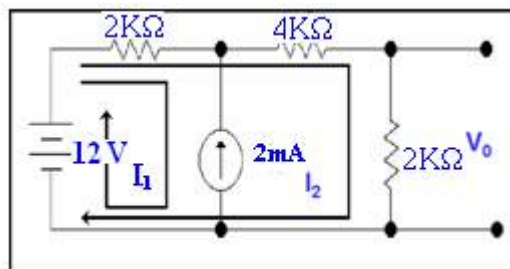
LECTURE 20

(Super Mesh Technique - Coupling equation)

Example: Calculate the voltage V_o .



Solution: we want to calculate the **voltage V_o** . The circuit can be redrawn as



Here

$$I_1 = -2\text{mA}$$

KVL equation for loop 2

$$2kI_2 + 4kI_2 + 2k(I_2 + I_1) = 12$$

$$6kI_2 + 2kI_2 + 2kI_1 = 12$$

$$8kI_2 - 4 = 12$$

$$8kI_2 = 16$$

$$I_2 = 2\text{mA}$$

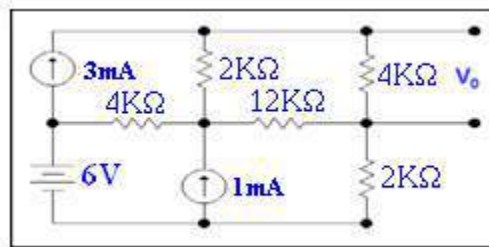
Now

$$V_o = 2k(I_2)$$

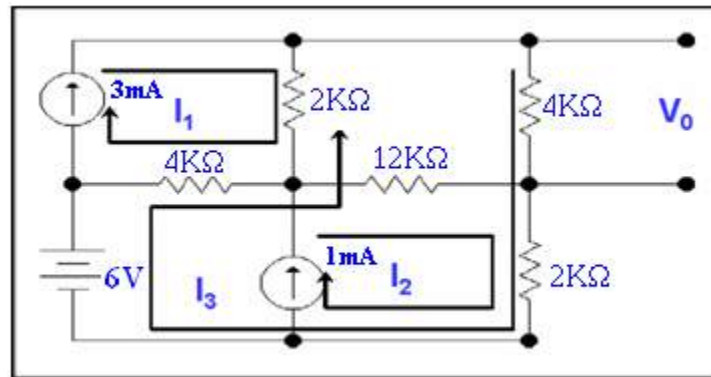
$$= 2k(2\text{m})$$

$$V_o = 4 \text{ volts}$$

Example: Calculate the voltage V_o .



Solution: We want to find the voltage V_o . The circuit can be redrawn as



Here

$$I_1 = 3\text{mA}$$

$$I_2 = 1\text{mA}$$

KVL for loop 3

$$4kI_3 + 2k(I_3 + I_2) + 4k(I_3 - I_1) + 2k(I_3 - I_1) = 6$$

$$4kI_3 + 2kI_3 + 2kI_2 + 4kI_3 - 4kI_1 + 2kI_3 - 2kI_1 = 6$$

$$12kI_3 + 2 - 12 - 6 = 12$$

$$12kI_3 = 22$$

$$I_3 = 11/6\text{mA}$$

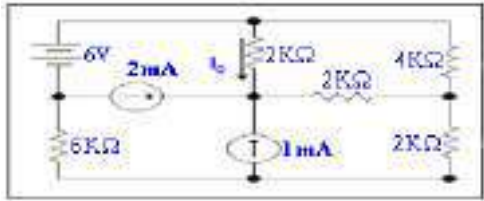
$$I_3 = 1.833\text{mA}$$

$$V_o = 4kI_3$$

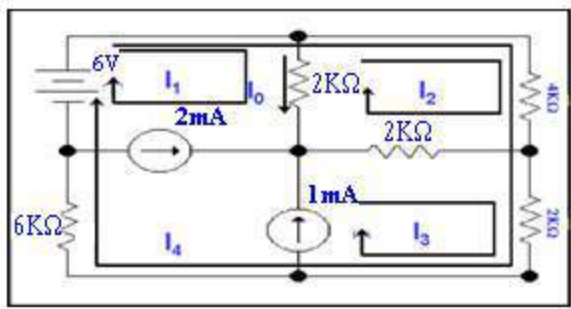
$$= 4k \times 1.833\text{mA}$$

$$V_o = 7.33 \text{ volts}$$

Example: Calculate the current I_o .



Solution: We want to find the current I_o . The circuit can be redrawn as



Here

$$I_1 = -2\text{mA}$$

$$I_3 = 1\text{mA}$$

Now KVL for **mesh 2**

$$\begin{aligned} 4k(I_2 + I_4) + 2k(I_2 - I_3) + 2k(I_2 - I_1) &= 0 \\ 8kI_2 + 4kI_4 - 2 + 4 &= 0 \\ 4kI_2 + 2kI_4 &= -1 \end{aligned}$$

Now KVL for **loop 4**

$$\begin{aligned} 4k(I_4 + I_2) + 2k(I_4 + I_3) + 6kI_4 - 6 &= 0 \\ 12kI_4 + 4kI_2 + 2 - 6 &= 0 \\ 12kI_4 + 4kI_2 - 4 &= 0 \\ 3kI_4 + 1kI_2 &= 1 \end{aligned}$$

Multiplying equation of **mesh2** by 3 and equation of **loop 4** by 2 and subtracting

$$\begin{aligned} 12kI_2 + 6kI_4 &= -3 \\ -2kI_2 + 6kI_4 &= -2 \\ \hline 10kI_2 &= -5 \\ I_2 &= -0.5\text{mA} \end{aligned}$$

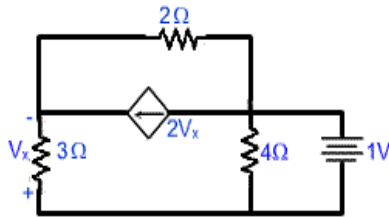
Now

$$\begin{aligned} I_o &= I_1 - I_2 \\ I_o &= -2 + 0.5 \end{aligned}$$

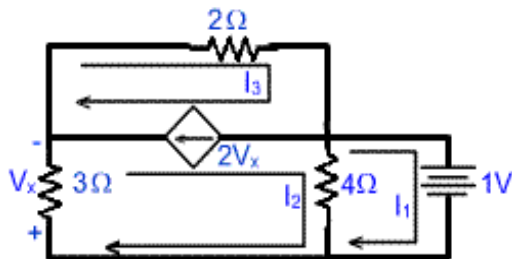
So

$$I_o = -1.5\text{mA}$$

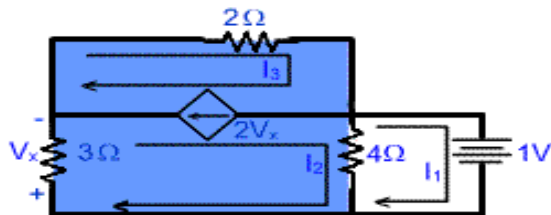
Example: Find the currents I_1, I_2 and I_3 .



Solution: We want to find the currents I_1, I_2 and I_3 . The circuit can be redrawn as



For mesh 1
 $4I_1 - 4I_2 + 1 = 0 \dots\dots\dots(1)$



For **Super mesh (mesh 2 and mesh 3)**
 $3I_2 + 2I_3 + 4I_2 - 4I_1 = 0 \dots\dots\dots(2)$

Coupling Equation
 $I_3 - I_2 = 2 V_X$

Here
 $V_X = 3I_2$

Solving equation 1
 $4I_1 - 4I_2 = -1$

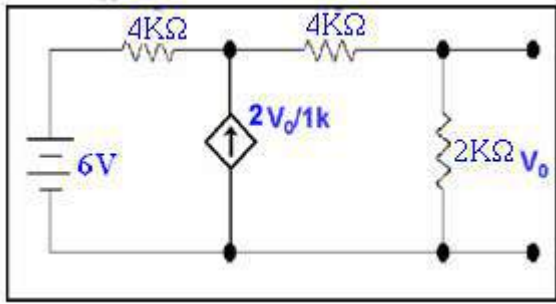
Solving equation 2
 $-4I_1 + 7I_2 + 2I_3 = -1$

For **Mesh 3**
 $-7I_2 + I_3 = 0 \dots\dots\dots(3)$

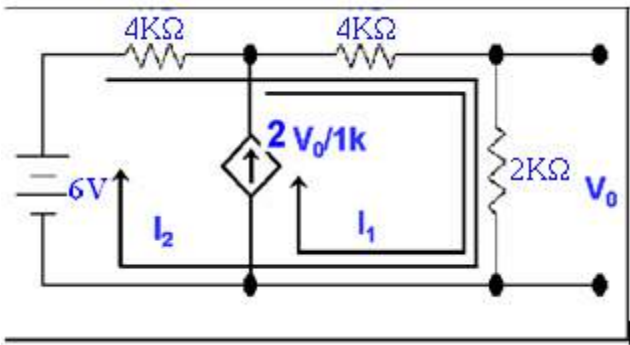
Solving equation 1, 2 and 3
 $I_1 = -308.8 \text{ mA}$
 $I_2 = -58.82 \text{ mA}$
 $I_3 = -411.8 \text{ mA}$

LECTURE 21
(Examples of Loop Analysis by using dependent sources - Coupling equation)

Example: Calculate the voltage V_o .



Solution: we want to calculate the **voltage V_o** . The circuit can be redrawn as



V_o can be given as

$$V_o = 2k(I_1 + I_2)$$

Now

$$\begin{aligned} I_1 &= 2V_o/1k \\ &= 2(2k(I_1 + I_2))/1k \\ &= 4(I_1 + I_2) \\ I_1 &= 4I_1 + 4I_2 \\ -3I_1 &= 4I_2 \\ I_1 &= -4/3 I_2 \end{aligned}$$

Now KVL for loop 2

$$\begin{aligned} 4kI_2 + 4k(I_1 + I_2) + 2k(I_1 + I_2) &= 6 \\ 4kI_2 + 4kI_1 + 4kI_2 + 2kI_1 + 2kI_2 &= 6 \\ 10kI_2 + 6kI_1 &= 6 \\ 5kI_2 + 3kI_1 &= 3 \end{aligned}$$

We know

$$I_1 = -4/3 I_2$$

So

$$\begin{aligned} 5kI_2 + 3k(-4/3)I_2 &= 3 \\ 5kI_2 - 4kI_2 &= 3 \\ I_2 &= 3mA \end{aligned}$$

So

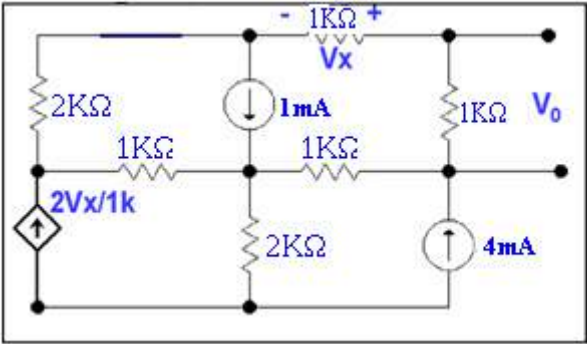
$$I_1 = (-4/3) I_2$$

$$I_1 = -4\text{mA}$$

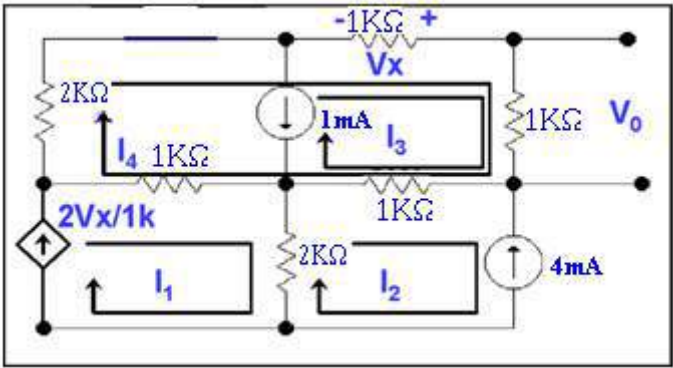
Now

$$\begin{aligned} V_0 &= 2k(I_1 + I_2) \\ &= 2k(-4\text{m} + 3\text{m}) \\ V_0 &= -2 \text{ volts} \end{aligned}$$

Example: Calculate the voltage V_0 .



Solution: We want to find the voltage V_0 . The circuit can be redrawn as



Here

$$\begin{aligned} I_2 &= -4\text{mA} \\ I_3 &= -1\text{mA} \end{aligned}$$

Now

$$V_x = -(I_3 + I_4)1k$$

and

$$\begin{aligned} I_1 &= 2V_x/1k \\ I_1 &= -2(I_3 + I_4) \\ &= 2 - 2I_4 \end{aligned}$$

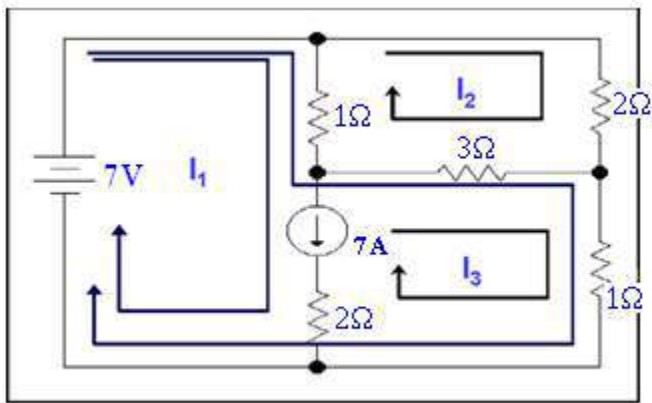
Now KVL for loop4

$$\begin{aligned} -1k(I_4 + I_3) + 1k(I_4 + I_3) + 1k(I_4 + I_3 - I_2) + 1k(I_4 - I_1) + 2kI_4 &= 0 \\ 2kI_4 - 2 \cdot 1kI_4 + 1 + 4 + 2kI_4 + 1k(I_4 - 2 + 2I_4) &= 0 \\ 6kI_4 + 1 &= 0 \\ 6kI_4 &= -1 \\ I_4 &= -1/6k \end{aligned}$$

Now

$$\begin{aligned} V_0 &= 1k(I_4 + I_3) \\ &= 1k(-1/6 - 1) \\ &= 1k(-1.166 - 1) \\ V_0 &= -1.166 \text{ volts} \end{aligned}$$

Example: Find loop currents.



Solution: We use the coupling equation technique to find loop currents to find the loop currents. The KVL equation for this **supermesh**

$$\begin{aligned} -7 + 1(I_1 - I_2) + 3(I_3 - I_2) + 1I_3 &= 0 \\ I_1 - 4I_2 + 4I_3 &= 7 \end{aligned} \tag{1}$$

And for mesh2

$$\begin{aligned} 1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) &= 0 \\ -I_1 + 6I_2 - 3I_3 &= 0 \end{aligned} \tag{2}$$

Coupling equation

$$I_1 - I_3 = 7 \tag{3}$$

solving (1) and (2)

$$\begin{aligned} 6I_1 - 24I_2 + 24I_3 &= 42 \\ -4I_1 + 24I_2 - 12I_3 &= 0 \end{aligned}$$

$$2I_1 + 12I_3 = 42 \tag{4}$$

Subtracting (3) and (4)

$$\begin{aligned} 2I_1 + 12I_3 &= 42 \\ 2I_1 - 2I_3 &= 14 \end{aligned}$$

$$\begin{aligned} 14I_3 &= 28 \\ I_3 &= 2 \text{ A} \end{aligned}$$

putting this in (1)

$$\begin{aligned} I_1 - 2 &= 7 \\ I_1 &= 9 \text{ A} \end{aligned}$$

and hence I_2 can be calculated by putting I_1 and I_3 in (1) or (2) from 1

$$\begin{aligned} 9 - 4I_2 + 8 &= 7 \\ -4I_2 &= -17 + 7 \\ 4I_2 &= 10 \\ I_2 &= 2.5 \text{ A} \end{aligned}$$

LECTURE 22

(Matrices and determinants)

MATRICES AND DETERMINANTS

are the coefficients of the independent variables. a_{ij} may be constant or functions of some parameter. A more convenient form may be obtained for the above equations by expressing them in matrix form

y_1

y_2

....

y_m

=

$a_{11} + a_{12} + a_{13} + + a_{1n}$

$a_{21} + a_{22} + a_{23} + + a_{2n}$

.....

$a_{m1} + a_{m2} + a_{m3} + + a_{mn}$

x_1

x_2

....

x_n

Or

$Y = A X$

Matrix $A=[a_{ij}]$ is called the characteristic matrix of the system; its order or dimension is denoted as $d(A) = m \times n$ where m is the number of rows and n is the number of columns. Attached to any matrix $A=[a_{ij}]$ is a certain scalar function of the a_{ij} , called the determinant of A. This number is denoted as

det A or |A| or Δ_A or

$a_{11} + a_{12} + a_{13} + ... + a_{1n}$

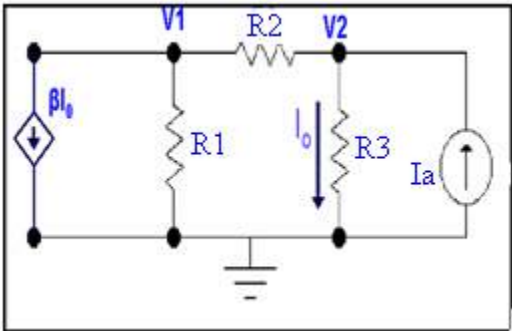
$a_{21} + a_{22} + a_{23} + ... + a_{2n}$

.....

$a_{m1} + a_{m2} + a_{m3} + ... + a_{mn}$

Applying these concepts to our analysis, the solution of simultaneous equations becomes very easy. Lets take some examples of KVL and KCL and apply this technique and we will see that the simplification of our equations will become extremely easy.

Example: Find current I_0 . By using matrices



Solution:

KCL at node 1

$\beta I_0 + V1/R1 + (V1 - V2)/R2 = 0$ ----- (1)

KCL at node 2

$(V2 - V1)/R2 + I_0 - I_a = 0$ ----- (2)

Where

$I_0 = V2/R3$ -----(A)

Simplifying these equations for node 1 and 2 put the value of equation A in 1 and 2 we have

$$\beta V_2/R_3 + V_1/R_1 + V_1/R_2 - V_2/R_2 = 0$$

$$(1/R_1 + 1/R_2) V_1 - (1/R_2 - \beta/R_3) V_2 = 0$$

For node 2

$$-(1/R_2) V_1 + (1/R_2 + 1/R_3) V_2 = I_a$$

Or in **matrix form**

$$\begin{bmatrix} (1/R_1 + 1/R_2) & -(1/R_2 - \beta/R_3) \\ -1/R_2 & (1/R_2 + 1/R_3) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_a \end{bmatrix}$$

Now let's determine the node voltage for the following given parameters

$$\beta = 2, R_2 = 6k, R_1 = 12k, R_3 = 3k, I_a = 2mA$$

Using these equations for the network yields

$$(1/4k) V_1 + (1/2k) V_2 = 0$$

$$-(1/6k) V_1 + (1/2k) V_2 = 2 \text{ mA}$$

Or in **matrix form**

$$\begin{bmatrix} 1/4k & 1/2k \\ -1/6k & 1/2k \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2m \end{bmatrix}$$

The circuit equations can be solved using matrix analysis. The general equation is

form of matrix

$$A X = Y \text{ or } G V = I \text{ we want to find } X \text{ or } V$$

$$\text{where in this case } V \text{ or } X = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, G \text{ or } A = \begin{bmatrix} 1/4k & 1/2k \\ -1/6k & 1/2k \end{bmatrix} \text{ and } I \text{ or } Y = \begin{bmatrix} 0 \\ 2m \end{bmatrix}$$

The solution of the matrix equation is

$$X = A^{-1} Y \text{ OR } V = G^{-1} I \text{ -----(B)}$$

To find A^{-1} or G^{-1}

We know that $A^{-1} = \text{Adj}(A) / |A|$ or $G^{-1} = \text{Adj}(G) / |G|$

Adjoint of the **coefficient matrix A** is

$$\text{Adj}(A) = \begin{bmatrix} 1/2k & -1/2k \\ 1/6k & 1/4k \end{bmatrix}$$

And the determinant will be

$$|A| = (1/4k)(1/2k) - (1/6k)(-1/2k)$$

$$= 5/24k^2$$

$$A^{-1} = \text{Adj}(A) / |A|$$

$$A^{-1} = \frac{\begin{bmatrix} 1/2k & -1/2k \\ 1/6k & 1/4k \end{bmatrix}}{\frac{5}{24k^2}}$$

by putting A^{-1} in equation (B) we have

Therefore

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 24k^2/5 \begin{bmatrix} 1/2k & -1/2k \\ 1/6k & 1/4k \end{bmatrix} \begin{bmatrix} 0 \\ 2m \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{24k^2}{5} \begin{bmatrix} (1/2k)(0) + (-1/2k)(2m) \\ (1/6k)(0) + (1/4k)(2m) \end{bmatrix}$$
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{24k^2}{5} \begin{bmatrix} 0 - 1 \\ 0 + \frac{1}{2} \end{bmatrix}$$
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{24}{5} \\ \frac{12}{5} \end{bmatrix}$$

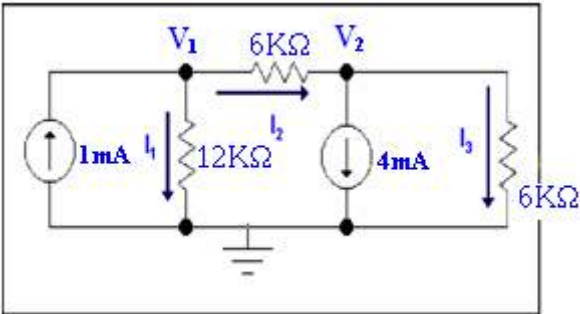
Hence

$V_1 = -24/5V$
 $V_2 = 12/5 V$

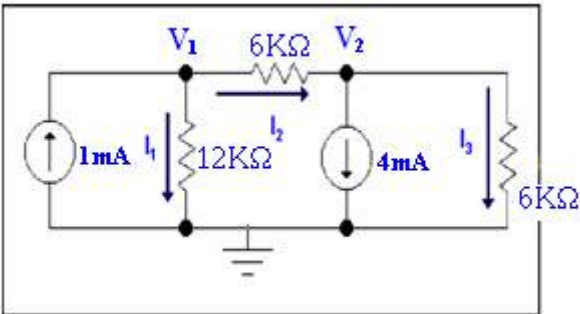
Hence I_0

$I_0 = V_2/R_3$
 $= (12/5)/3k$
 $= 4/5k$
 $= 4/5mA$
 $I_0 = 0.8mA$

Example: Find current I_1 , I_2 and I_3 . By using matrices.



Solution: We want to calculate the currents I_1 , I_2 and I_3 . By using matrices proceed as



KCL at **node 1**
 $-1m + (1/12k)(V_1 - 0) + (1/6k)(V_1 - V_2) = 0$
 $(1/12k + 1/6k)V_1 - (1/6k)V_2 = 1mA$ ----- (A)

KCL for **node 2**
 $-(1/6k)(V_1 - V_2) + 4m + (1/6k)(V_2 - 0) = 0$
Which can be expressed as
 $-(1/6k)V_1 + (1/6k + 1/6k)V_2 = -4mA$ ----- (B)

Simplifying these two equations (A) and (B)
 $V_1/4k - V_2/6k = 1mA$
 $-V_1/6k + V_2/3k = -4mA$

Or in **matrix form**

$$\begin{bmatrix} 1/4k & -1/6k \\ -1/6k & 1/3k \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1m \\ -4m \end{bmatrix}$$

The circuit equations can be solved using matrix analysis. The general form of matrix equation is

$A X = Y$ or $GV=I$ we want to find X or V

where in this case V or $X = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, G or $A = \begin{bmatrix} 1/4k & -1/6k \\ -1/6k & 1/3k \end{bmatrix}$ and I or $Y = \begin{bmatrix} 1m \\ -4m \end{bmatrix}$

The solution of the matrix equation is

$$X = A^{-1}Y \text{ OR } V = G^{-1}I \text{ ----- (C)}$$

To find A^{-1} or G^{-1} we know that $A^{-1} = \text{Adj}(A) / |A|$ or $G^{-1} = \text{Adj}(G) / |G|$

Ad joint of the **coefficient matrix A** is

$$\text{Adj}(A) = \begin{bmatrix} 1/3k & 1/6k \\ 1/6k & 1/4k \end{bmatrix}$$

And the determinant will be

$$|A| = (1/3k)(1/4k) - (-1/6k)(-1/6k)$$

$$= 1/18k^2$$

$$A^{-1} = \text{Adj}(A) / |A|$$

$$A^{-1} = \frac{\begin{bmatrix} 1/3k & 1/6k \\ 1/6k & 1/4k \end{bmatrix}}{\frac{1}{18k^2}}$$

by putting A^{-1} in equation (C) we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 18k^2 \begin{bmatrix} 1/3k & 1/6k \\ 1/6k & 1/4k \end{bmatrix} \begin{bmatrix} 1m \\ -4m \end{bmatrix}$$

By solving the above equation we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -6 \\ -15 \end{bmatrix}$$

Hence

$$V_1 = -6V$$

$$V_2 = -15V$$

Knowing the voltages we can determine all currents using **OHM's LAW**

$$I_1 = V_1/12k$$

$$= -6/12k$$

$$= -1/2mA$$

$$I_1 = -0.5mA$$

Now I_2 can be calculated as

$$I_2 = (V_1 - V_2)/6k$$

$$= -6 - (-15)/6k$$

$$I_2 = 3/2mA$$

Now for I_3

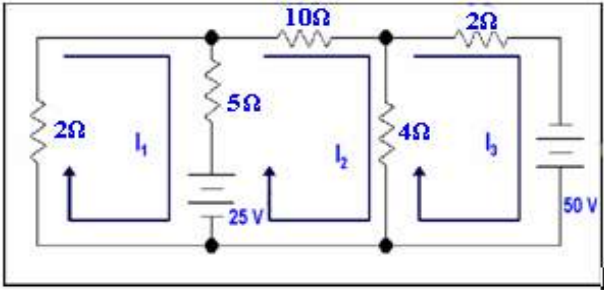
$$I_3 = V_2/6k$$

$$= -15/6k$$

$$I_3 = -5/2mA$$

LECTURE 23
(Superposition Theorem and examples)

Example: Find current I_1 , I_2 and I_3 By using Cramer's rule.



Solution: We will use Cramer's rule to solve this example

KVL equation for loop 1

$$\begin{aligned} 2I_1 + 5(I_1 - I_2) &= -25 \\ 2I_1 + 5I_1 - 5I_2 &= -25 \\ 7I_1 - 5I_2 &= -25 \text{ ----- (A)} \end{aligned}$$

KVL equation for loop 2

$$\begin{aligned} 10I_2 + 4(I_2 - I_3) + 5(I_2 - I_1) &= 25 \\ 10I_2 + 4I_2 - 4I_3 + 5I_2 - 5I_1 &= 25 \\ -5I_1 + 19I_2 - 4I_3 &= 25 \text{ ----- (B)} \end{aligned}$$

KVL equation for loop 3

$$\begin{aligned} 2I_3 + 4(I_3 - I_2) &= 50 \\ 2I_3 + 4I_3 - 4I_2 &= 50 \\ -4I_2 + 6I_3 &= 50 \text{ ----- (C)} \end{aligned}$$

We will write equation A ,B and C In matrix form

$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \\ 50 \end{bmatrix}$$

The determinant of the coefficient matrix is

$$\begin{aligned} |A| &= \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix} \\ |A| &= 536 \end{aligned}$$

By Cramer's rule

$$\begin{aligned} I_1 &= \frac{\begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} \\ I_1 &= (-700) \div 536 \\ I_1 &= -1.31 \text{ A} \end{aligned}$$

By Cramer's rule

$$I_2 = \begin{vmatrix} 7 & -25 & 0 \\ -5 & 25 & -4 \\ 0 & 50 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$$I_2 = 1700/536$$

$$I_2 = 3.17A$$

By Cramer's rule

$$I_3 = \begin{vmatrix} 7 & -5 & -25 \\ -5 & 19 & 25 \\ 0 & -4 & 50 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}$$

$$I_3 = 5600/536$$

$$I_3 = 10.45 A$$

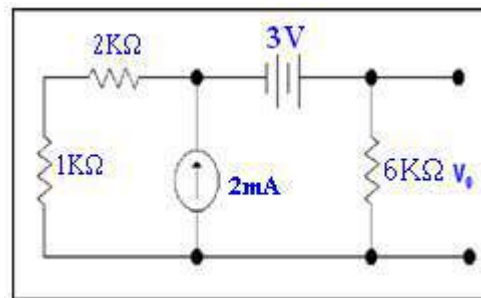
Superposition Theorem :

The principle of superposition, which provides us with the ability to reduce a complicated problem to several easier problems – each containing only a single independent source – states that

“In any linear circuit containing multiple sources, the current or voltage at any point in the circuit may be calculated as the algebraic sum of the individual contributions of each source acting alone.”

When determining the contributions due to independent sources, any remaining current sources are made zero by replacing them by open circuit and any voltage sources are made zero by replacing them by short circuit.

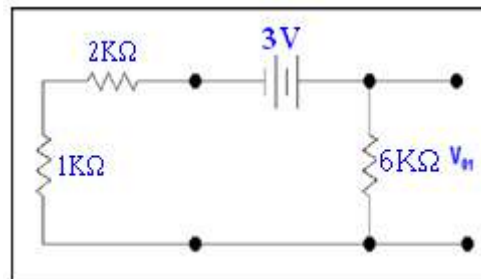
Example: Calculate V_o by applying principle of superposition.



Solution:

Applying principle of superposition, we will take effect of the sources one by one.

Only voltage source is acting



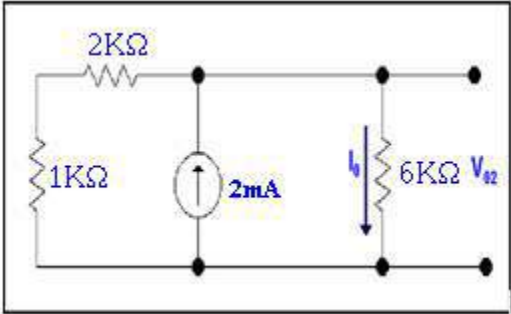
Applying voltage division rule

$$V_{R1} = \frac{R_1 \times V_1}{R_1 + R_2}$$

$$V_{01} = V_{6k} = (6 \times 3) / (3 + 6)$$

$$V_{01} = 2 \text{ volts}$$

Only current source is acting



Let the current through 6k resistor is I_0
by applying current division rule

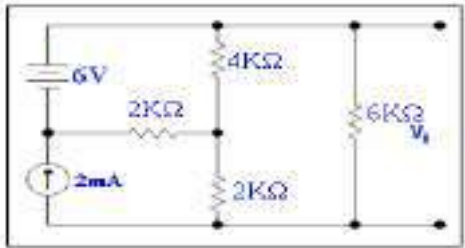
$$I_{R1} = \frac{R_2 \times I(t)}{R_1 + R_2}$$

$$I_0 = (3 \times 2)/9 = 2/3 \text{ mA}$$
$$V_{02} = I_0 \times 6k$$
$$= 2/3m \times 6k = 4V$$

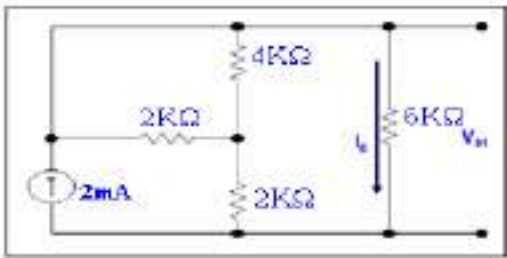
So V_0 will be

$$V_0 = V_{01} + V_{02}$$
$$= 2V + 4V$$
$$V_0 = 6V$$

Example: Calculate V_0 by applying principle of superposition.



Solution: We want to calculate the voltage V_0 . Superposition theorem can be applied as
Only current source is acting



Let I_0 be the current following through the 6k resistor then

$$2k \parallel 4k = (2 \times 4)/(2+4) = 8/6 = (4/3)k$$

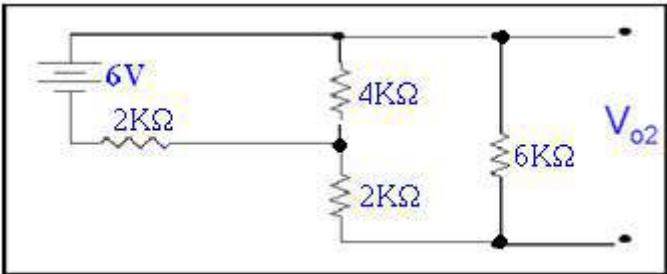
$(4/3)k$ is in series to 2k

$\therefore (4/3)k + 2k \parallel 6k$, hence applying current division rule

$$I_{R1} = \frac{R_2 \times I(t)}{R_1 + R_2}$$

$$I_o = \frac{(4/3)+2}{(4/3+2)+6} 2mA = \frac{20}{3(28/3)} = \frac{20}{28}$$
$$V_{o1} = \frac{20}{28} 6 = \frac{30}{7} V$$

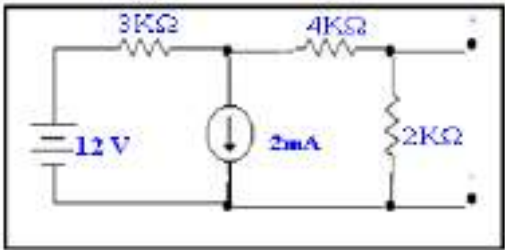
Making only voltage source to act



Or it can be redrawn as

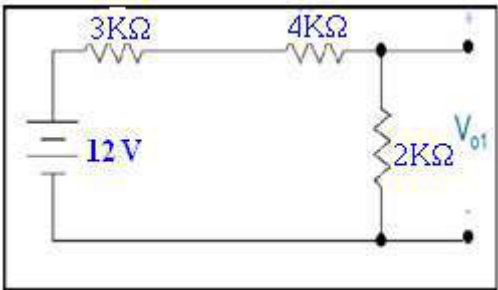
$$R_{ab} = 8k \parallel 4k = \frac{8 \times 4}{8 + 4} = \frac{8}{3} k$$
$$V_{ab} = \frac{\frac{8}{3} k}{\frac{8}{3} k + 2k} 6 = \frac{24}{7}$$
$$V_{o2} = \frac{24}{7} \times \frac{6k}{6k + 2k} = \frac{18}{7} \text{ Volts.}$$
$$V_o = V_{o1} + V_{o2} = \frac{30}{7} + \frac{18}{7} = \frac{48}{7} \text{ Volts}$$

Example: Calculate V_o by applying principle of superposition.



Solution:

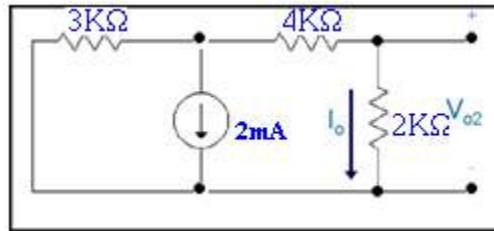
We want to calculate V_o .
When only voltage source is acting



Applying voltage division rule

$$V_{R1} = \frac{R_1 \times V_1}{R_1 + R_2}$$
$$V_{o1} = (12 \times 2) / (7 + 2) = 24/9V$$

When only current source is acting



Let I_o flows through $2k\Omega$. applying current division rule

$$I_{R1} = \frac{R_2 \times I(t)}{R_1 + R_2}$$

$$I_o = (-2)3/(6+3) = -6/9 = -2/3 \text{ mA}$$

$$V_{o2} = 2 \times -2/3 = -4/3 \text{ Volts}$$

Therefore,

$$\begin{aligned} V_o &= V_{o1} + V_{o2} \\ &= 24/9 - 4/3 \\ V_o &= 4/3 \text{ Volts} \end{aligned}$$

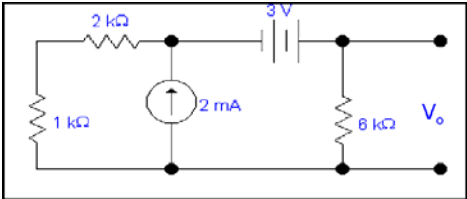
LECTURE 24
(Source Transformation and examples)

Source Transformation:

If we have any source embedded within a network, say this source is a current source having a value I & there exists a resistance having a value R , in parallel to it. We can replace it with a voltage source of value $V=IR$ in series with same resistance R . The reverse is also true that is a voltage source V , in series with a resistance R can be replaced by a current source having a value $I= V/R$

In parallel to the resistance R . Parameters within circuit, for example an output voltage remains unchanged under these transformations.

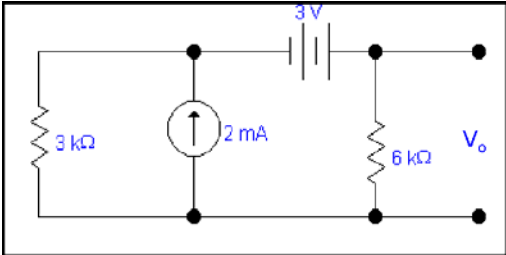
Example: Calculate the voltage V_o using source transformation method.



Solution:

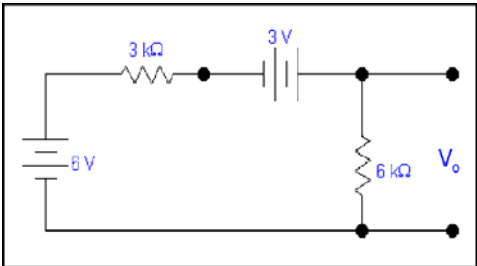
We want to calculate the voltage V_o using source transformation method. We proceed as

1k is in series with 2k so combined effect =3k

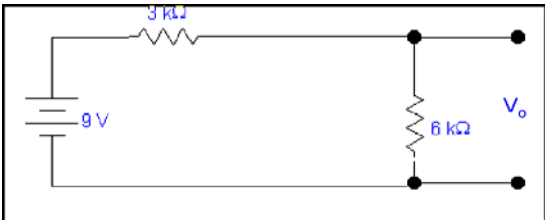


Now 2mA source is in parallel with 3k resistor. So it can be changed to a voltage source of value = $2\text{m} \times 3\text{k}$ (**by ohm's Law**) = 6Volts.

3k resistor will become in series with this source as shown in the circuit below



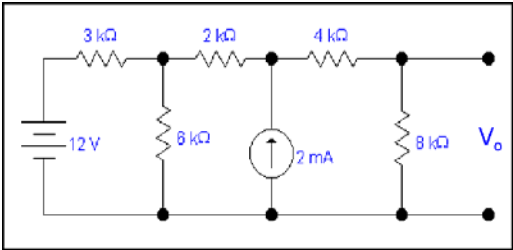
Positive terminal of the 6 volts battery is connected with the negative terminal of 3 volts battery so they will be summed up as shown in the circuit below



Applying voltage division rule

$$V_{R1} = \frac{R_1 \times V_1}{R_1 + R_2}$$
$$V_o = (6 \times 9)/9$$
$$= 6 \text{ Volts}$$

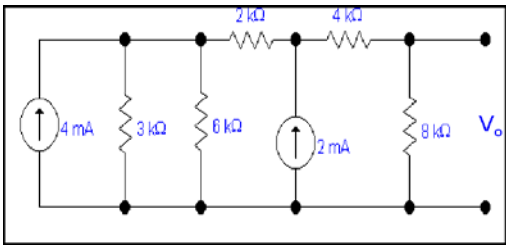
Example: Calculate the voltage V_o using source transformation method.



Solution:

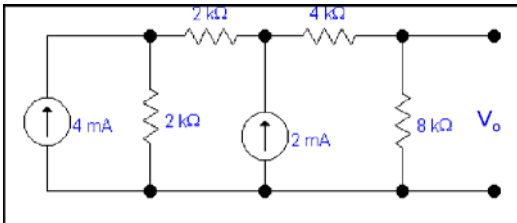
We want to calculate the voltage V_o using source transformation method. We proceed as
3k is in series with 12 volts battery. So it can be converted into a current source of value $=12/3k$ (by ohm's Law)
 $=4mA$

Now voltage source has transformed in the current source as shown below

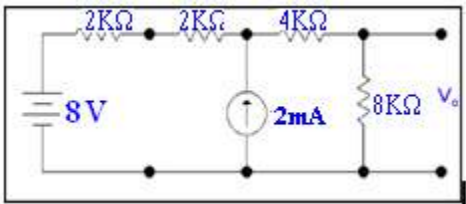


Now 3k resistor is in parallel with 6k resistor so
 $3k||6k = (3k \times 6k)/3k + 6k$
 $= 2k$

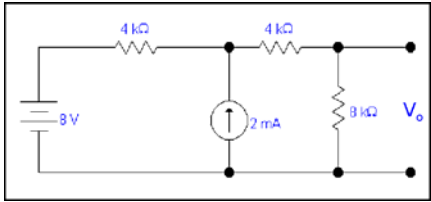
Now we will replace 3k and 6k resistance with 2k as shown in the circuit below



4mA source is parallel with 2k resistor. So it can be converted into a voltage source. By using ohm's Law we have the value of voltage source = 8 volts which is in series with a resistor of 2k, so our modified circuit will be



In the above circuit 2k is in series with 2k so modified circuit will be

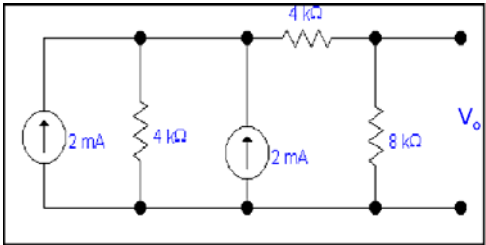


8V source is in series with 4k resistor. So it can be converted into a current source of value $I = \frac{8V}{4k} = 2mA$ (by using Ohm's Law)

$$I = 2mA$$

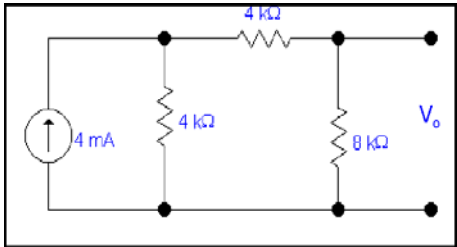
2mA current source is in parallel to 4k

Our modified circuit will be as

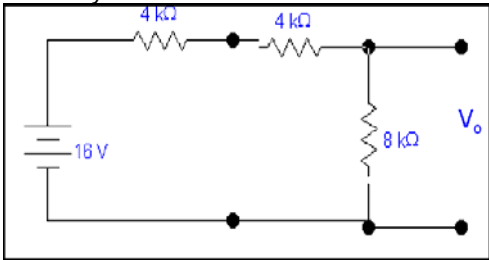


In the above circuit two current sources are parallel to each other, so current sources will add up to give value of 4mA.

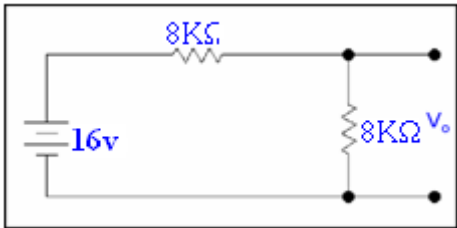
Our modified circuit will be as



In the above circuit 4mA source is in parallel with 4k resistor .So it can be converted into a voltage source of value 16volts by Ohm's Law. Our modified circuit will be as



In the above circuit 4k and 4k are in series so our modified circuit will be



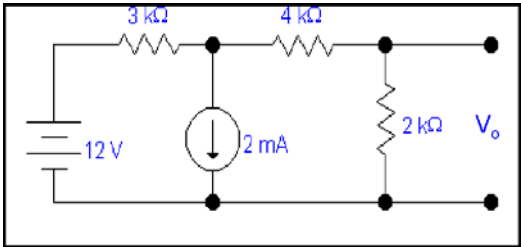
Now applying voltage division rule

$$V_{R1} = \frac{R_1 \times V_1}{R_1 + R_2}$$

$$V_0 = \frac{8k}{16k} \times 16V$$

$$V_0 = 8volts$$

Example: Calculate the voltage V_o using source transformation method.



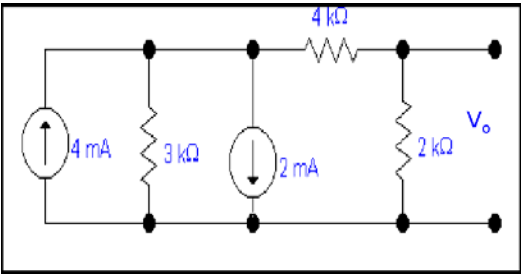
Solution:

We want to calculate the voltage V_o using source transformation method. We

Proceed as

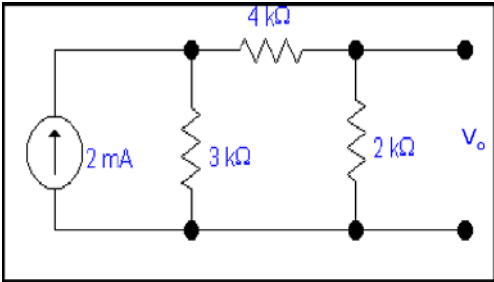
12 volt source is in series with 3k resistor. So it can be converted into current source of value $I = 12/3k = 4mA$ (by Ohm's Law)

In modified circuit 4mA current source and resistor 3k will be in parallel. So modified circuit will be as



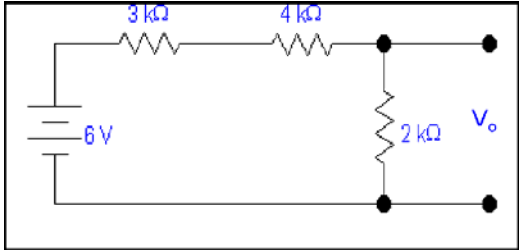
Now we will combine current sources as $4mA - 2mA = 2mA$, so our modified

circuit will be as,

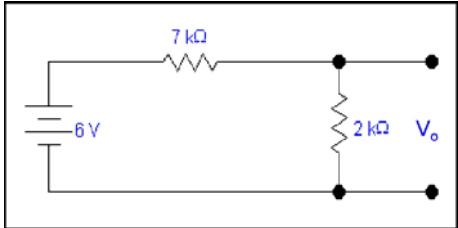


2mA source is in parallel with 3k source so it can be converted into a voltage source of value $V = 2 \times 3 = 6V$ (by Ohm's Law)

Our modified circuit will be as,



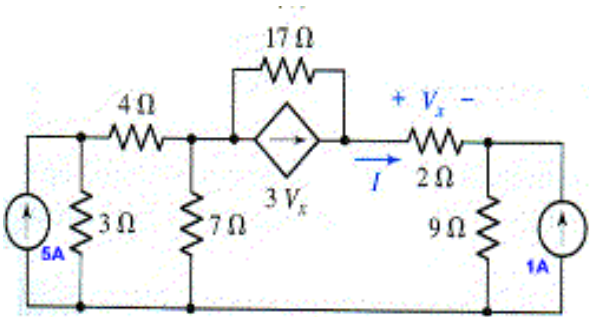
In the above circuit 3k is in series with 4k so modified circuit will be as



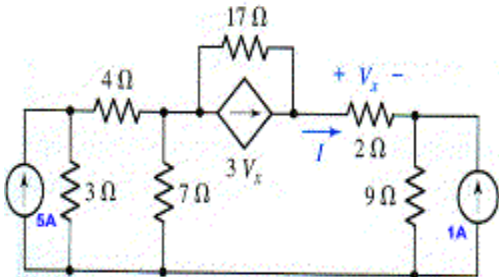
Now applying voltage division rule

$$V_{R1} = \frac{R_1 \times V_1}{R_1 + R_2}$$
$$V_0 = 2k/9k \times 6$$
$$= 12/9$$
$$V_0 = 4/3 \text{ volts}$$

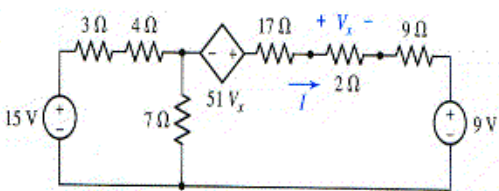
Example: Calculate the current I using source transformation method.



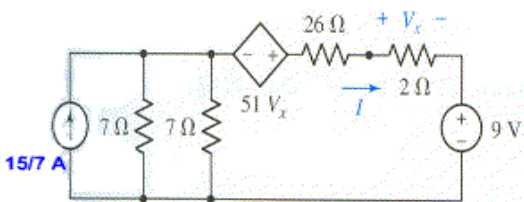
Solution: we want to calculate the current I by source transformation.



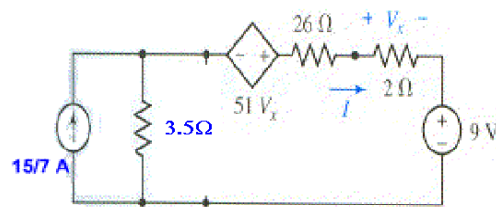
3 Ohms resistor is in parallel with 5A source. So it can be converted into a voltage source 15V.



3 Ohms resistor 4 Ohms are in series and the resultant is in series with 15V so it can be converted into a current Source.

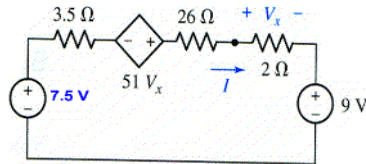


$$7 \parallel 7 = 7 \times 7 / 7 + 7 = 49/14 = 3.5 \text{ ohm}$$



Current source is in parallel with 3.5 ohms resistance. So it can be converted into a Voltage source by using Ohm's Law

$$V = IR = 15/7 \times 3.5 = 7.5 \text{ Volts}$$



The current I can be found using KVL

$$-7.5 + 3.5I - 51V_x + 28I + 9 = 0 \text{ ----- (A)}$$

Where

$$V_x = 2I$$

putting the value of V_x in (A) we have

$$-7.5 + 3.5I - 51(2I) + 28I + 9 = 0$$

$$3.5I - 102I + 28I + 1.5 = 0$$

$$-70.5I + 1.5 = 0$$

$$I = 1.5/70.5$$

$$I = 21.28 \text{ mA}$$

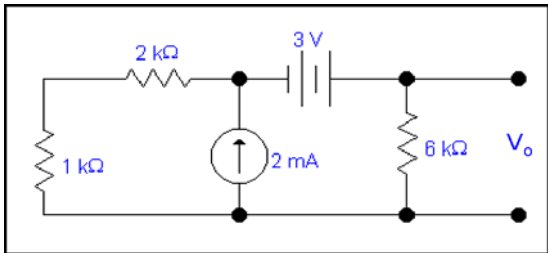
LECTURE 25
(Thevenin's Theorem)

THEVENIN'S THEOREM:

In solving any problem by Thevenin's theorem. We will follow the steps as stated below

- (1) First of all we will remove the load resistor(The resistor we want to calculate the unknown quantity that is voltage, current or power) from the circuit leaving behind an open circuit.
- (2) Calculating the V_{th} at open terminals of the open circuit by any method.
- (3) Calculating the R_{th} (The total resistance in the Thevenin's circuit) by open circuiting all current sources and short circuiting all voltage sources.
- (4) After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.

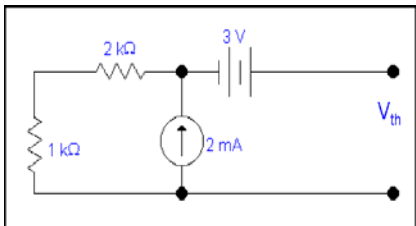
Example: Calculate the voltage V_o by using Thevenin's theorem .



Solution:

We want to calculate V_o by using Thevenin's theorem .we will follow the steps given earlier.

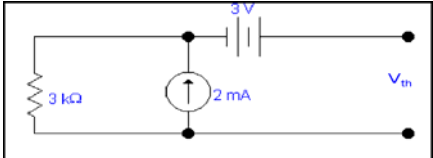
First step: Removing R_L



Here R_L is 6k resistor at which we want to calculate the voltage V_o .

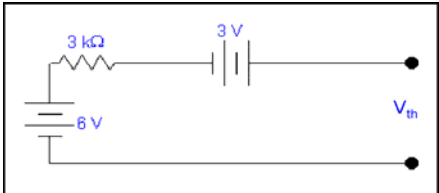
Second step: Calculating V_{th}

1k is in series with 2k so modified circuit will be as

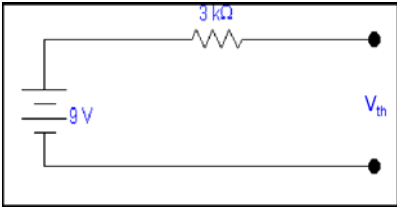


3k is in parallel with 2mA source, so by source transformation
Now 2mA source is in parallel with 3k resistor. So it can be changed to a
voltage source of value = $2m \times 3k$ (**by ohm's Law**)
= 6 Volts.

3k resistor will become in series with this source as shown in the circuit below

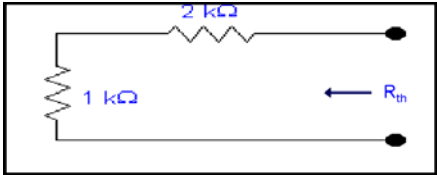


Now the combined effect of these two source will be 9 volts.



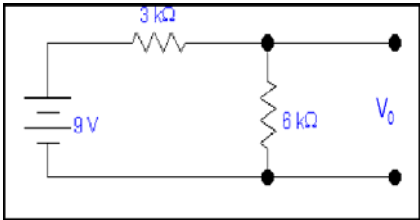
So $V_{th} = 9 \text{ volts}$

Third step: Calculating R_{th}



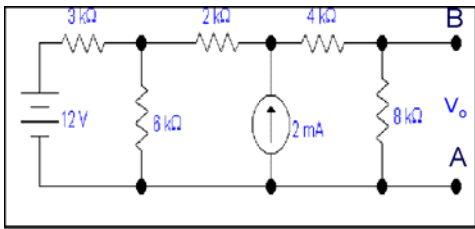
$R_{th} = 3k$

Fourth step: Calculating the unknown quantity.
After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.



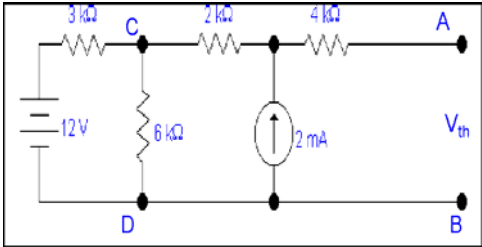
$$V_o = (6k/9k) \times 9$$
$$V_o = 6 \text{ volts}$$

Example: Calculate the voltage V_o by using Thevenin's theorem .



Solution: We want to calculate V_o by using Thevenin's theorem .we will follow the steps given earlier.

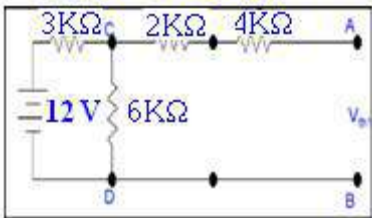
First step: Removing R_L
Here R_L is 8k resistor at which we want to calculate the voltage V_o .



Second step: Calculating V_{th}

Now we will follow the **superposition method** to calculate V_{th} .

Only voltage source is acting

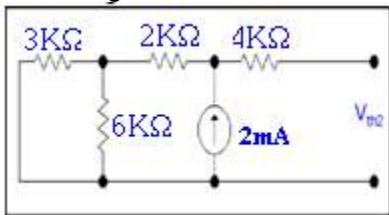


No current in 'CA' branch so voltage drop across 4k and 2k resistor is zero so
 $V_{CD} = V_{th1}$

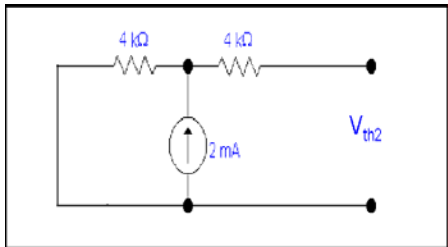
Applying voltage division rule

$$V_{CD} = V_{th1} = V_{6k} = (12 \times 6) / (3 + 6) = 8 \text{ volts}$$

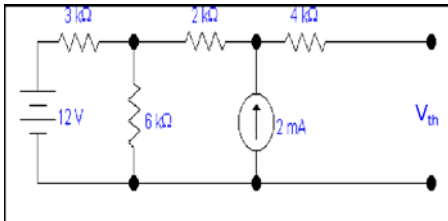
Only current source is acting



$$3k \parallel 6k = (3k \times 6k) / (3k + 6k) = 2k$$
$$2k \text{ is in series with } 2k = 4k$$



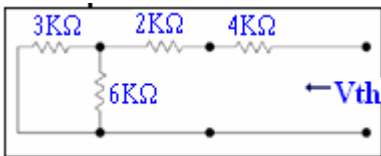
Current through open circuit is zero by ohm's law so
 $V_{th2} = 4k \times 2m$
 $V_{th2} = 8 \text{ volts}$



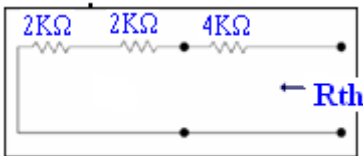
So

$$V_{th} = V_{th1} + V_{th2} = 8 + 8 = 16 \text{ volts}$$

Third step: Calculating R_{th}

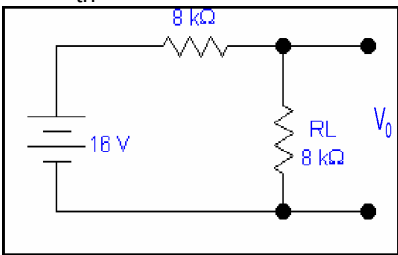


$3k \parallel 6k = (3k \times 6k) / 9k$
 $= 2k$



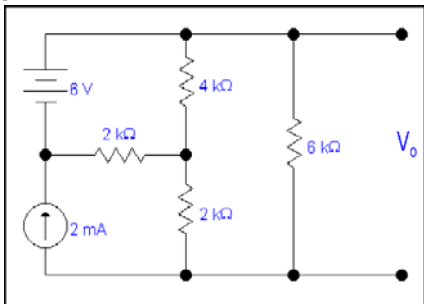
$R_{th} = 2k + 2k + 4k$
 $R_{th} = 8k$

Fourth step: Calculating unknown quantity.
After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.

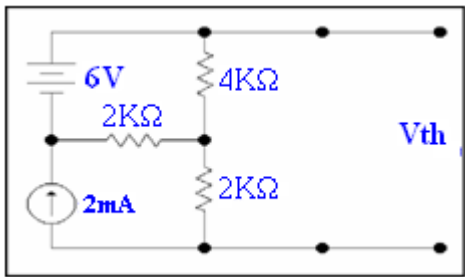


$V_0 = (8k / 16k) \times 16$
 $V_0 = 8 \text{ volts}$

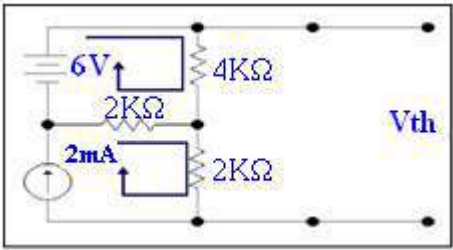
Example: Calculate the voltage V_0 by using Thevenin's theorem .



Solution:
We want to calculate V_0 by using Thevenin's theorem .we will follow the steps given earlier.
First step: Removing R_L
Here R_L is 6k resistor at which we want to calculate the voltage V_0 .



Second step: Calculating V_{th}



Apply KVL to the circuit

Here

$$I_2 = 2mA$$

For loop 1

$$4kI_1 + 2k(I_1 - I_2) = 6$$

$$4kI_1 + 2kI_1 - 2kI_2 = 6$$

$$6kI_1 - 2kI_2 = 6$$

Putting value of I_2

$$6kI_1 - 4 = 6$$

$$I_1 = 10/6k$$

$$I_1 = 1.6mA$$

Now voltage across 4k resistor

$$V_{4k} = 1.6m \times 4k = 20/3$$

$$V_{4k} = 6.66volts$$

Voltage across 2k resistor

$$V_{2k} = 2m \times 2k$$

$$V_{2k} = 4volts$$

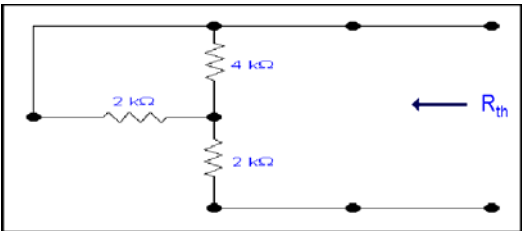
So

$$V_{th} = V_{4k} + V_{2k}$$

$$= 20/3 + 4 = 6.66 + 4$$

$$V_{th} = 10.66 \text{ volts}$$

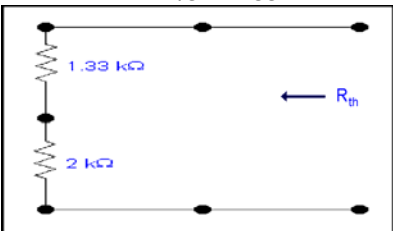
Third step: Calculating R_{th}



$$4k || 2k = 4k \times 2k / 6k$$

$$= 8/6 k$$

$$= 4/3 k = 1.33 k$$



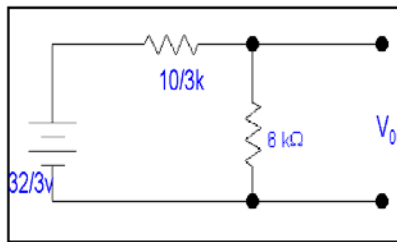
They are in series so

$$R_{th} = 1.33k + 2k = 10/3k$$

$$R_{th} = 3.33k$$

Fourth step: Calculating unknown quantity.

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these resistances.

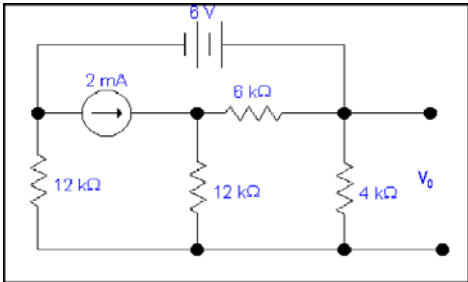


$$\begin{aligned}
 V_0 &= 32/3 \times 6 \times 1/(10/3+6) \\
 &= 64 \times 3/10 + 18 \\
 &= 192/28 \\
 \mathbf{V_0} &= \mathbf{48/7 \text{ volts}}
 \end{aligned}$$

LECTURE 26

(Examples of Thevenin's Theorem - Thevenin's Theorem and Dependent sources with examples)

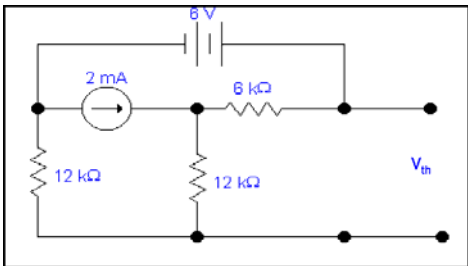
Example: Calculate the voltage V_o by using Thevenin's theorem .



Solution:

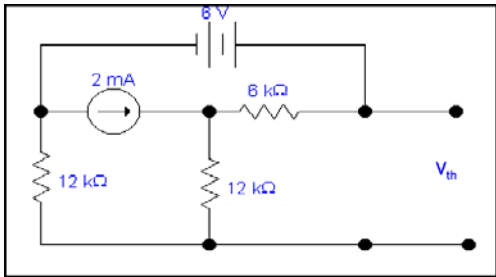
We want to calculate V_o by using Thevenin's theorem .we will follow the following steps .

First step: Removing R_L

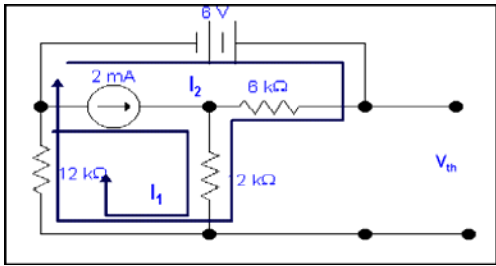


Here R_L is $4\text{ k}\Omega$ resistor at which we want to calculate the voltage V_o .

Second step: Calculating V_{th}



We want to calculate V_{th} . Apply KVL in two loops to calculate the individual loop currents.



Here

$$I_1 = 2\text{ mA}$$

KVL for the super mesh

$$\begin{aligned} -6 + 6kI_2 + 12k(I_1 + I_2) + 12k(I_1 + I_2) &= 0 \\ -6 + 6kI_2 + 12kI_2 + 12kI_2 + 24kI_1 &= 0 \\ 30kI_2 - 6 + 24k(2m) &= 0 \\ 30kI_2 - 6 + 48 &= 0 \end{aligned}$$

$$\begin{aligned} I_2 &= -42/30mA \\ I_2 &= \mathbf{-1.4mA} \end{aligned}$$

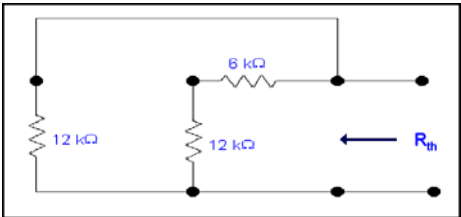
Now we will calculate the voltage across 6k and 12 k to find the value of V_{th}
Therefore,

$$\begin{aligned} V_{6k} &= 6kI_2 \\ &= 6k(-1.4m) \\ \mathbf{V_{6k} &= -8.4volts} \\ V_{12k} &= 12k(I_1 + I_2) \\ &= 12k(2m - 1.4m) \\ \mathbf{V_{12k} &= 7.2volts} \end{aligned}$$

So,

$$\begin{aligned} V_{th} &= V_{6k} + V_{12k} \\ &= -8.4 + 7.2 \\ \mathbf{V_{th} &= -1.2volts} \end{aligned}$$

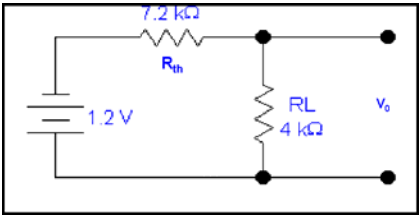
Third step: Calculating R_{th}



6k is in series with 12k .The resultant of these two is in parallel with 12k.
 $(6k+12k)||12k = 18k \times 12k/(12k+18k)$
 $\mathbf{R_{th} = 7.2k}$

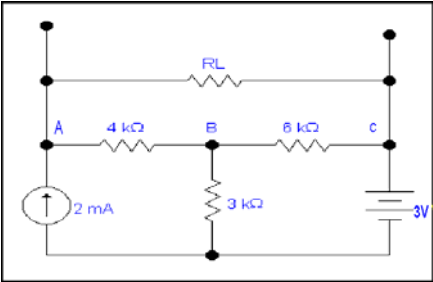
Fourth step: Calculating the unknown quantity.

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.



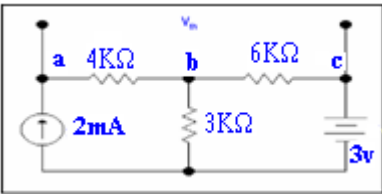
$$\begin{aligned} V_0 &= -1.2 \times 4k/(7.2k + 4k) \\ \mathbf{V_0 &= -0.4285 volts} \end{aligned}$$

Example: Calculate the value of R_L and the maximum power dissipation across it by Thevenin's Theorem.



Solution: We want to calculate R_L and the maximum power across it by using Thevenin's theorem .we will follow the steps given earlier.

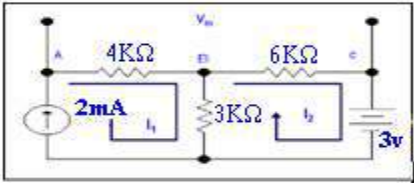
First step: Removing R_L



To remove R_L to calculate V_{th} .In this case our R_{th} is our R_L .

Second step: Calculating V_{th}

We want to calculate V_{th} . Apply KVL in two loops to calculate the individual loop currents.



Here

$$I_1 = 2\text{mA}$$

Applying KVL to loop 2

$$6kI_2 + 3k(I_2 - I_1) + 3 = 0$$

$$9kI_2 - 3kI_1 + 3 = 0$$

$$9kI_2 - 6 + 3 = 0$$

$$I_2 = 1/3 \text{ mA}$$

$$I_2 = 0.33\text{mA}$$

Now we will calculate the voltage across 4k and 6k to find the value of V_{th}
Therefore,

$$V_{AB} = 4k(I_1) \\ = 4k(2\text{mA})$$

$$V_{AB} = 8 \text{ volts}$$

$$V_{BC} = 6k(I_2) \\ = 6k \times 0.33\text{mA}$$

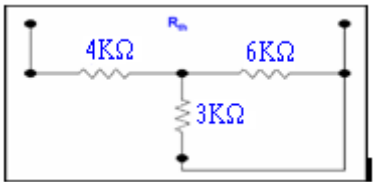
$$V_{BC} = 2 \text{ volts}$$

So

$$V_{th} = V_{AB} + V_{BC} \\ = 8 + 2$$

$$V_{th} = 10 \text{ volts}$$

Third step: Calculating R_{th}



To calculate R_{th} . Short circuiting the voltage source and open circuiting the current source.

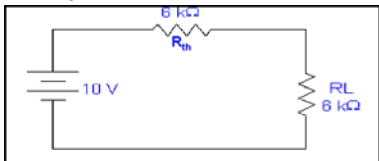
3k is in parallel with 6k and 4k is series with these two so

$$3k || 6k + 4k = 3k \times 6k / (3k + 6k) + 4k$$
$$= 2k + 4k$$
$$= 6k = R_{th} = R_L$$

$R_{th} = R_L = 6k$

Fourth step: Calculating unknown quantity.

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.



For maximum power dissipation

$R_L = R_{th} = 6k$

$P_L = I^2 R$ ----- (A)

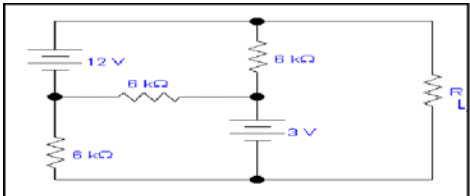
From Ohm's law $V=IR$ and $I = V/R$
so $I = 10/12$ put in A we have

$P_L = (10/12k)^2 (6k)$

$= (0.83)^2 \times 6$

$P_L = 4.1 \text{ mW}$

Example: Calculate the value of R_L and the maximum power dissipation across it by Thevenin's Theorem.

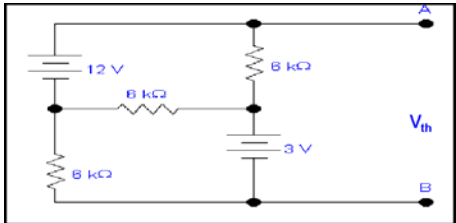


Solution:

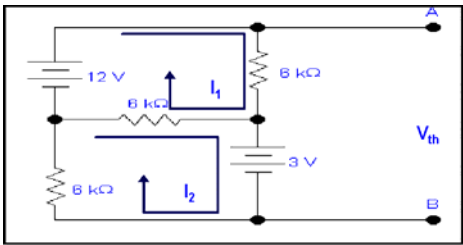
We want to calculate R_L and the maximum power across it by using Thevenin's theorem .we will follow the following steps .

First step: Removing R_L

To remove R_L to calculate V_{th} .In this case our R_{th} is our R_L .



Second step: Calculating V_{th}



Apply KVL to the circuit

For loop1

$$\begin{aligned} -12 + 6I_1 + 6(I_1 - I_2) &= 0 \\ -12 + 12I_1 - 6I_2 &= 0 \\ 2I_1 - I_2 &= 2 \text{ ----- (A)} \end{aligned}$$

For loop 2

$$\begin{aligned} 12I_2 - 6I_1 + 3 &= 0 \\ 4I_2 - 2I_1 + 1 &= 0 \\ 2I_1 &= 4I_2 + 1 \end{aligned}$$

put in equation (A) we have

$$\begin{aligned} 4kI_2 + 1 - 1kI_2 &= 2 \\ 3kI_2 &= 1 \\ I_2 &= 0.33\text{mA} \end{aligned}$$

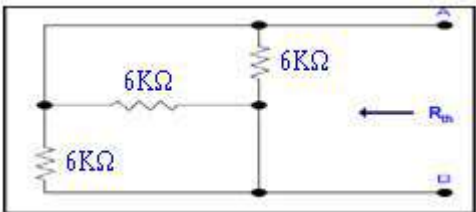
from equation (A)

$$\begin{aligned} I_1 &= (12 + 2)/2 \\ &= 0.33 + 2/2 \\ I_1 &= 1.166\text{mA} \end{aligned}$$

Now voltage across 6k resistor

$$\begin{aligned} V_{R6k} &= 6kI_1 \\ &= 6k(1.166\text{mA}) \\ V_{R6k} &= 7\text{volts} \\ V_{th} &= 7\text{V} + 3\text{V} \\ V_{th} &= 10\text{ volts} \end{aligned}$$

Third step: Calculating R_{th}



We want to calculate R_{th} . 6k is in parallel with 6k .The resultant is again parallel with third 6k resistor.

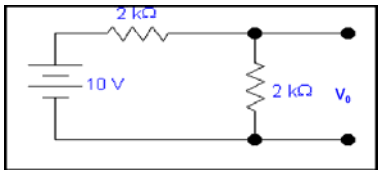
$$\begin{aligned} 6k||6k &= (6k \times 6k)/(6k + 6k) \\ &= 3k \\ 3k||6k &= 6k \times 3k/(6k + 3k) \\ &= 2k \end{aligned}$$

So

$$R_{th} = R_L = 2k$$

Fourth step: Calculating unknown quantity.

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these resistances.



$$V_{RL} = 10 \times 2k/4k$$
$$= 5 \text{ volts}$$

To calculate the power dissipation we have

$$P_L = V^2/R = 25/2k$$
$$P_L = 12.5mW$$

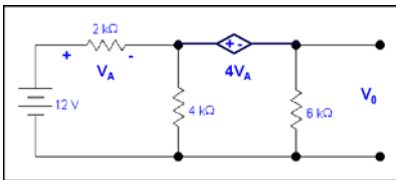
THEVENIN'S THEOREM AND DEPENDENT SOURCES:

Working with dependent sources is different from working with independent sources while applying Thevenin's theorem .

While calculating R_{th} we can simply open circuit current sources and short circuit voltage sources. Because the voltage or current of the dependent sources is dependent on the voltage or current of these independent spources.

While calculating R_{th} we will short circuit the open terminals of the Thevenin circuit and will calculate the I_{sc} and then divide V_{th} with I_{sc} to calculate R_{th} .

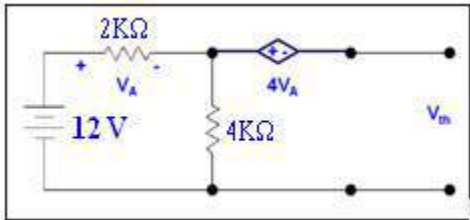
Example: Calculate the voltage V_o by using Thevenin's theorem .



Solution:

We want to calculate V_o by using Thevenin's theorem .we will follow the following steps .

First step: Removing R_L



Here R_L is 6k resistor at which we want to calculate the voltage V_o .

Second step: Calculating V_{th}

Voltage across 4k resistor

$$V_{4k} = (4k/6k) 12$$

$$V_{4k} = 8 \text{ volts}$$

Voltage across 2k resistor

$$V_{2k} = (2k/6k) 12$$

$$V_A = V_{2k} = 4 \text{ volts}$$

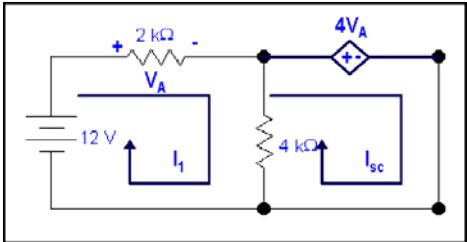
$$V_{th} = 8 - 4V_A$$

$$= 8 - 4(4)$$

$$= 8 - 16$$

$$V_{th} = - 8 \text{ volts}$$

Third step: When we use dependent sources we will use the following technique. In the above circuit output which is opened after removing the load resistance now we will replace this open circuit with short circuit. As in the circuit below.



We will first find I_{sc} to calculate R_{th} .

Here

$$V_A = 2kI_1$$

Applying KVL to loop 1

$$\begin{aligned} -12 + 2kI_1 + (I_1 - I_{sc})4k &= 0 \\ -12 + 2kI_1 + 4kI_1 - 4kI_{sc} &= 0 \\ 6kI_1 - 4kI_{sc} &= 12 \end{aligned}$$

For other loop

$$\begin{aligned} (I_{sc} - I_1)4k + 4V_A &= 0 \\ 4kI_{sc} - 4kI_1 + 4(2kI_1) &= 0 \\ 4kI_{sc} - 4kI_1 + 8kI_1 &= 0 \\ I_1 &= -I_{sc} \end{aligned}$$

Putting in equation for loop1

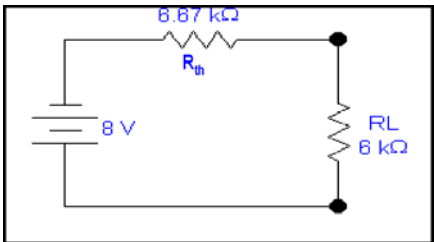
$$\begin{aligned} 3I_1 - 2I_{sc} &= 6 \\ 3(-I_{sc}) - 2I_{sc} &= 6 \\ I_{sc} &= -6/5 \text{ mA} \end{aligned}$$

So

$$\begin{aligned} R_{th} &= V_{th}/I_{sc} \\ &= -8/(-6/5 \text{ m}) \\ R_{th} &= 6.67k \end{aligned}$$

Fourth step:

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these resistances.



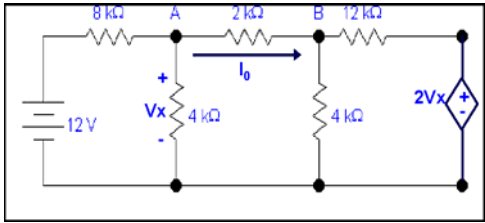
So

$$\begin{aligned} V_0 &= (6k/6k + 6.67k) \times 8 \\ &= 6k/12.67k \times 8 \\ V_0 &= 3.78 \text{ volts} \end{aligned}$$

LECTURE 27

(Thevenin's Theorem and Dependent sources with examples - Norton's Theorem with examples)

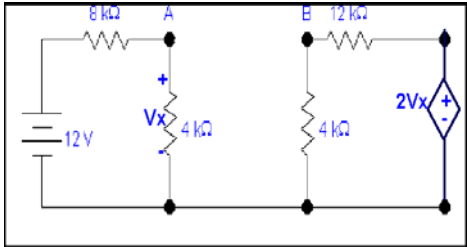
Example: Calculate the current I_o by using Thevenin's theorem .



Solution:

We want to calculate I_o by using Thevenin's theorem .we will follow the following steps .

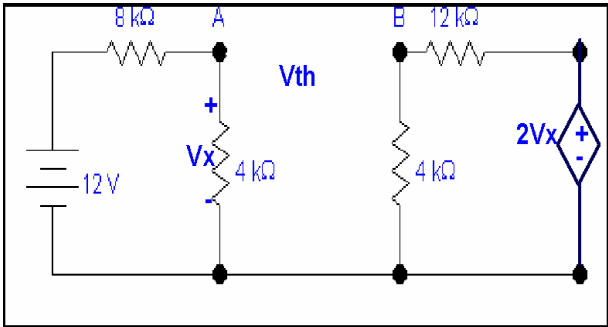
First step: Removing R_L to calculate V_{th}



Removing R_L to calculate V_{th} .

Here R_L is 2k resistor at which we want to calculate the current I_o .

Second step: Calculating V_{th}



Now we will calculate the voltage V_{th} .

Voltage at node A

$$V_x = (4 \times 12) / (4 + 8) \\ = 4 \text{ volts} = V_A$$

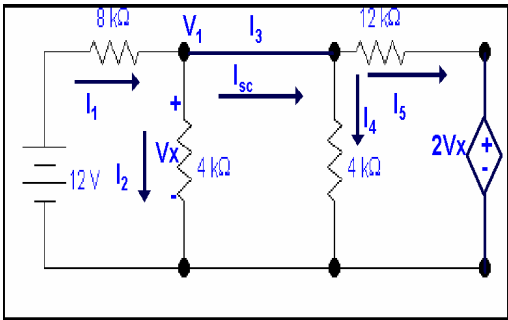
Now voltage at node B

$$V_B = (4 \times 2V_x) / 12 + 4 \\ V_B = (4 \times 8) / 16 \quad \text{where } V_x = 4 \text{ volts} \\ = 2 \text{ volts}$$

So

$$V_{AB} = 4 - 2 = 2 \text{ volts} = V_{th}$$

Third step: Calculating R_{th}



Now to find I_{sc} .

For node 1

$$(V_1 - 12)/8k + V_1/4k + V_1/4k + (V_1 - 2V_x)/12k = 0$$

From the fig $V_x = V_1$

$$(V_1 - 12)/8k + V_1/4k + V_1/4k + (V_1 - 2V_1)/12k = 0$$

$$3V_1 - 36 + 6V_1 + 6V_1 + 2V_1 - 4V_1 = 0$$

$$V_1 = 36/13$$

$$I_1 = (12 - V_1)/8k \\ = (12 - 36/13)/8k \\ = 15/13 \text{ mA}$$

$$I_1 = 1.15 \text{ mA}$$

$$I_2 = (36/13)/4k \\ = 9/13$$

$$I_2 = 0.692 \text{ mA}$$

$$I_4 = (36/13)/4 \\ I_4 = 9/13 \text{ mA} = 0.692 \text{ mA}$$

$$I_5 = (36/13 - 2(36/13))/12k \\ = -3/13 \text{ mA}$$

$$I_5 = 0.23 \text{ mA}$$

$$I_3 = I_{sc} = I_5 + I_4 \\ = 0.692 - 0.23$$

$$I_3 = 0.46 \text{ mA}$$

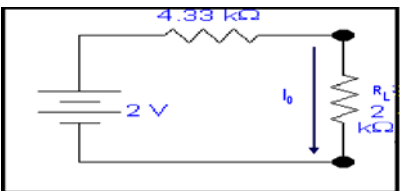
So

$$R_{th} = V_{th} / I_{sc} \\ = 2 / 0.46 \text{ mA}$$

$$R_{th} = 4.33 \text{ k}$$

Fourth step: Calculating the unknown quantity.

After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.

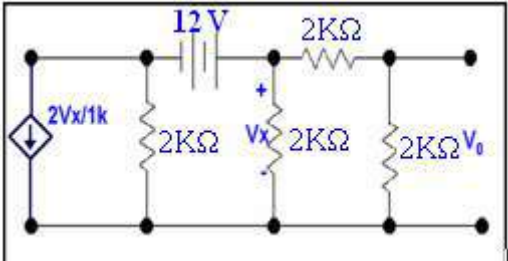


So

$$I_0 = 2 / (2k + 4.33k)$$

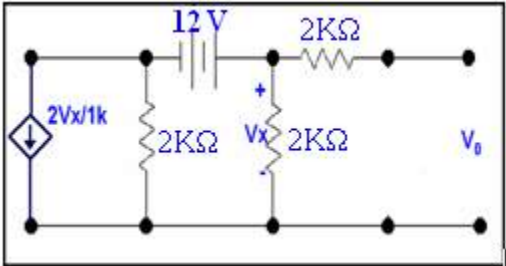
$I_0 = 0.315\text{mA}$

Example: Calculate the voltage V_o by using Thevenin's theorem .



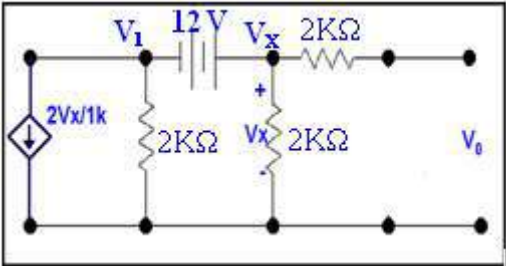
Solution: We want to calculate V_o by using Thevenin's theorem .we will follow these steps .

First step: Removing R_L



To remove R_L to calculate V_{th} .

Second step:



$V_x = 12 + V_1$

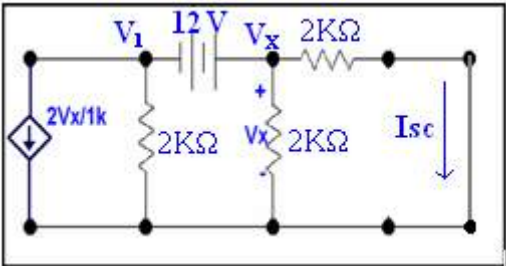
For node 1

$V_1/2k + 2V_x/1k + V_x/2k = 0$
 $V_1 + 4V_x + V_x = 0$

Putting the value of V_1

$V_x - 12 + 4V_x + V_x = 0$
 $6V_x = 12$
 $V_x = 2\text{volts} = V_{th}$

Third step: Calculating R_{th}

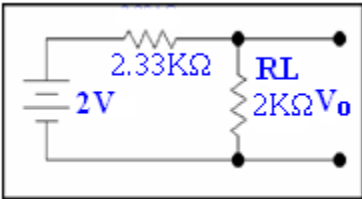


There is dependent source in the circuit we will short circuit the open circuit so we have here

$V_x = V_1 + 12$

$$\begin{aligned} V/2k + 2Vx/1k + Vx/2k + Vx/2k &= 0 \\ Vx - 12 + 4Vx + Vx + Vx &= 0 \\ 7Vx &= 12 \\ I_{sc} &= 12/7 \times 1/2 \\ &= 6/7 \text{ mA} \\ R_{th} &= V_{th} / I_{sc} \\ &= 2/(6/7) \text{ m} \\ R_{th} &= 7/3 \text{ k} = 2.33 \text{ k} \end{aligned}$$

Fourth step: Calculating unknown quantity.
After calculating V_{th} and R_{th} , re-inserting the load resistance R_L in the circuit in series with R_{th} and considering the V_{th} as a battery in series with these two resistances.

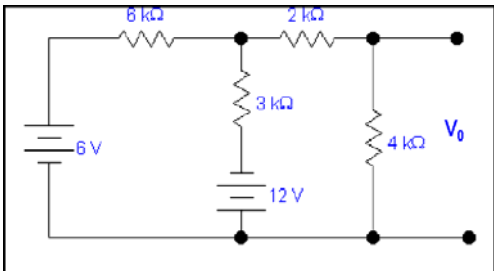


$$\begin{aligned} V_0 &= 2 \times 2k/2k + 2.33k \\ V_0 &= 0.92 \text{ volts} \end{aligned}$$

NORTON'S THEOREM

- Working with Norton's theorem is same as working with Thevenin's theorem .
- We will follow the same four steps.
- But in second step instead of finding V_{th} we will find I_{Nor} by short circuiting the open terminals of the circuit.
- In fourth step our circuit will consist of a current source of value equal to I_{Nor} and the Norton resistance will become parallel with R_L .

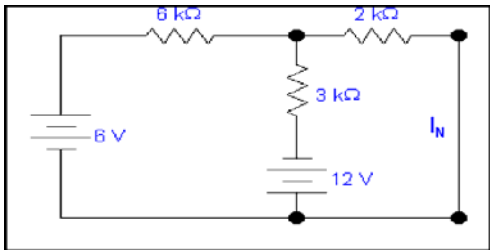
Example: Calculate the voltage V_0 by using Norton's theorem .



Solution:

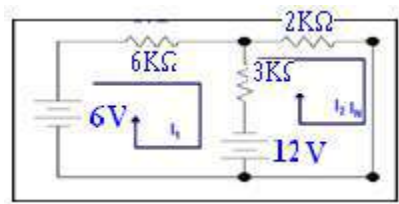
We want to calculate V_0 by using **Norton's theorem**.we will follow these steps .

First step: Replacing R_L with a short circuit to find I_N .



Here R_L is 4k resistor.

Second step:



We want to calculate I_N .

KVL for loop1

$$6kI_1 + 3k(I_1 - I_2) - 12 - 6 = 0$$
$$9kI_1 - 3kI_2 = 18$$

KVL for loop 2

$$2kI_2 + 12 + 3k(I_2 - I_1) = 0$$
$$-3kI_1 + 5kI_2 = -12$$

Solving equations for loop1 and loop2

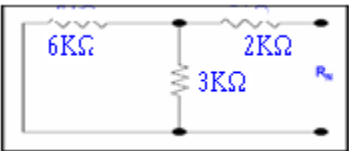
$$I_2 = -1.5\text{mA}$$

So,

$$I_N = -1.5\text{mA}$$

Third step: Calculating R_N

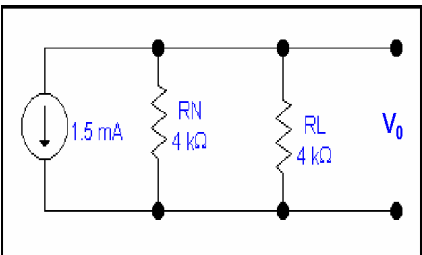
To calculate R_N we will short circuit all voltage sources. Now 3k is in parallel with 6k and 2k is in series with them.



$$6k || 3k + 2k = (6k \times 3k) / (6k + 3k) + 2k$$
$$= 2k + 2k$$
$$= 4k = R_N$$

Fourth Step:

After calculating I_N and R_N , re-inserting the load resistance R_L in the circuit in parallel R_N and considering the I_N current source parallel with these two resistances.



Current through $R_L = (-1.5\text{mA} \times 4k) / 8k$ (using current division rule)

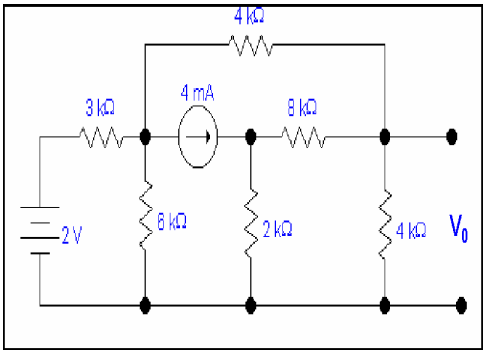
$$I_0 = -0.75\text{mA}$$

SO

$$V_0 = (-0.75\text{mA})(4k)$$

$V_0 = -3\text{volts}$

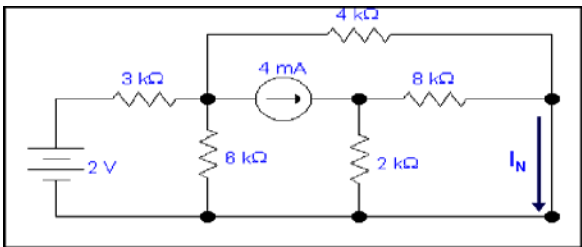
Example: Calculate the voltage V_0 by using Norton's theorem .



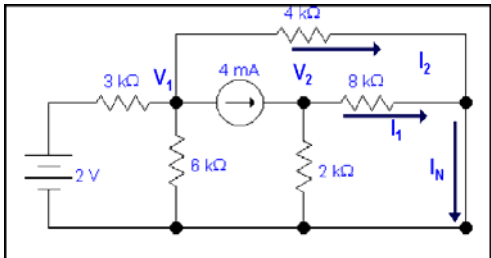
Solution:

We want to calculate V_0 by using **Norton's theorem**.we will follow these steps .

First step: Replacing R_L with a short circuit to find I_N . Here R_L is 4k resistor.



Second step:



$I_{sc} = I_1 + I_2$

For node 1

$$\begin{aligned} V_1/4k + (V_1 - 2)/3k + V_1/6k + 4m &= 0 \\ 3V_1 + 4V_1 - 8 + 2V_1 + 48 &= 0 \\ 9V_1 + 40 &= 0 \\ \text{Or } V_1 &= -4.44V \end{aligned}$$

For node 2

$$\begin{aligned} V_2/2k + V_2/8k - 4 &= 0 \\ 4V_2 + V_2 - 32 &= 0 \\ 5kV_2 &= 32 \\ V_2 &= 32/5 \\ V_2 &= 6.4V \end{aligned}$$

Now

$$I_1 = V_2/8k$$

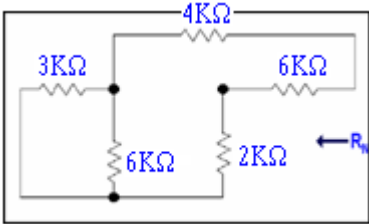
$$\begin{aligned} &= 6.4/8k \\ \mathbf{I_1} &= \mathbf{0.8\ mA} \\ I_2 &= V_1/4k \\ &= 4.44/4k \\ &= -1.11mA \end{aligned}$$

So,

$$\begin{aligned} I_N &= I_1 + I_2 \\ I_N &= (1.11 + 0.8) \\ \mathbf{I_N} &= \mathbf{-0.31mA} \end{aligned}$$

Third step: Calculating R_N

To calculate R_N we will short circuit all voltage sources and open the current sources.

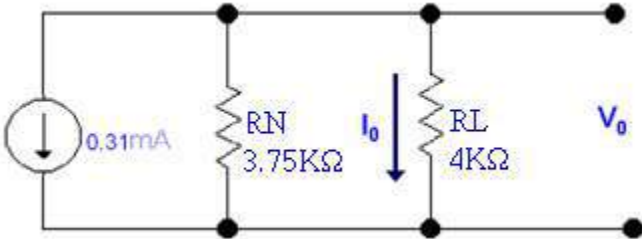


For R_N

$$\begin{aligned} 3k \parallel 6k &= 2k \\ 2k \text{ is in series with } 4k &= 2k + 4k \\ &= 6k \\ 8k \text{ is in series with } 2k &= 8k + 2k \\ &= 10k \\ 10k \parallel 6k &= 10 \times 6/16 \\ &= 60/16 \\ \mathbf{R_N} &= \mathbf{3.75k} \end{aligned}$$

Fourth Step:

After calculating I_N and R_N , re-inserting the load resistance R_L in the circuit in parallel R_N and considering the I_N current source parallel with these two resistances.



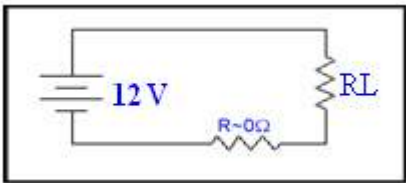
To calculate V_0

$$\begin{aligned} I_0 &= 0.31m \times 3.75k \times 1/(4k+3.75k) \\ &= 0.14mA \\ V_0 &= 4k \times 0.14m \\ \mathbf{V_0} &= \mathbf{0.56\ Volts} \end{aligned}$$

LECTURE 28

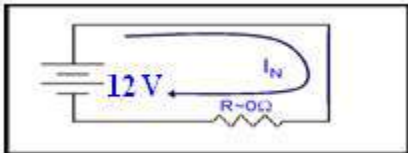
(Norton's Theorem with examples - Linearity Principle)

Example: Find Norton equivalent circuit.



Solution:

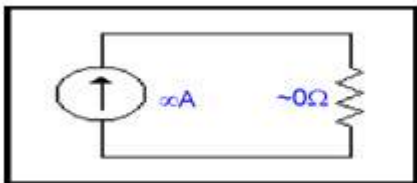
First step: Removing R_L replace open circuit with short circuit



by ohm's Law we have

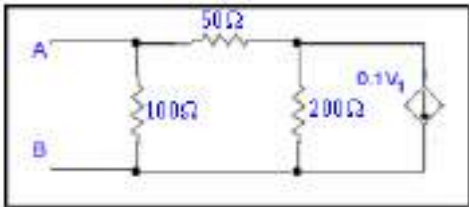
$I_N=12/-0 \rightarrow \infty A$

Now we will insert the resistance $R=-0$ parallel to current source.



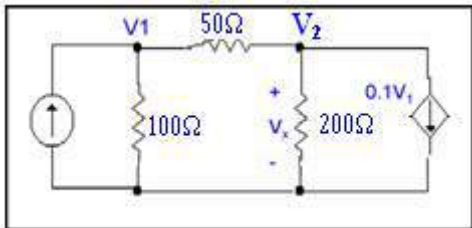
So we have the Norton's equivalent circuit.

Example: Calculate Norton's current through the circuit.



Solution:

Inject a 1A current source into the port and define V_x across 200Ω so that



For node 1

$1=(V_1/100)+(V_1-V_x)/50$

$100=V_1+2V_1-2V_x$

$3V_1-2V_x=100 \text{ ----- (A)}$

For node 2

$-0.1V_1=(V_x/200)+(V_x-V_1)/50$

$-20V_1=V_x+4V_x-4V_1$

$5V_x + 16V_1 = 0 \text{ -----(B)}$

Solving simultaneously A and B we have

~~$10V_x + 32V_1 = 0$~~

$10V_x + 15V_1 = 500$

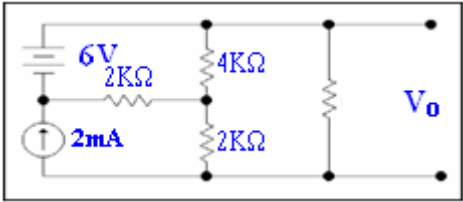
$47V_1 = 500$

$V_1 = 10.64V$

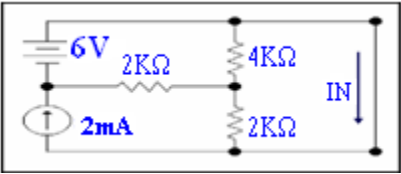
Also $R_{th} = R_n = V_1 / I_A = 10.64\Omega$

Since no independent source is involved in the circuit, hence $I_N = 0$

Example: Calculate the voltage V_o by using Norton's theorem , do not use loop and node methods. Use super postion method.



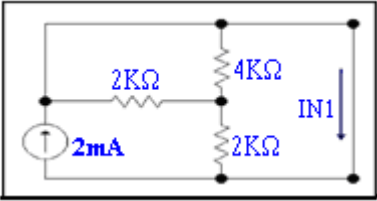
Solution:
We want to calculate V_o by using **Norton's theorem**.we will follow these four steps .
First step: Replacing R_L with a short circuit to find I_N .



Here R_L is 6k resistor.

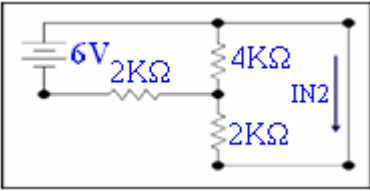
Second step:
We cant use loop or node methods, so lets use super position method to calculate I_N
To apply super position method we will remove all circuits one by one i.e. after removing voltage source we will replace it with short circuit and current source with open circuit.
Hint: Don't remove all circuits simultaneously.

Only current source is acting.



Due to short circuit all current will follow through the short circuit so
 $I_{N1} = 2mA \text{ -----(A)}$

Only voltage source is acting.



4k is in parallel with 2k resistor which in return is in series with 2k resistor .So total resistance.

$R = (4k || 2k) + 2k$
 $= 8/6 k + 2k$

So

$R = 3.33\text{ k}$

$I_{N2} = 6/3.33\text{k}$

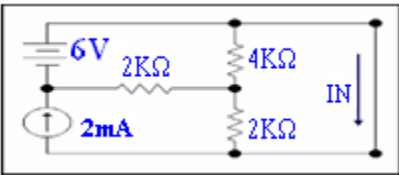
$I_{N2} = 1.80\text{mA} \text{ ----- (B)}$

total I_N from both sources so from equation A and B we have

$I_N = I_{N1} + I_{N2}$

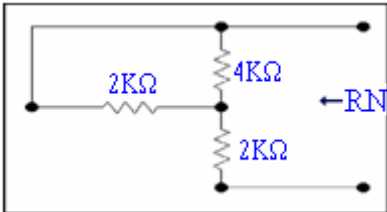
$= 2\text{mA} + 1.80\text{mA}$

$= 3.80\text{mA}$



Third step: Calculating R_N

To calculate R_N we will short circuit all voltage sources and open circuit all current sources.



4k is in parallel with 2k. The combined effect of these is in series with 2k.

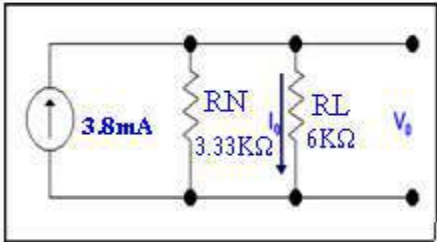
$4\text{k} || 2\text{k} + 2\text{k} = 1.33 + 2\text{k}$

$= 3.33\text{k}$

$= R_N$

Fourth Step:

After calculating I_N and R_N , re-inserting the load resistance R_L in the circuit in parallel R_N and considering the I_N current source parallel with these two resistances. So our Norton equivalent circuit will be.



by current divider rule we have

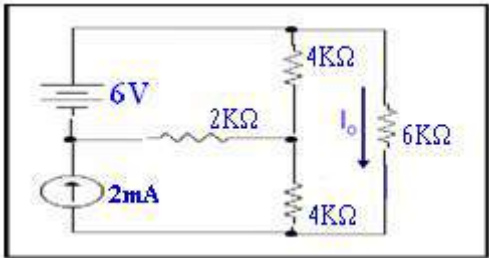
$I_0 = (3.80\text{mA})(3.33) \times 1/9.33\text{k}$

$= 1.356\text{ mA}$

By ohm's Law we have

$V_0 = 6\text{k} \times 1.35 = 8.143\text{ volts}$

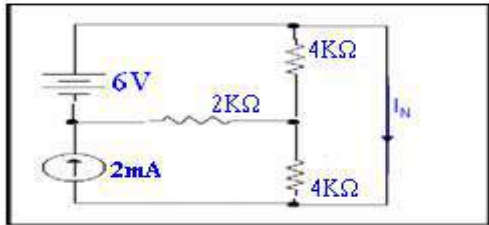
Example: Calculate the current I_o by using Norton's theorem .



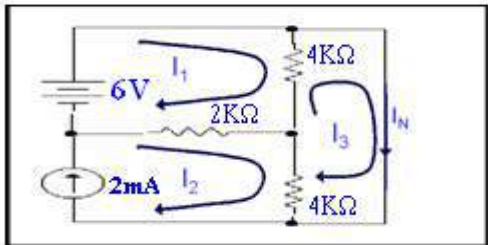
Solution:

We want to calculate I_o by using **Norton's theorem**.we will follow these steps .

First step: Replacing R_L with a short circuit to find I_N . Here R_L is 6k resistor.



Second step: Calculate current I_N



From loop 2, we can write

$$I_2=2mA$$

For loop 1

$$4k(I_1-I_N)+2k(I_1-I_2)-6=0$$

$$4kl_1-4kl_N+2kl_1-2kl_2-6=0$$

$$6kl_1-4kl_N-2kl_2=6$$

Putting the value of I_2

$$6kl_1-4kl_N-2k(2m) = 6$$

$$6kl_1-4kl_N = 10$$

For Loop 3

$$4k(I_N-I_2)+4k(I_N-I_1)=0$$

$$4kl_N-4kl_2+4kl_N-4kl_1=0$$

$$-4kl_1+8kl_N=8$$

Solving equations for loops 1 and 3

$$12kl_1-8kl_N=20$$

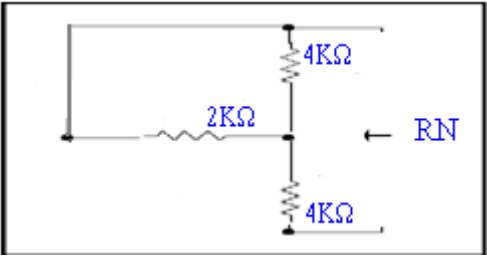
$$-12kl_1+24kl_N=24$$

$$16kl_N=44$$

$$I_N = 2.75 \text{ mA}$$

Third step: Calculating R_N

To calculate R_N we will short circuit all voltage sources and open the current sources.

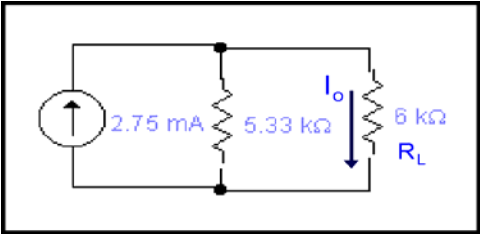


For R_N

$$2k \parallel 4k + 4k = ((2k \times 4k) / (2k + 4k)) + 4k$$
$$R_N = 5.33k\Omega$$

Fourth Step:

After calculating I_N and R_N , re-inserting the load resistance R_L in the circuit in parallel R_N and considering the I_N current source parallel with these two resistances.



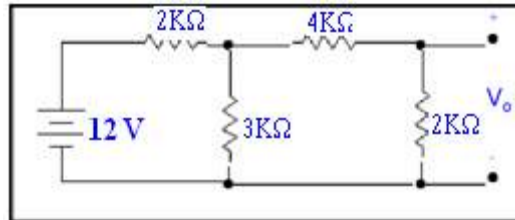
$$I_o = (2.75 \text{ m}) (5.33 \text{ k}) / (6 + 5.33) \text{ k}$$
$$I_o = 1.29 \text{ mA}$$

Linearity Principle

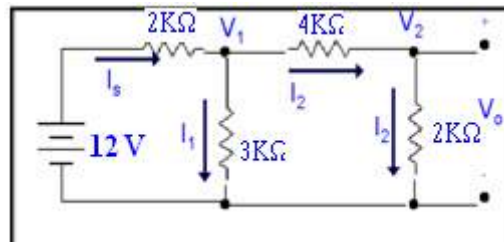
In this technique we assume the unknown quantity and analyze the circuit in reverse manner. Until we reach the source which is producing all voltages or currents and calculate its value.

Now by comparing this value with the original value of the source we calculate the exact unknown quantity of the circuit.

Example: Calculate V_o by linearity principle.

**Solution:**

We want to calculate V_o by linearity principle. Let us assume that V_o is 1 Volt.



Our assumption is $V_o = 1V$

Therefore, $V_2 = 1V$

Hence $I_2 = 1/2k = 0.5mA$

Also, $V_{4k} = (0.5m)(4k) = 2V$

Now, $V_1 = V_{4k} + V_2 = 2 + 1 = 3V$

Then, $I_1 = 3/3k = 1mA$

Also, $I_s = I_2 + I_1 = 0.5m + 1m = 1.5mA$

$V_{2k} = I_s(2k) = 2k(1.5m) = 3V$

$V_s = V_{2k} + V_1 = 3 + 3 = 6V$

When V_o is 1, source voltage is 6V, but original source voltage is 12 V, hence output voltage will be 2 Volts.

LECTURE 29

(Basic semiconductor concepts)

INTRINSIC SILICON:

- A crystal of pure or intrinsic silicon has a regular lattice structure. Where the atoms are held at their fixed positions by bonds, called Covalent bonds, formed by four valence electrons associated with four each silicon atom.
- At sufficiently low temperature these covalent bond are intact and not (very few) free electrons are available to conduct electric current.
- Thermal ionization results in free electrons and holes in equal numbers and hence equal concentrations. These free electrons and holes move randomly through silicon crystal structure, and in this process some electrons may fill some of the holes. This process is called **RECOMBINATION**
- It results in disappearance of free electrons and holes. The recombination rate is proportional to the number of free electrons and holes, which in turn is determined by ionization rate.

CONDUCTION BAND:

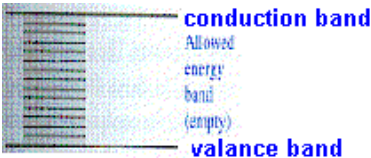
In semiconductors and insulators, there is a band gap above the valence band, called a conduction band. In a semiconductor, the range of electron energy, higher than that of the valence band, sufficient to make the electrons free to move from atom to atom under the influence of an applied electric field and thus constitute an electric current.

Valence BAND:

AT 0°C the outermost electrons of an atom of a semiconductor material (such as silicon and germanium) are present in valence-orbits. Associated with these orbits is a band of energies that is termed as valence band of energy.

BAND GAP:

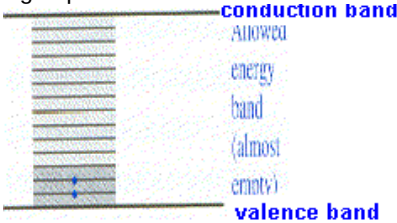
The difference of energy between valence band and conduction band is called band gap energy.



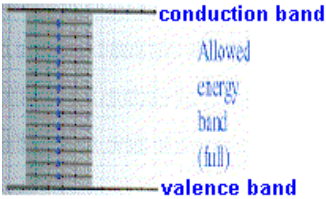
In the above fig upper line is denoted as conduction band while lower line is valence band. There are no electrons available in these energy bands.

Drift

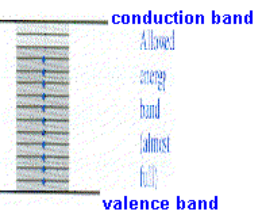
The process whereby charged particles move under the influence of electric field.



In the above fig there are few electrons present in the near by valence band if we apply some electric field to such a material then these electrons due to applied electric field may drift hence we can observe some activity into electrical property of the material therefore some flow of current also observe and we say this is a conductor.



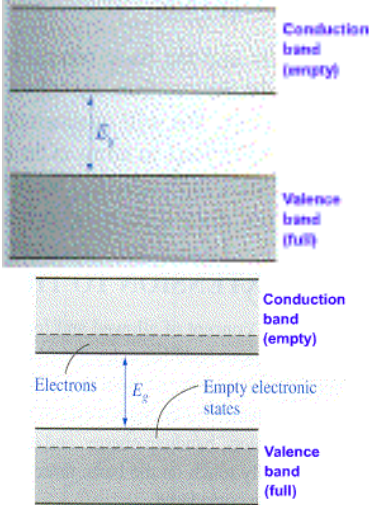
In the above fig all the energy levels are fully filled with electron if we apply any electric field to such a material then these electrons have no space to drift hence such material act as insulator. Insulators are such material which offers maximum resistance to the flow of electrons and their conductivity is very low.



in the above fig almost all energy levels are full except two levels are kept empty if we apply electric field to such a material then these electrons have some space to drift therefore we will observe some flow of current through such a material so these material are called conductors. Conductors are such materials which offers zero resistance to the flow of electrons and their conductivity is maximum.

BAND GAP ENERGY:

It is the energy required to excite an electron from its valence band to a conduction band and denoted as E_g . as show in the fig below



Diffusion

The process of flow of particles from a region of high concentration to a region of low concentration.

Diffusion Current

The current that results from the diffusion of charged particles.

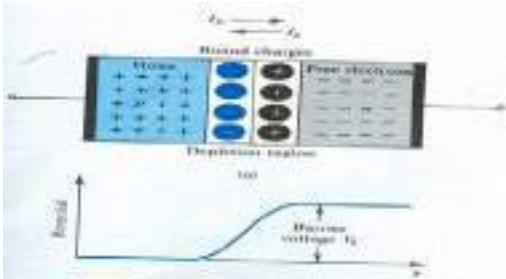
Drift Velocity

The average velocity of charged particles in the presence of an electric field.

DOPED SEMICONDUCTORS

- Doped semiconductors are the materials in which carrier of one kind (electrons or holes) predominate.
- Doped silicon in which the majority of charge carriers are negatively charged electrons is called n type.
- While silicon doped so that majority of charge carriers are positively charged holes is called p type.
- Doping of silicon to turn it into p type or n type is achieved by a small number of impurity atoms.
- For instance, introducing impurity atoms of a pentavalent element such as phosphorus results in n type silicon.

NO BIAS

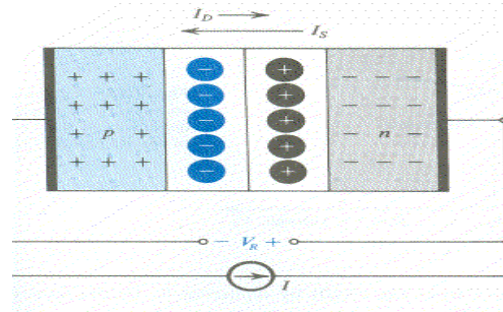


THE DIFFUSION CURRENT I_D

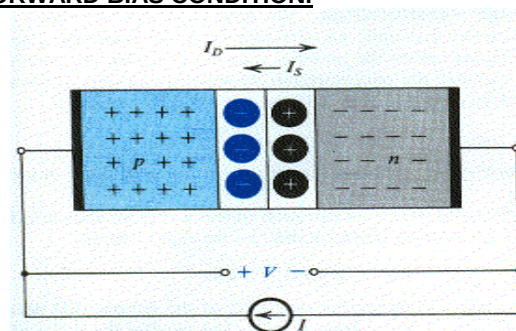
- Because the concentration of holes is high in the p region and low in the n region, holes diffuse across the junction from the p region to the n region.
- Similarly, electrons diffuse the junction from n side to p side.
- These two current components add together to form the diffusion current I_D . Whose direction is from p side to n side, as indicated in the figure.

THE DEPLETION REGION

- The electrons that diffuse across the region quickly recombine with some of the majority holes present in the p region and thus disappear from the scene.
- This results also in disappearance of some majority holes, causing some of the bound negative charge to be uncovered (i.e. no longer neutralized by holes). Thus in the p material close to the junction that is depleted of free electrons.
- This p region will contain uncovered bound negative charge, as indicated in the figure.
- From the above it follows that a carrier depletion region will exist on both sides of the junction.
- With n side of this region positively charged and p side negatively charged. This carrier depletion region or simply, depletion region is also called the SPACE CHARGE REGION.

THE P-N JUNCTION UNDER REVERSE BIAS CONDITION.

- The pn junction is excited by a constant current source I in the reverse direction. To avoid breakdown, I kept smaller than I_S .
- Note that the depletion layer widens and the barrier voltage increases by V_R volts, which appears between the terminals as reverse voltage.
- The current I will be carried by electrons flowing in the external circuit from the n material to p material (that is, in direction opposite to that of I).
- This will cause electrons to leave the n material and holes to leave p material. Thus reverse current I will result in an increase in the width of, and the charge stored in the depletion layer. Which will increase the voltage across depletion region?

THE P-N JUNCTION UNDER FORWARD BIAS CONDITION.

- The pn junction is excited by a constant current source supplying a current I in forward direction. The depletion layer narrows and barrier voltage decreases by V volts, which appears as an external voltage in the forward direction.
- This current causes majority carriers to be supplied to both sides of the junction by the external circuit. Holes to the p material and electrons to the n material.
- These majority carriers will neutralize some of the uncovered, causing less charge to be stored in the depletion region. Thus the depletion layer narrows and the depletion barrier voltage reduces.
- This reduction in voltage cause more electrons to move from n side to p side and more holes to move from p side to n side. So that diffusion currents increases until equilibrium is achieved.

LECTURE 30

(PN Junction Diode - Ideal Diode - Ideal Diode as a Rectifier)

INTRODUCTION TO THE PN- JUNCTION DIODE.

The schematic symbol for the pn-junction diode is shown in the fig.

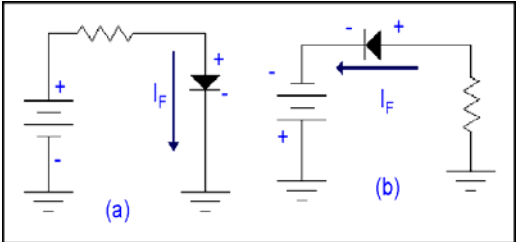


A diode will conduct when it meets the condition that the voltage difference between the anode and the cathode exceeds the barrier voltage of approximately 0.3 volts for a germanium diode and 0.7 for a silicon diode.

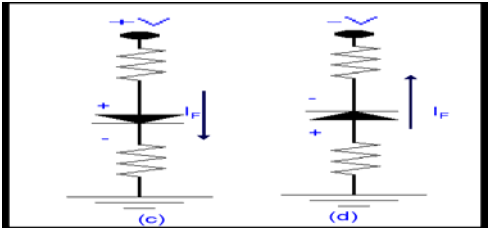
THE FORWARD BIASED PN-JUNCTION (DIODE)

When positive terminal of a battery is connected to the anode or P-side and negative terminal to the cathode or to the N-side of a diode (PN-junction), the diode is said to be **FORWARD-BIASED**.

In the figures to follow it may be noted that the arrow points to the more negative potential in each case. In **fig. a** below the positive terminal of a battery through a resistor is connected the anode whereas the cathode is attached to the neutral terminal and now the condition of the forward biased is fulfilled. In **fig. b** below the negative terminal of a battery is connected to the cathode whereas the anode is attached through a resistor to the neutral terminal and now the condition of the forward biased is fulfilled where I_F shows the forward conventional current.



In **fig. c** below the positive volts has been applied to the anode of the diode through a resistance and cathode terminal of the diode is connected to the neutral terminal through resistance which will fulfill the condition of forward biased. Where I_F shows the forward conventional current.

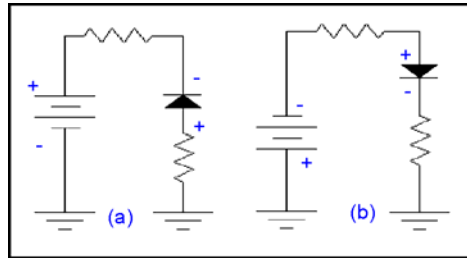


In **fig. d** above the -ve volts has been applied to the cathode of the diode through a resistance and anode terminal of the diode is connected to the neutral terminal through resistance which will fulfill the condition of forward biased. Where I_F shows the forward conventional current will be establish in the diode.

THE REVERSE - BIASED PN-JUNCTION (DIODE)

When positive terminal of a battery is connected to the cathode or N-side and negative terminal to the anode or to the P-side of a diode (PN-junction), the diode is said to be **REVERSED - BIASED**.

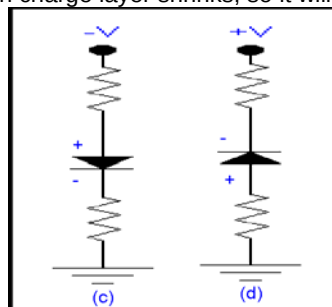
A PN-junction diode is reverse biased when the n type material (cathode) is more positive than the p type material (anode). This causes the depletion region to widen and prevent current. A diode will not conduct when the arrow points to the more positive of the diode potentials.



In **fig. a** the negative of the battery is connected to the anode through a resistor and positive terminal of the battery is connected the cathode therefore no current will flow and we can say that diode is not existing and it will act as an open circuit.

In **fig. b** the negative terminal of the battery is connected to the anode through a resistor and positive terminal of the battery is connected the cathode.

The reverse – biased diode behaves opposite to the forward-biased diode. This means in case of a reverse-biased diode, majority carrier current does not flow and instead of that only minority carrier current can flow. Moreover, the depletion charge layer will expand or enlarged compared to the forward –biased diode where the depletion charge layer shrinks, so it will act as an open circuit.



In **fig. c** the negative voltage has been applied to the anode of the diode through a resistance and cathode terminal of the diode is connected to the neutral terminal through resistance. Which will fulfill the condition of reversed biased.

In **fig. d** the positive voltage has been applied to the cathode of the diode through a resistance and anode terminal of the diode is connected to the neutral terminal through resistance. Which will fulfill the condition of reversed biased

Here we can say that the reverse-biased PN-junction can't support majority carrier current but it will allow the minority carrier current to flow across the junction. This minority carrier current is called as reverse current and is much smaller than the forward majority carrier current of the forward biased PN-junction.

The Ideal Diode

The diode can be considered to be a one Way Street that is it conducts electricity well in one direction but hardly any in the opposite direction. An ideal diode has no resistance in the forward direction and infinite resistance in the reverse direction.

An ideal diode is like a light switch in your home. When the switch is closed, the circuit is completed; and the light turns on. When the switch is open, there is no current and the light is off.

However, the diode has an additional property; it is unidirectional, i.e. current flows in only one direction (anode to cathode internally). When a forward voltage is applied, the diode conducts; and when a reverse voltage is applied, there is no conduction. A mechanical analogy is a rat chat, which allows motion in one direction only.

- The ideal diode may be considered the most fundamental non linear circuit element.
- It is a two terminal device and the i-v characteristic is shown

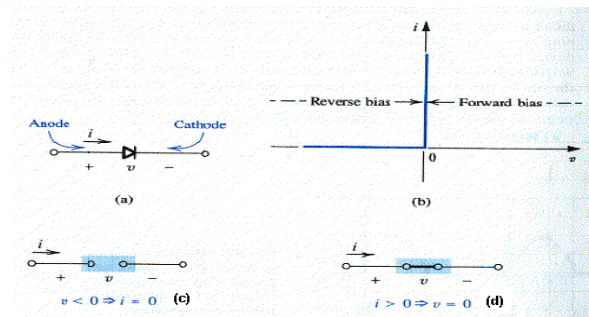


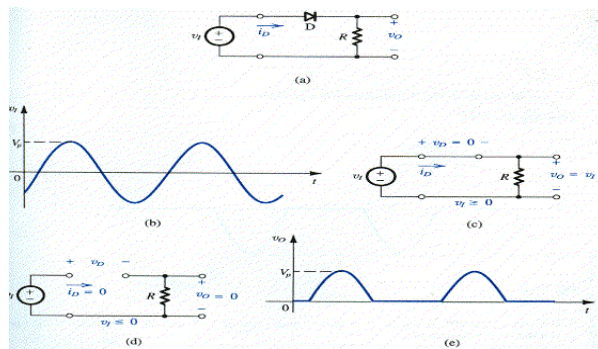
Fig a is diode circuit symbol, fig b is i - v characteristic fig c is equivalent circuit in the reverse direction and fig d is the equivalent circuit in the forward direction.

In fig b the terminal characteristic of the ideal diode can be interpreted as. If a negative voltage is applied to the diode, no current flows and the diode behaves as an open circuit as shown in the fig c. Diodes operated in this mode are said to be reverse –biased. An ideal diode has zero current when operated in the reverse direction and is said to be cut off.

On the other hand, if a positive voltage is applied to the ideal diode, zero voltage drop appears across the diode .In other words ideal diode behaves as a short circuit in the forward direction as show in the fig d , it passes any current with zero voltage drop. A forward –conducting diode is said to be turned on.

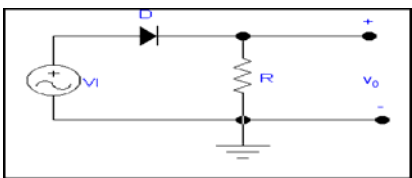
The Ideal Diode as a Rectifier

Rectification is generally defined as the process of converting an alternating current to a unidirectional current. A rectifier device conducts current substantially in one direction only. An ideal rectifier diode would be an open circuit in one direction and short circuit in the other direction. It also would not dissipate power during the rectification process.

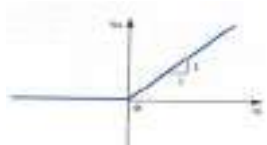


The rectifier circuit shown in the fig a . This circuit consists of series connection of a diode D and a resistor R . Let the input voltage be sinusoid as shown in fig b and assume the diode to be ideal. During the positive half cycle of the input sinusoid ,the positive v_i will cause current to flow through the diode in its forward direction. It follows that the diode voltage V_D will be very small i.e ideally zero. Thus the circuit will have the equivalent show in the fig (c) and the output voltage V_O will be equal to the input voltage v_i . On the other hand ,during the negative half cycle of v_i the diode will not conduct .Thus the circuit will have the equivalent show in the fig (d), and V_O will be zero .Thus, the output voltage will have the waveform shown in fig (e).Note that when V_i alternates in polarity and has a zero average value , V_O is unidirectional and has a finite average value or a dc component. Thus the circuit of fig (a) rectifier the signal and hence is called a rectifier.

Example: For the circuit draw transfer characteristic v_o VS v_i



Solution:

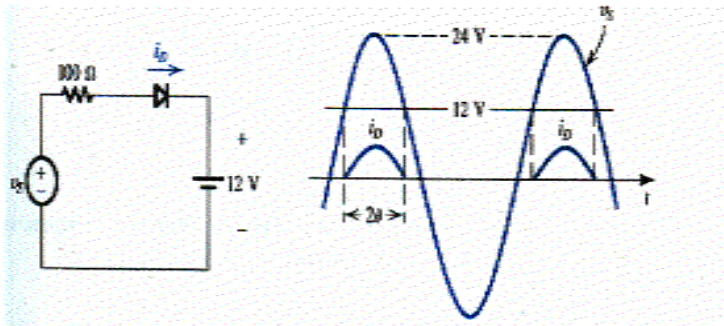


LECTURE 31

(Terminal characteristics of the Junction diodes - Forward bias region)

Example

Fig shows a circuit for charging a 12 volts battery. If v_s is a sinusoid with 24 volts peak amplitude, find the fraction of each cycle during which the diode conducts also find the peak value of the diode current and the maximum reverse biases voltage that appears across the diode.



Solution :

The diode conducts when v_s exceeds 12 volts as shown in the previous fig. The conduction angle is 2θ , where θ is given by

$$\begin{aligned} 24\cos\theta &= 12 \\ \cos\theta &= 1/2 \\ \theta &= \cos^{-1}(1/2) \end{aligned}$$

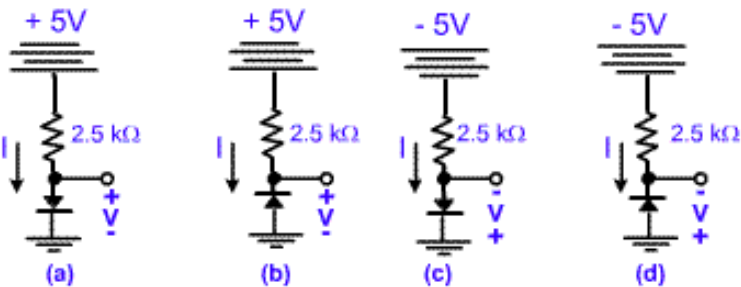
Thus $\theta = 60^\circ$ and the conduction angle is 120° or one third of the cycle.

The peak value of the diode current is given by

$$\begin{aligned} I_d &= 24 - 12/100 \\ &= 0.12\text{ A} \end{aligned}$$

The maximum reverse voltage across the diode occurs when v_s is at its negative peak and is equal to $24 + 12 = 36$ volts

Example: Assuming the diodes to be ideal .Find the value of I and V in the circuit shown in figure below.



Solution :

We are using ideal diode here. In fig (a) positive 5 Volt battery is connected to the anode of the diode through a resistance of $2.5\text{ k}\Omega$. The cathode terminal of the diode is connected with the neutral terminal. The diode is in forward biased condition. From the fig we see that all components are in series so the same current will flow.

so by Ohm's Law we have

$$\begin{aligned} I &= 5/2.5 \\ I &= 2\text{mA} \end{aligned}$$

diode acting as short circuit so,

$$V = 0\text{ volts}$$

In fig (b) the circuit is same only the diode has reversed. 5 Volt battery is connected to the cathode of the diode through a resistance of $2.5\text{ k}\Omega$. The anode terminal of the diode is connected with the neutral terminal. Now the diode is in reversed biased condition so, it will act as an open circuit therefore no current will flow.

$$I = 0\text{mA}$$

whole voltage of 5 volts will appear at positive terminal.

$$V = 5\text{volts}$$

In fig (c) negative 5 Volt battery has been connected to the anode of the diode through a resistance of 2.5 k ohm. The cathode terminal of the diode is connected with the neutral terminal. The diode is in reversed biased condition and it will act as an open circuit therefore current will flow.

$I = 0\text{mA}$

whole voltage of 5 volts will appear

$V = -5\text{volts}$

In fig (d) the circuit is same only the diode has reversed. negative 5 Volt battery has been connected to the cathode of the diode through a resistance of 2.5 k ohm. The anode terminal of the diode is connected with the neutral terminal. Now the diode is in forward biased condition so, it will act as an short circuit therefore current will flow.

so by Ohm's Law we have

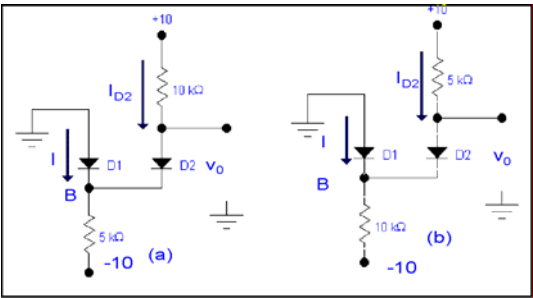
$I = 5/2.5$

$I = 2\text{mA}$

diode acting as short circuit so,

$V = 0\text{ volts}$

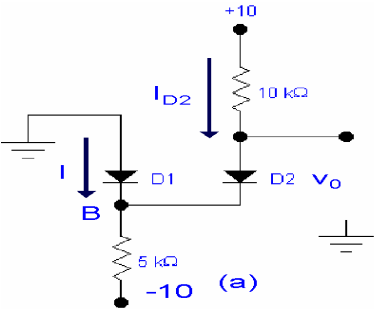
Example Assuming the diodes to be ideal, find the values of I and V_0 in the circuits.



Solution :

For the circuit in the fig (a) we shall assume that both diodes are conducting. It follows that $V_B = 0$ and $V_0 = 0$. The current through D_2 can now be determined from

$$I_{D2} = (10 - 0)/10$$
$$= 1\text{mA}$$



Writing node equation at B,

$I + 1 = 0 - (-10)/5$

Results in $I = 1\text{mA}$. Thus D_1 is conducting as originally assumed, and the final result is

$I = 1\text{mA}$

and

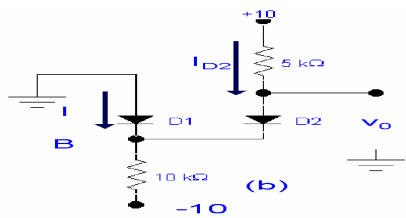
$V = 0\text{ volts}$

For the circuit in fig (b), if we assume that both diodes are conducting , then $V_B = 0$

and $V_0 = 0$. The current in D_2 is obtained

from

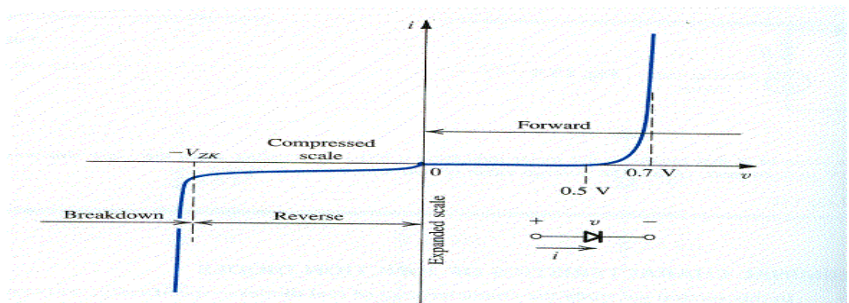
$$I_{D2} = (10 - 0)/5 = 2\text{mA}$$



$I + 2 \text{ m} = 0 - (-10)/10\text{k}$
Which yields $I = -1\text{mA}$. Since this is not possible , our assumption is not correct.
we start again, assuming that D_1 is off and D_2 is on.
The current I_{D2} is given by
$$I_{D2} = 10 - (-10)/15$$
$$= 1.33\text{mA}$$

and the voltage at node B is
$$V_B = -10 + 10 \times 1.33 = 3.3 \text{ volts}$$

Thus D_1 is reversed biased as assumed, and the final result is $I = 0$ and $V_0 = 3.3 \text{ volts}$
TERMINAL CHARACTERISTICS OF THE JUNCTION DIODES



As indicated the characteristic curve consist of three distinct regions:
(1) The forward bias region, determined by $v > 0$
(2) The reverse bias region determined by $v < 0$
(3) The breakdown region determined by $v < -V_{ZK}$
These three regions of operations described in the following.
THE FORWARD BIAS REGION:
● The forward bias or simply forward region is entered when the terminal voltage v is positive. In the forward region the i - v relationship is closely approximated as
$$i = I_s (e^{v/nV_T} - 1)$$

● In this equation I_s is a constant for a given diode at a given temperature and it is directly proportional to the area of the diode.
● The Voltage V_T is a constant and given as
$$V_T = kT/q$$

where
 k = Boltzmann,s constant
 $= 1.38 \times 10^{-23} \text{ joules/kelvin}$
 T = absolute temperature in kelvin
 q =magnitude of the electronic charge
 $= 1.60 \times 10^{-19} \text{ coulombs}$
● At room temperature (20°C) the value of V_T is 25.2mV.
● In the diode equation the constant n has a value between 1 and 2 depending on the material and physical structure of the diode.

- For appreciable current I in the forward direction, $i \gg I_s$. The diode equation can be modified as

$$i = I_s e^{v/nVT}$$

This expression can be given alternatively in algorithmic form

$$v = nV_T \ln(i/I_s)$$

- Let us consider the forward i - v relationship and evaluate the current I_1 corresponding to the diode voltage V_1 :

$$I_1 = I_s e^{V_1/nVT}$$

Similarly, if the voltage V_2 the diode current I_2 will be

$$I_2 = I_s e^{V_2/nVT}$$

These two equations can be combined to produce

$$I_2/I_1 = e^{(V_2 - V_1)/nVT}$$

Which can be rewritten as

$$V_2 - V_1 = nVT \ln(I_2/I_1)$$

EXAMPLE:

A silicon diode said to be 1mA device displays a forward voltage of 0.7 V at a current of 1mA. Evaluate the junction scaling constant I_s in the event that n is either 1 or 2. What scaling constant would apply for a 1A

diode of the same manufacture that conducts 1A at 0.7V.

SOLUTION:

Since

$$i = I_s e^{v/nVT}$$

Then

$$I_s = i e^{-v/nVT}$$

for 1mA diode:

$$\text{if } n=1 : I_s = 10^{-3} e^{-700/25}$$

$$I_s = 6.9 \times 10^{-16} \text{ A}$$

$$\text{If } n=2 : I_s = 10^{-3} e^{-700/50}$$

$$I_s = 8.3 \times 10^{-10} \text{ A}$$

The diode conducting 1A at 0.7V corresponds to 1000mA in parallel with a total junction area 1000 times greater. Thus I_s is also 1000 times greater.

LECTURE 32

(Reverse bias region - Break down region - Analysis of diode circuit)

Example: Consider a silicon diode with $n=1.5$. Find the change in voltage if the current changes from 0.1mA to 10mA.

Solution:

$$i = I_s e^{v/nV_T}$$

$$I_1 = I_s e^{v_1/nV_T}$$

$$I_2 = I_s e^{v_2/nV_T}$$

$$I_1/I_2 = e^{(v_1-v_2)/nV_T}$$

$$v_1-v_2 = nV_T \ln(I_1/I_2)$$

here,

$$V_T = 25 \text{ mV}$$

$$= 1.5 \times 25 \ln(10/0.1)$$

so change in the voltage will be

$$v_1 - v_2 = \mathbf{172.7 \text{ mV}}$$

Example: A silicon junction diode with $n=1$ has $v = 0.7$ volts at $i = 1\text{mA}$. Find the voltage drop at $i = 0.1\text{mA}$ and $i = 10\text{mA}$.

Solution:

$$v_2 - v_1 = nV_T \ln(i_2/i_1)$$

$$v_1 = 0.7\text{V}$$

$$i_1 = 1\text{mA}$$

and $n=1$ thus

$$v_2 = 0.7 + V_T \ln(i_2)$$

for

$$i_2 = 0.1\text{mA}$$

$$V_T = 25 \text{ mV} = 0.025 \text{ volts}$$

$$v_2 = 0.7 + 0.025 \ln(0.1)$$

$$\mathbf{v_2 = 0.64 \text{ volts}}$$

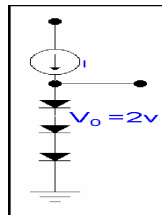
For

$$i_2 = 10\text{mA}$$

$$v_2 = 0.7 + 0.025 \ln(10)$$

$$\mathbf{v_2 = 0.76 \text{ volts}}$$

Example: The circuit in the fig. utilize three identical diodes having $n=1$ and $I_s = 10^{-14} \text{ A}$. Find the value of the current I to obtain an output voltage $V_o = 2\text{V}$, if a current of 1mA is drawn away, what is the change in output voltage.



Solution:

For $V_o = 2\text{V}$, so the voltage drop across each diode is $2/3\text{V}$.

Thus I must be

$$I = I_s e^{v/nV_T}$$

$$V_T = 25 \text{ mV} = 0.025 \text{ volts}$$

$$= 10^{-14} \times \frac{2}{3} \times 0.025$$

$$= 3.8 \text{mA}$$

If a current of 1mA is drawn away from the terminals by means of a load, the current through the diodes reduces to

$$3.8 - 1 = 2.8$$

Thus the voltage across each diode changes by

$$\Delta V = v_2 - v_1 = nV_T \ln(i_2/i_1)$$

$$\Delta V = nV_T \ln(2.8/3.8)$$

$$= -7.63 \text{mV}$$

The total decrease in V_0

$$V_0 = 3 \times 7.63$$

$$= 22.9 \text{mV}$$

Example: A particular diode conducts 1A at a junction voltage of 0.65 volts and 2A at a junction voltage 0.67 volts. What are its values of n and I_s , what current will follow if its junction voltage is 0.7volts?

Solution:

$$I = I_s e^{V_D/nV_T} \text{ ----- (A)}$$

V_D is a diode voltage.

$$V_T = 25 \text{ mV} = 0.025 \text{ volts}$$

$$1 = I_s e^{0.65/n(0.025)} \text{ ----- (B)}$$

$$2 = I_s e^{0.67/n(0.025)} \text{ ----- (C)}$$

Taking natural log of (B) and (C) and subtracting we have

$$\ln(1) = \ln(I_s e^{0.65/n(0.025)}) \text{ ----- (B)}$$

$$\ln(2) = \ln(I_s e^{0.67/n(0.025)}) \text{ ----- (C)}$$

Subtracting (B) and (C) we have

$$- \ln 2 = - 0.02/n(0.025)$$

$$n = 0.02/0.025 \ln 2$$

$$n = 1.154$$

put the value of n in equation B we have

$$I_s = e^{-0.65/1.154(0.025)}$$

$$= 1.64 \times 10^{-10}$$

so from equation A we have

$$I_D = 1.64 \times 10^{-10} e^{0.7/1.154(0.025)}$$

$$I_D = 1.64 \times 10^{-10} \times 3.4 \times 10^{10}$$

$$I_D = 5.66 \text{A}$$

THE REVERSE BIAS REGION:

The reverse bias region of operation is entered when the diode voltage is made negative. The previous equation indicates that if v is negative and a few times larger than V_T in magnitude the exponential term becomes negligibly small as compared to unity and diode current becomes

$$i = -I_s$$

that is, the current in the reverse direction is constant and equal to I_s . This constancy is the reason behind the term saturation current.

A good part of the reverse current is due to the leakage effect.

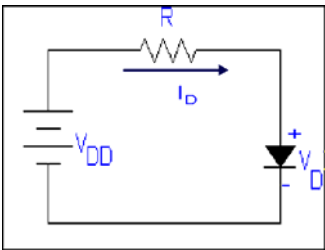
The leakage currents are proportional to the junction area, just as I_s . Its dependence on temperature is however, different from that of I_s .

THE BREAKDOWN REGION:

- The break down region is entered when the magnitude of the reverse voltage exceeds a threshold value specific to a particular diode and is called the **BREAKDOWN VOLTAGE**.

- It is the voltage at the knee of i-v curve and is denoted by V_{ZK} .
- In the break down region reverse current increases rapidly , while the associated change in voltage is very small this fact is used in voltage regulation.

ANALYSIS OF DIODE CIRCUITS:



Consider the circuit shown in the fig. We wish to analyze this circuit to determine the diode current I_D and voltage V_D .

The diode is obviously biased in forward direction.

Assuming that V_{DD} is greater than 0.5V or so, the diode current will be much greater than I_s and we can represent i-v characteristics by the exponential relationship resulting in

$$I_D = I_s e^{V_D/nV_T} \quad (1)$$

The other that governs circuit operations is obtained by writing a KVL equation resulting in

$$I_D = (V_{DD} - V_D) / R \quad (2)$$

if we put $I_D=0$ in equation 2 we will get

$$V_{DD} = V_D$$

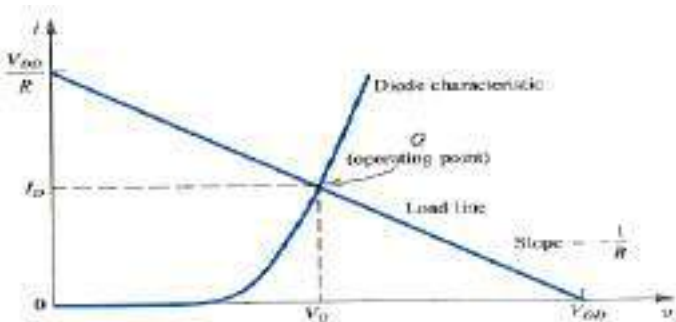
if we put $V_D = 0$

$$I_D = V_{DD} / R$$

We draw current along Y-axis and voltage across X-axis

GRAPHICAL ANALYSIS:

- Graphical analysis is performed by plotting the relationships of equations (1) and (2) on the i-v plane .The solution can then be obtained as the point of intersection of the two graphs.

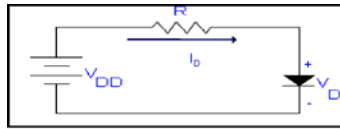


- A sketch of the graphical construction is shown in the fig. The curve represents the exponential diode equation and the straight line represents the equation (2). Such a straight line is known as load line . The load line intersects the diode curve at point Q, which represents the operating point of the circuit. It co-ordinates gives values of I_D and V_D

ITERATIVE ANALYSIS:

- Equations (1) and (2) can be solved using a simple iterative procedure, as illustrated in the following example

EXAMPLE: Determine the current I_D and the diode voltage V_D for the circuit in the fig. with $V_{DD} = 5V$ and $R = 1k$. Assume that the diode has a current of $1mA$ at a voltage of $0.7V$ and its voltage drop changes by $0.1V$ for every decade change in current.



SOLUTION:

To begin the iteration, we assume that $V_D = 0.7V$ and the equation (2) determines the current

$$\begin{aligned} I_D &= \frac{V_{DD} - V_D}{R} \\ &= \frac{5 - 0.7}{1} \\ &= 4.3mA \end{aligned}$$

Now from our previous knowledge

$$V_2 - V_1 = nV_T \ln(I_2/I_1)$$

$$\begin{aligned} \log_{10} x &= (\ln x) / (\ln 10) \text{ where } \ln 10 = 2.3 \\ \text{so } 2.3 \log_{10} x &= \ln x \\ \text{now the equation of log to the base 10 form will be as} \\ V_2 - V_1 &= 2.3nV_T \log_{10} I_2/I_1 \end{aligned}$$

For our case, $2.3 nV_T = 0.1\text{volts}$

$$V_2 = V_1 + 0.1 \log_{10} I_2/I_1$$

Substituting $V_1 = 0.7V$, $I_1 = 1mA$ and $I_2 = 4.3mA$

$$V_2 = 0.763V$$

Thus the results for the first iteration are

$$I_D = 4.3mA \text{ and } V_D = 0.763V$$

The second iteration proceeds in the similar manner

$$I_D = \frac{V_{DD} - V_D}{R}$$

Here $V_D = 0.763V$

$$I_D = \frac{5 - 0.763}{1}$$

$$= 4.237mA$$

$$V_2 = 0.763 + 0.1 \log_{10}(4.237/4.3)$$

$$= 0.762V$$

Thus the second iteration yields

$$I_D = 4.23mA \text{ and } V_D = 0.762V$$

Since these values are not much very different from the values obtained after the first iteration, no further iterations are necessary and the

Solution is $I_D = 4.23mA$ and $V_D = 0.762V$

LECTURE 33

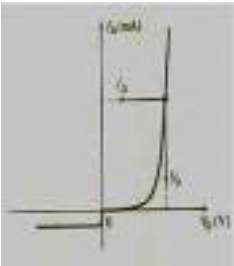
(DC or Static Resistance-The constant voltage drop model - AC or Dynamic Resistance)

DC OR STATIC RESISTANCE:

- When diode is subjected to a dc voltage its operating point on the characteristic curve does not change with time. Hence the resistance of the diode can be found easily

$$r_D = V_D / I_D$$

- It may also be seen on the curve that the resistance in the reverse bias is quite high.

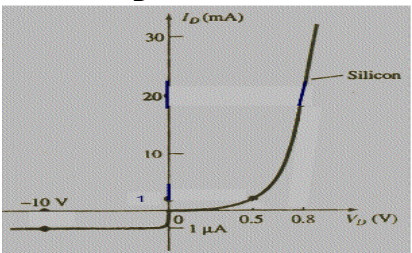


Example: Determine the resistance level for the diode having characteristics as shown in the graph below at

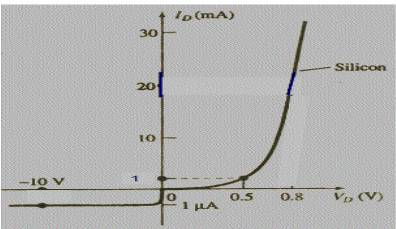
(1) $I_D = 1\text{mA}$

(2) $I_D = 20\text{mA}$

(3) $V_D = -10\text{V}$



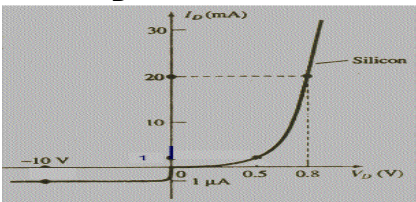
SOLUTION:



- (1) From the curve
For $I_D = 1\text{mA}$, $V_D = 0.5\text{volts}$

$$R_D = 0.5/1\text{m}$$

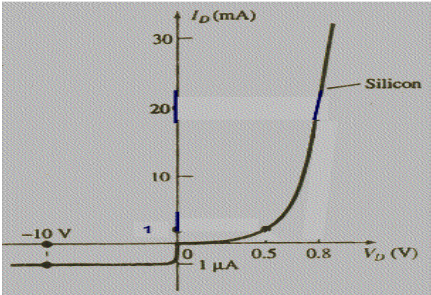
$$R_D = 500 \text{ ohms}$$



- (2) For $I_D = 20\text{mA}$, $V_D = 0.8\text{volts}$

$$R_D = 0.8/20\text{m}$$

$$R_D = 40\text{ohms}$$



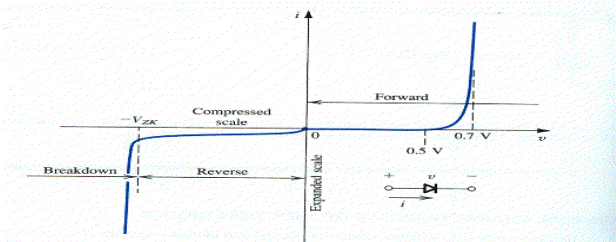
(3) For $V_D = -10V,$ $I_D = -1\text{microA}$

$R_D = 10/10^{-6}$

$R_D = 10\text{Mohms}$

THE CONSTANT VOLTAGE DROP MODEL:

- We use a vertical straight line to approximate the fast rising part of the exponential curve, as shown in the fig.



Ideal Model:

We have seen that the diode behaves essentially as a **switch**: on when forward biased, off when reverse biased. The ideal diode characteristic is shown in Figure B. Corresponding to this is the ideal diode model, Figure C.

Figure B: Ideal diode characteristic.

✓ Ideal-diode volt-ampere characteristic

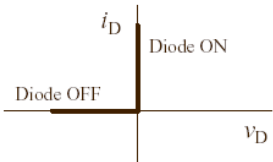
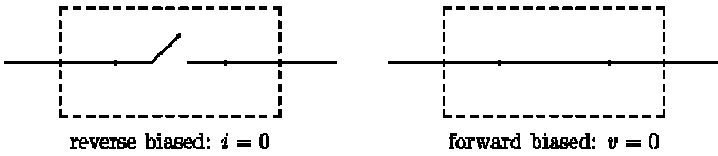


Figure C: Ideal diode model.



When the diode is reverse biased, we replace it with an open switch, and when the diode is forward biased, we use a closed switch.

PRACTICAL MODEL:

In practice we find that there is a voltage drop of about 0.7 V across the diode (silicon; germanium is 0.3 V) when it is forward biased, and so it is often useful to include this voltage drop in circuit analysis. Accordingly, the **practical model** is the ideal model with the addition of a voltage source in the forward bias model. Figure D shows the practical model characteristic. The practical model is shown in Figure E.

Figure D: Practical diode characteristic.

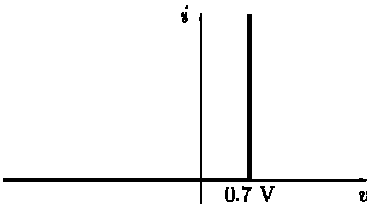
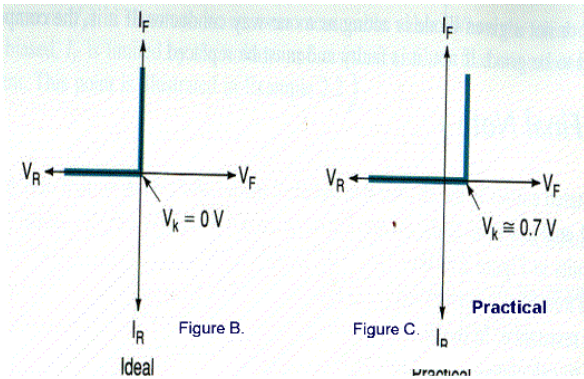
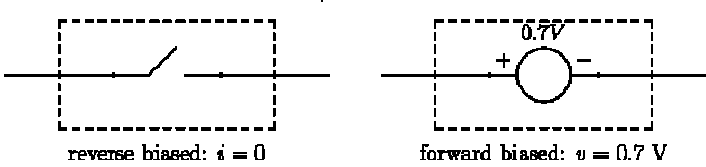
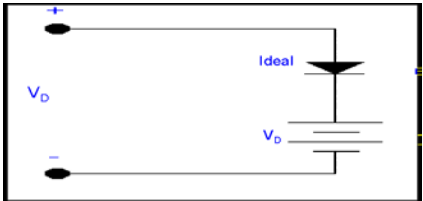


Figure E: Practical diode model.

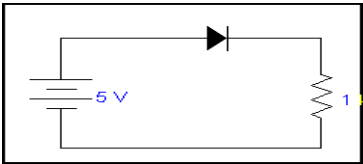


- The resulting model simply says that a forward conducting diode exhibits a constant voltage drop V_D . The value of V_D is usually taken to be as 0.7 volts in case of silicon diode. The constant voltage drop model can be represented by the equivalent model shown in the fig.



THE EFFECT OF THE FORWARD VOLTAGE DROP ON CIRCUIT ANALYSIS

To illustrate this effect consider this circuit,



According to the KVL, the sum of the component voltages in the circuit must be equal to the applied voltage. By formula

$$V_s = V_F + V_R$$

V_s = voltage of source

V_F = forward voltage across the diode

V_R = voltage drop across 1 ohm resistance

if we consider diode is a silicon so, $V_F = 0.7$ volts and arrange this equation for V_R then

$$V_R = V_s - 0.7V$$

According to ohms law

$$I_T = V_R / R$$

Putting the value of V_R from this equation

$$I_T = V_s - 0.7/R_1$$

Putting the value of R_1 we have

$$= 5 - 0.7/1$$

$$= 4.3A$$

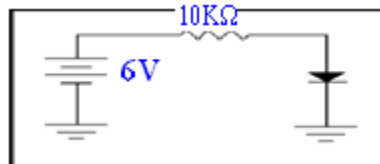
If we consider this diode to be **ideal** than no voltage drop will occur across it and we will get

$$V_s = V_R$$

$$V_F = 0$$

$$I_T = 5mA$$

Example: Find voltage across R_1 . Consider silicon diode so that 0.7 V drop is assumed across it.



Solution:

By KVL

$$V_s = V_R + V_f$$

$$V_R = V_s - 0.7$$

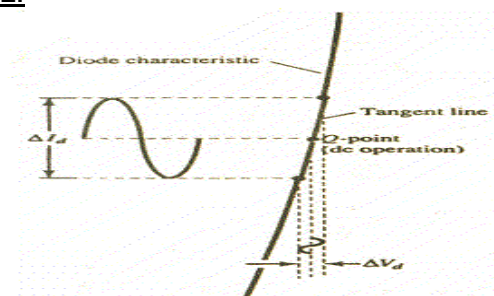
$$V_R = 6 - 0.7$$

$$= 5.3 \text{ volts} \approx 5 \text{ Volts.}$$

THE CONSTANT VOLTAGE DROP MODEL:

- (1) Diode current will remain at zero until the knee voltage is reached.
- (2) Once the applied voltage reaches the value of V_K , the diode turns on and the forward conduction occurs.
- (3) As long as the diode conducting, the value of V_F is approximately equal to V_K . In other words the value of V_F is assumed to be 0.7volts regardless of the value of I_F .

AC OR DYNAMIC RESISTANCE:

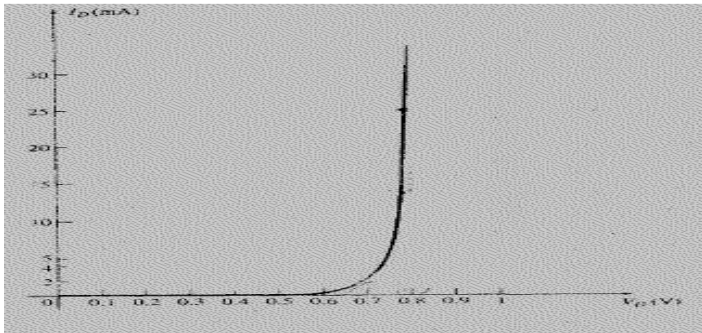


$$r_D = \Delta V_D / \Delta I_D$$

Δ shows a finite change in the quantity. The resistance in the vertical rise region is small, where as ac resistance is much larger at low current levels.

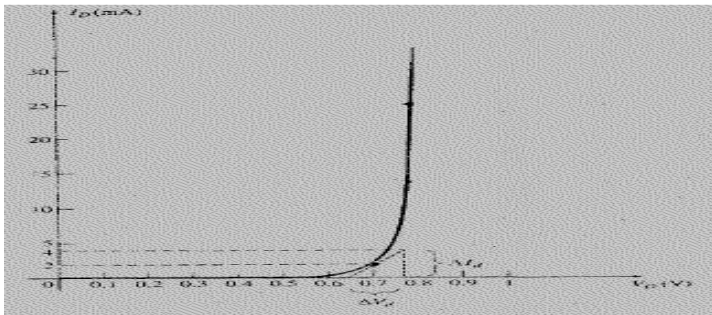
LECTURE 34
(Small Signal Model and its applications)

EXAMPLE: Find the AC resistance for the curve shown at
(1) $I_D = 2\text{mA}$
(2) $I_D = 25\text{mA}$

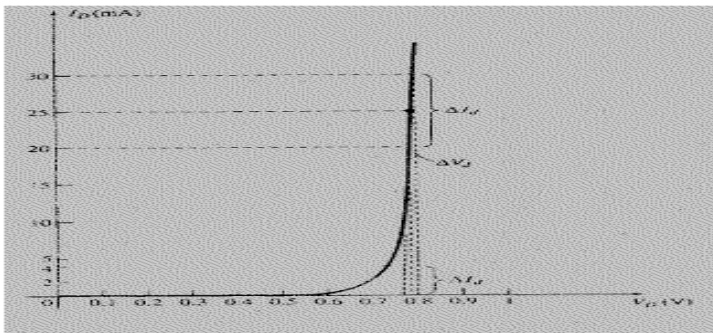


SOLUTION:

(1) Choosing a swing of pulse minus 2mA
For
 $I_D = 4\text{mA}, V_D = 0.76\text{V}$
And $I_D = 0\text{mA}, V_D = 0.65\text{V}$
 $\Delta I_D = 4 - 0 = 4\text{mA}$
 $\Delta V_D = 0.76 - 0.65 = 0.11\text{V}$
 $r_D = 0.11/4\text{mA}$
 $= 27.5\text{ ohms}$



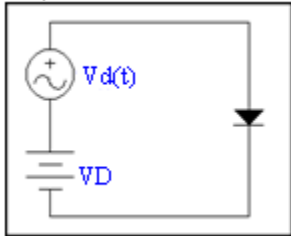
(2) Choose a swing of pulse minus 5mA
 $\Delta I = 30 - 20 = 10\text{mA}$



$\Delta V_D = 0.8 - 0.78$
 $= 0.02\text{V}$
 $r_D = 0.02/10$
 $= 10\text{ ohms}$

SMALL SIGNAL MODEL

In some applications circuit is supplied to a bias to operate in the forward region and also a small ac signal is super imposed on dc quantity consider the circuit shown



When there is no ac, diode voltage is V_D so that

$$I_D = I_S e^{V_D / nV_T}$$

and when signal $V_d(t)$ is applied, then instantaneous voltage $V_d(t)$ is
$$v_D(t) = V_D + v_d(t)$$

so that instantaneous diode current will be

$$i_D(t) = I_S e^{v_D / nV_T}$$

Substituting for v_D

$$i_D(t) = I_S e^{(V_D + v_d) / nV_T}$$

$$\text{Or } = I_S e^{V_D / nV_T} \cdot e^{v_d / nV_T}$$

$$\text{Or } i_D(t) = I_D e^{v_d / nV_T} \text{ ----- (A)}$$

If the amplitude of the signal is sufficiently smaller than 1, so that $v_d / nV_T \ll 1$ expanding the previous equation in the form of a taylor series i.e

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

Above expression is a taylor series expansion of exponential function e^x
in the equation (A) $x = v_d / nV_T$, so we solve the equation (A) upto two terms by taylor series

$$e^{v_d / nV_T} = (1 + v_d / nV_T)$$

put this value in equation (A) we have

$$\begin{aligned} i_D(t) &= I_D (1 + v_d / nV_T) \\ &= I_D + I_D / nV_T v_d \text{ ----- (B)} \end{aligned}$$

Or we may write

$$I_D / nV_T v_d = i_d \text{ put in equation (B) we have}$$

$$i_D = I_D + i_d$$

From previous equation we may see that the quantity I_D / nV_T has the dimensions of conductance given in mho's and is called diode small signal conductance. The inverse of it is called small signal resistance or incremental resistance given by

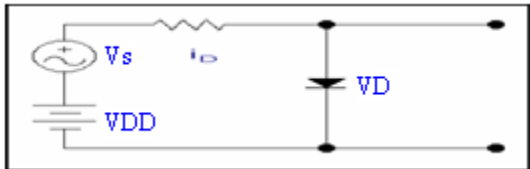
$$r_d = nV_T / I_D$$

$$r_d \propto 1 / I_D$$

nV_T are constants and are provided by the manufacturer.

APPLICATION:

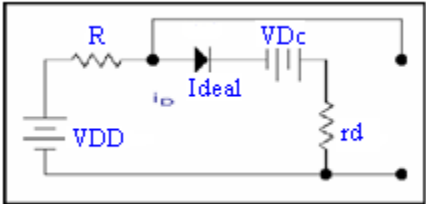
Consider the circuit as shown, for analysis purposes, we can split the circuit into two parts that is ac and dc



DC source having the value of VDD

we replace the ac and replace diode with constant drop model

For dc analysis the Circuit will be



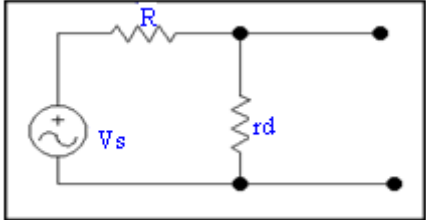
in case of DC we consider only DC current I_D is flowing and no effect of the ideal diode because this ideal diode is forward biasing and it results in short circuit therefore

From KVL

$$V_{DD} - I_D R - V_{D0} - I_D r_d = 0$$
$$V_{DD} = I_D R + V_{D0} + I_D r_d \text{ ----- (A)}$$

in case of DC only DC current I_D is flowing the

For ac analysis we will remove DC sources which are the part of the original circuit and also remove the DC source which appear in the previous effective circuit and we also remove the ideal circuit so our circuit will be as



For ac analysis

$$v_s = i_d (R + r_d) \text{ ----- (B)}$$

by combining (A) and (B)

Overall analysis is

$$V_{DD} + v_s = I_D R + V_{D0} + I_D r_d + i_d (R + r_d)$$
$$= I_D (R + r_d) + i_d (R + r_d) + V_{D0}$$
$$V_{DD} + v_s = (R + r_d)(I_D + i_d) + V_{D0} \text{ ----- (C)}$$

But

$$I_D + i_d = i_D$$

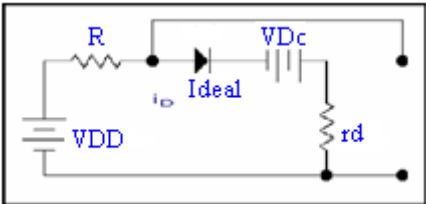
therefore

$$V_{DD} + v_s = (R + r_d)(I_D + i_d) + V_{D0}$$

separating the dc and signal quantities on both sides of equation (C)

$$V_{DD} = I_D R + V_{D0}$$

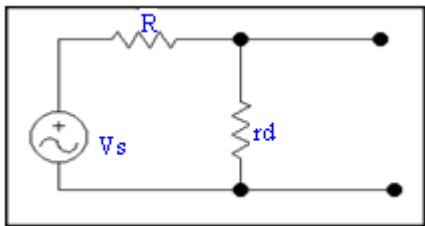
which is represented by the circuit in the figure below



and for the signal

$$v_s = i_d (R + r_d)$$

which is represented by the circuit in the figure below



However, if we carefully see the ac equation circuit, it is nothing more than a voltage divider.
Hence the diode signal voltage will be

$$V_d = V_s \frac{r_d}{(r_d + R)}$$

EXAMPLE: Find the value of the diode small signal resistance r_d at bias current of 0.1, 1 and 10mA.

Assume $n = 1$

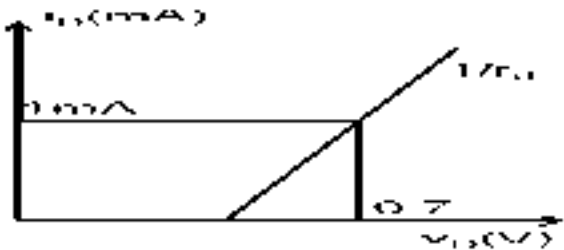
SOLUTION:

$$\begin{aligned} r_d &= n \frac{V_T}{I} \\ \text{where } V_T &= 25\text{m Volt} \\ &= 1 \times 25\text{m} / I \end{aligned}$$

$$\begin{aligned} \text{for } I &= 0.1\text{mA}, & r_d &= 250\text{ohms} \\ I &= 1\text{mA}, & r_d &= 25\text{ohms} \\ I &= 10\text{mA}, & r_d &= 2.5\text{ohms} \end{aligned}$$

EXAMPLE: For the diode that conducts 1mA at a forward voltage drop of 0.7 V and whose $n = 1$. Find the equation of the straight line tangent at $I_D = 1\text{mA}$.

SOLUTION:



$$\begin{aligned} \text{Slope} &= 1/r_d \\ &= I/nV_T \\ &= 1\text{mA}/1 \times 25\text{mV} \\ &= 1/25\text{ohms} \end{aligned}$$

Now on V_D axis it will be

$$\begin{aligned} &= 0.7 - 1\text{m}(25) \\ &= 0.7 - 0.025 \\ &= 0.675\text{ V} \end{aligned}$$

So
by the equation of straight line $y - y_1 = m (x - x_1)$

$$\begin{aligned} \text{here } m &\text{ is a slope } = 1/25\text{ohms} \\ i_D &= 1/25 (V_D - 0.675) \\ V_D - 0.675 - i_D(25) &= 0 \end{aligned}$$

Which is the required equation of straight line.

EXAMPLE: Consider a diode with $n = 2$ biased at 1mA. Find the change in current as a result of changing the voltage by

- (a) – 20 mV (b) – 10 mV (c) – 5mV
(d) +5mV (e) +10mV (f) +20mV

In each case do calculations

- (1) using small signal model
(2) using the exponential model.

SOLUTION:

For small signal model we have

$$\Delta v = r_d \Delta i$$

$$\Delta i = \Delta v / r_d$$

But

$$r_d = n V_T / I$$

$$= 2 \times 25\text{mV} / 1\text{mA}$$

$$= 50\Omega$$

$$\Delta i = \Delta v / 50$$

Now from exponential model

$$i = I_S e^{v / nVT}$$

Also we know that

$$I_D + \Delta i = I_D e^{\Delta v / nVT}$$

$$I_D + \Delta i = I_D e^{\Delta v / nVT}$$

$$\Delta i = I_D (e^{\Delta v / nVT} - 1)$$

but

$$I_D = 1\text{mA} \text{ thus}$$

$$\Delta i = (e^{\Delta v / nVT} - 1)$$

(a) For $V = -20\text{ mV}$

$$(1) \Delta i = -20/50$$

$$= -0.4\text{mA}$$

$$(2) \Delta i = e^{-20\text{mV}/50\text{mV}} - 1$$

$$= 0.33\text{mA}$$

(d)

$$\Delta v = 5\text{mV}$$

$$(1) \Delta i = 5/50 = 0.10\text{mA}$$

$$(2) \Delta i = e^{5/50} - 1$$

$$= 0.11\text{mA}$$

(f)

$$\Delta v = 20\text{mV}$$

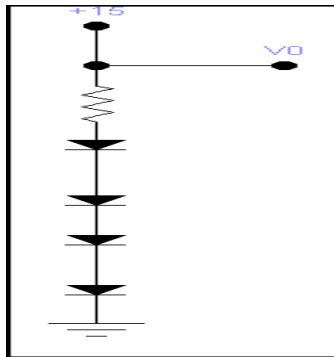
$$(1) \Delta i = 20/50$$

$$= .40\text{mA}$$

$$(2) \Delta i = e^{20/50} - 1$$

$$= 0.49\text{mA}$$

EXAMPLE: Design the circuit in the fig. so that $V_0 = 3V$ when $I_L = 0$, and V_0 changes by 40mV per 1mA of load current. Find the value of R . (assume four diodes are identical) relative to a diode with 0.7 drop at 1 mA current. Assume $n = 1$



SOLUTION:

$V_0 = 3V$, when $I_L = 0$, therefore each diode should exhibit a drop of 0.75V. If $I_L = 1mA$, then V_0 changes by 40mV and a change due to each diode is 10mV.

Hence

$$r_d = 10mV/1mA \\ = 10 \text{ Ohms}$$

but

$$r_d = nV_T / I_D \\ 10 = 1 \times 25m / I_D \\ I_D = 2.5mA$$

Hence

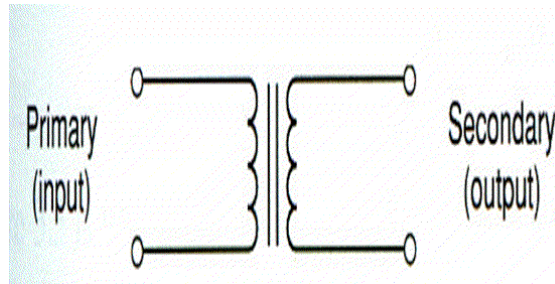
$$15 - 3 - I_D R_D = 0 \\ R = (15 - 3) / I_D \\ = (15 - 3) / 2.5m \\ = 4.8k \text{ Ohms.}$$

LECTURE 35

(Transformers - Secondary voltage and current - Transformer Input/Output phase relationship)

TRANSFORMERS:

- Transformers are not semiconductor devices; however, they play an integral role in the operations of most of power supplies.
- The basic schematic symbol for the transformer is as shown in figure,

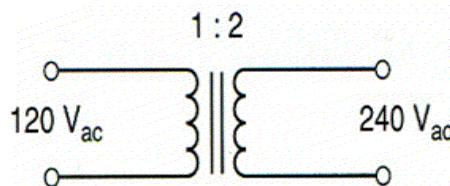


- The transformer basically consists of two inductors, which are in close proximity to each other.
- However, they are not physically connected.
- The device consists of two windings called the primary and secondary windings.
- The input to the transformer is applied to the primary.
- The output is taken from the secondary of the device.
- An alternating voltage is applied to the primary which induces an alternating voltage in the secondary.
- However, keep in mind that primary and secondary windings are physically isolated.
- A transformer can be designed in three modes.

1. STEP UP TRANSFORMER:

The step up transformer provides a secondary voltage which is greater in amplitude as compared to primary.

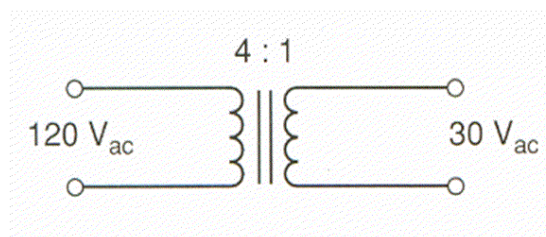
For example, a step up transformer may provide a 240 Vac out put against 120 Vac at the in put. A step up transformer is shown



2. STEP DOWN TRANSFORMER:

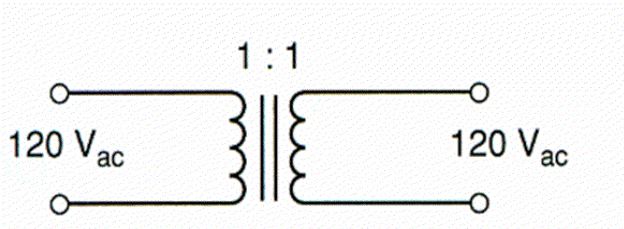
The step down transformer provides a secondary voltage which is less than the primary voltage.

For example, a transformer shown in the figure provides 30Vac at the out put against 120Vac input, as shown



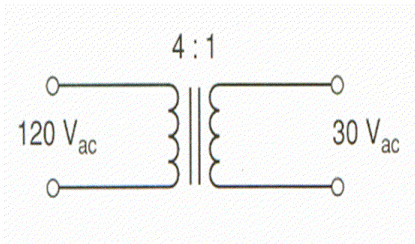
3.ISOLATION TRANSFORMER:

An isolation transformer provides an out put voltage that is equal to the input voltage.
These are used to electrically isolate power supplies from the power lines.



TURN RATIO:

- The turn ratio of a transformer is the ratio of number of turns in the primary to the number of turns in the secondary.
- For example, the step down transformer shown has a turn ratio of 4:1, which means there are four turns in the primary against each turn in the secondary.
-



- The turn ratio of the transformer is equal to the voltage ratio of the two components, so that

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

Where

- N2=Number of turns in the secondary winding.
- N1= Number of turns in the primary winding.
- V2=Voltage in the secondary.
- V1=Voltage to the primary.

CALCULATING SECONDARY VOLTAGE

When turn ratio and the primary voltage is known, the secondary voltage can be found as

$$V_2 = (N_2/N_1)V_1$$

For example the step down transformer in the previous fig. has 120 Vac input. The secondary voltage can be found as

$$\begin{aligned} V_2 &= (N_2/N_1) V_1 \\ &= 1/4(120Vac) \\ V_2 &= 30Vac \end{aligned}$$

CALCULATING SECONDARY CURRENT:

Ideally, transformers are 100 percent efficient. This means that the ideal transformer can transfer 100 percent of its input power to the secondary. So that by formula we may write

$$P_2 = P_1$$

Since power is equal to the product of voltage and current

$$V_2 I_2 = V_1 I_1$$

and

$$I_1/I_2 = V_2/V_1$$

The current ratio is inverse to the voltage ratio. This means that

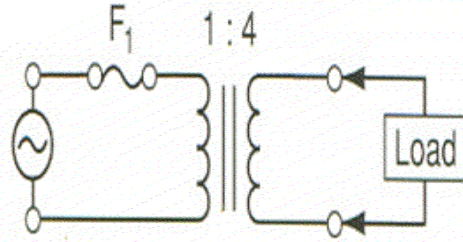
- (1) For step down transformer, $I_2 > I_1$
- (2) For step up transformer, $I_2 < I_1$

- In other words, current varies opposite to the variation in voltage. If voltage increases, current decreases and vice versa.
Since the Voltage ratio of a transformer is equal to the turn ratio therefore, the previous equation can be written as

$$\begin{aligned} I_1/I_2 &= N_2/N_1 \\ I_2 &= (N_1/N_2)I_1 \end{aligned}$$

EXAMPLE:

The fuse in the figure is used to limit the current in the primary of the transformer. Assuming that the fuse limits the value of I_1 to 1A, what is the limit on the value of the secondary current.

**SOLUTION:**

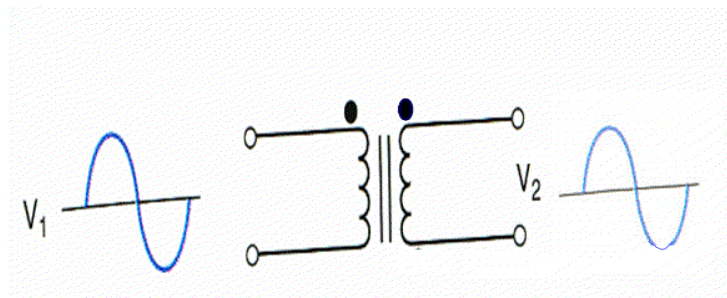
The maximum secondary current is found using the limit on I_1 and the turn's ratio as follows:

$$\begin{aligned} I_2 &= N_1/N_2 I_1 \\ &= (1/4)1A \\ I_2 &= 250mA \end{aligned}$$

If the secondary current tries to exceed the 250mA limit, the primary current will exceed its limit and blow off the fuse.

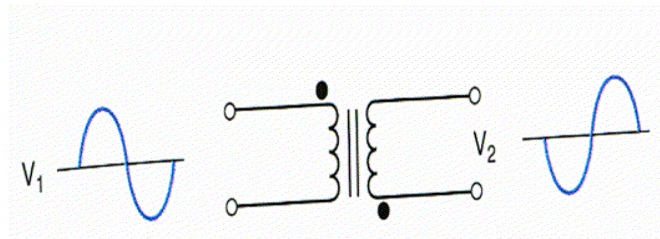
TRANSFORMER INPUT/OUTPUT PHASE RELATIONSHIP:

- In the schematic symbol, there are two dots : one on the top side of the primary and the other on top of secondary. In this case we will be working with a transformer which does not produce any phase difference between the input and out put.
-



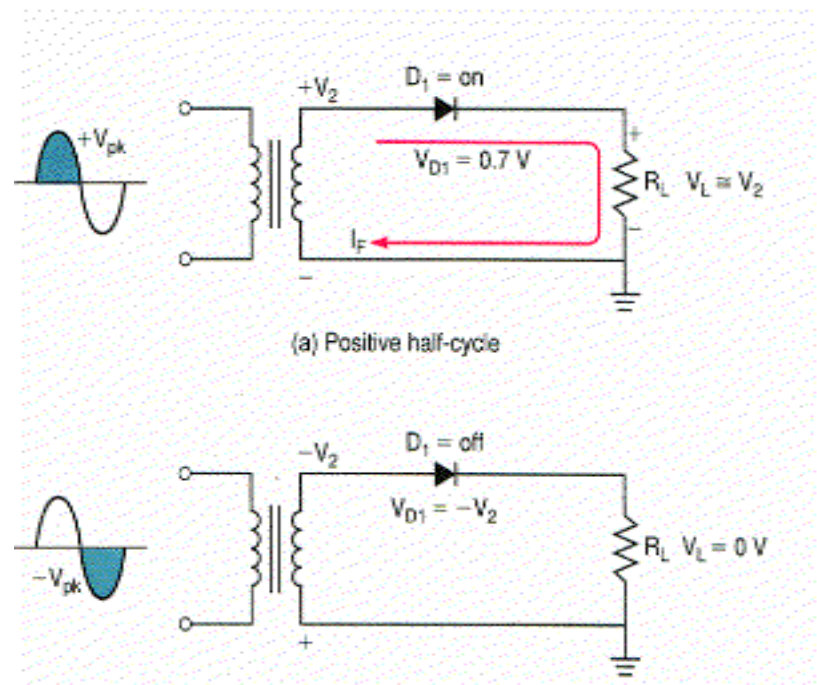
- In the schematic symbol, there are two dots : one on the top side of the primary and one on the bottom side of the secondary, in this case

We are working with a transformer whose output voltage is 180° out of phase with its input voltage as shown in fig.



HALF WAVE RECTIFIERS:

- The half wave rectifier is made up of a diode and a resistor as shown in the fig. The half wave rectifier is used to eliminate either positive or negative part of the input.



Negative Half Wave Rectifiers:

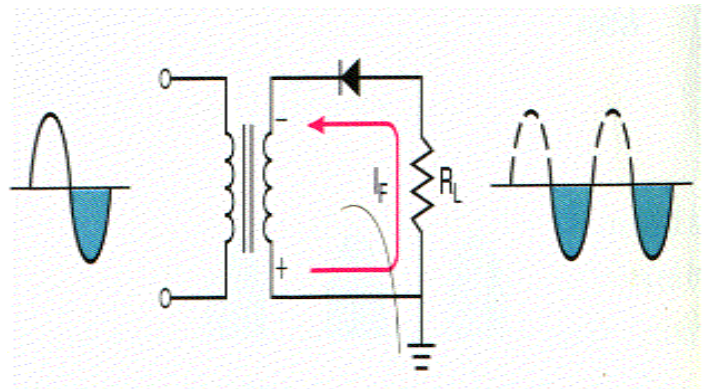


Fig. shows a half wave rectifier with diode direction is reversed. In this circuit the diode will conduct on the negative half cycle of the input, and

$$V_L = V_2.$$

The diode will be reversed biases for the positive half cycle of the input and

$$V_D = V_2.$$

As a result positive half cycle of the input is eliminating. The operating principle of the negative half wave rectifier is same as the positive half wave rectifiers. The only difference is the polarity of the output will be reversed.

LECTURE 36
(Half Wave Rectifiers)

Negative Half Wave Rectifiers:

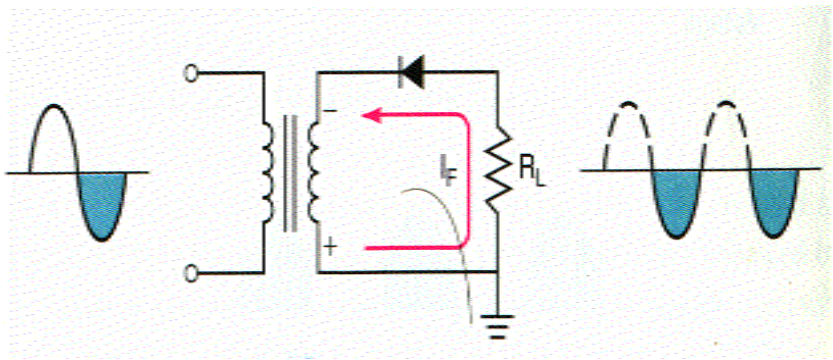


Fig. shows a half wave rectifier with diode direction is reversed. In this circuit the diode will conduct on the negative half cycle of the input, and

$$V_L = V_2.$$

The diode will be reversed biases for the positive half cycle of the input and

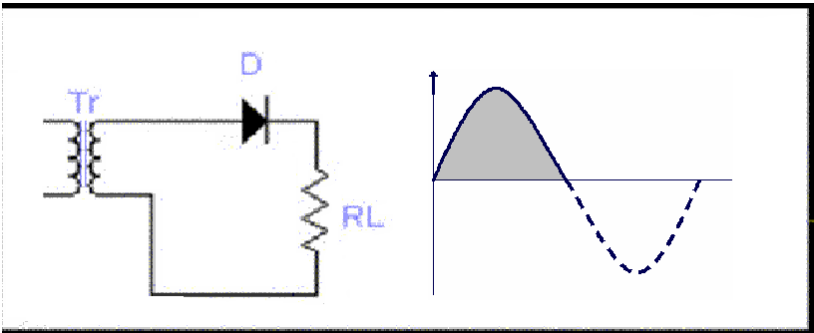
$$V_D = V_2.$$

As a result positive half cycle of the input is eliminating. The operating principle of the negative half wave rectifier is same as the positive half wave rectifiers. The only difference is the polarity of the output will be reversed.

GENERAL RULES:

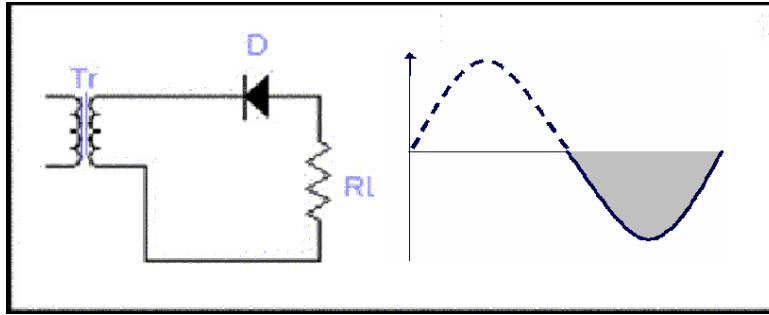
- (1) When the diode points towards the load (R_L), the output from the rectifier will be positive.

The point made so far is summarized in the fig below.



- (2) When the diode points toward the transformer, the output from the rectifier will be negative.

The point made so far is summarized in the fig below.

**CALCULATING LOAD VOLTAGE AND CURRENT VALUES.**

The load voltage in half wave rectifier can be found as

$$V_{L(pk)} = V_{2(pk)} - V_F$$

$V_{2(pk)}$ is the secondary voltage of the transformer, found as

$$V_{2(pk)} = (N_2/N_1) V_1$$

where

N_2/N_1 = (The turn ratio of transformer secondary turns to primary turn)

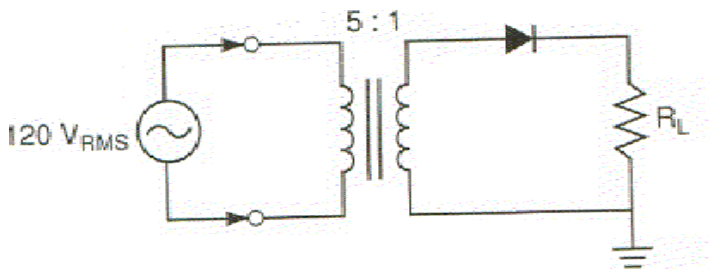
$V_1(pk)$ = (The peak transformer primary voltage)

Above equation assumes that the transformer is given as a peak value.

More often than not, source voltages are given as **rms** values. When this is the case, the source voltage is converted into peak value as follows:

$$V_{pk} = V_{rms} / 0.707$$

EXAMPLE: Determine the peak load voltage for the circuit shown in the fig.

**SOLUTION:**

First, the ac input to the transformer is converted to a peak value, as follows

$$\begin{aligned} V_1(pk) &= V_{1rms}/0.707 \\ &= 120/0.707 \\ &= 169.7V_{pk} \end{aligned}$$

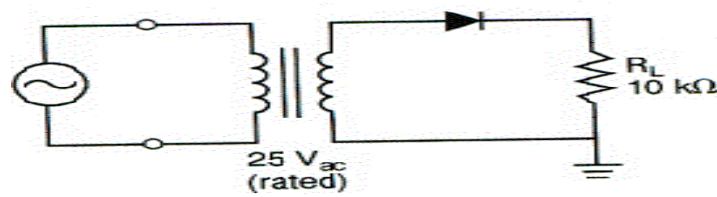
Now, the voltage values in the secondary circuit are found as

$$\begin{aligned} V_{2(pk)} &= N_2/N_1(V_1_{pk}) \\ &= 1/5(169.7) \\ V_{2(pk)} &= 33.94V_{pk} \end{aligned}$$

And

$$\begin{aligned} V_{Lpk} &= V_{2(pk)} - V_F \\ \text{diode is a silicon diode} \\ &= 33.94 - 0.7 \\ V_{Lpk} &= 33.24V_{pk} \end{aligned}$$

EXAMPLE: Determine the peak load voltage for the circuit shown in the fig.



SOLUTION:

The transformer is shown to have a $25V_{ac}$ rating. This value of V_2 is converted into peak form as

$$V_{2(pk)} = V_{2(rms)} / 0.707$$

$$= 25V_{rms} / 0.707$$

$$V_{2(pk)} = 35.36V_{pk}$$

Now the value of V_{pk} is found as

$$V_{L(pk)} = V_{2(pk)} - 0.7$$

$$= 35.36 - 0.7$$

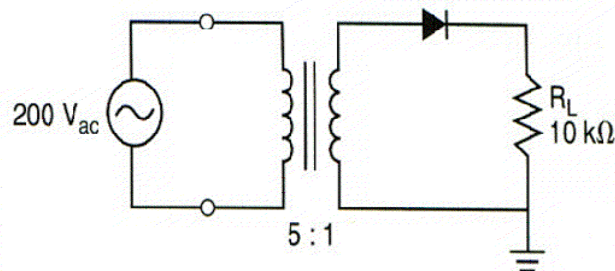
$$V_{L(pk)} = 34.66V_{pk}$$

CALCULATING LOAD CURRENT:

Once the peak voltage is determined, the peak load current is found as

$$I_{L(pk)} = V_{L(pk)} / R_L$$

EXAMPLE: What is the peak load current for the circuit shown in fig.



SOLUTION:

The input voltage is given an rms value. This value is converted to a peak value as follows:

$$V_{1(pk)} = V_{1(rms)} / 0.707$$

$$= 200V_{ac} / 0.707$$

$$V_{1(pk)} = 282.9V_{pk}$$

Now, the load voltage and current are found, after finding peak voltage, as

$$V_{2(pk)} = N_2/N_1 V_{1(pk)}$$

$$= (1/5)(282.9V_{pk})$$

$$V_{2(pk)} = 56.6V_{pk}$$

Finally, the load voltage and current and current values are found as:

$$V_{L(pk)} = V_{2(pk)} - V_F$$

$$= 56.6 - 0.7$$

$$V_{L(pk)} = 55.9V_{pk}$$

and the current will be

$$I_{L(pk)} = V_{L(pk)} / R_L$$

$$= 55.9 \text{ V}_{pk} / 10 \text{ k}$$

$$I_{L(pk)} = 5.59 \text{ mA}_{pk}$$

AVERAGE LOAD VOLTAGE AND CURRENT:

- Since rectifiers are used to convert ac to dc, V_{ave} is a very important value. For a half wave rectifier, V_{ave} is found as

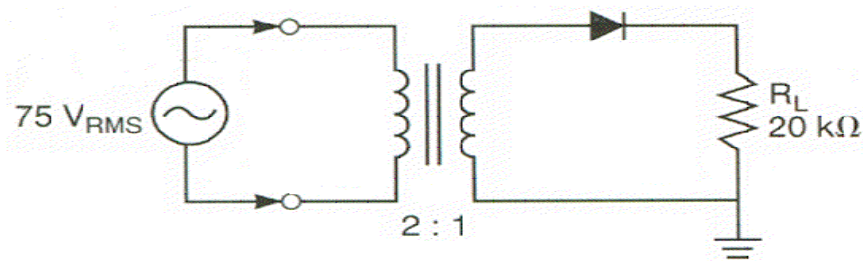
$$V_{ave} = V_{pk} / \pi$$

Another form of this equation is

$$V_{ave} = 0.318(V_{pk}) \text{ (half wave rectified)}$$

Where $0.318 = 1/\pi$. Either of these equations can be used to determine the dc equivalent load voltage for a half wave rectifier. Let's take some examples to illustrate this

EXAMPLE: Determine the value of V_{ave} for the circuit shown in fig.



SOLUTION:

$$V_{1(pk)} = V_{1(rms)} / 0.707$$

$$= 75 \text{ V}_{ac} / 0.707$$

$$V_{1(pk)} = 106.1 \text{ V}_{pk}$$

$$V_{2(pk)} = N_2 / N_1 V_{1(pk)}$$

$$= 1/2 (106.1)$$

$$V_{2(pk)} = 53.04 \text{ V}_{pk}$$

$$V_{L(pk)} = 53.04 - 0.7$$

$$V_{L(pk)} = 52.34 \text{ V}_{pk}$$

$$V_{ave} = V_{pk} / \pi$$

$$= 52.34 / \pi$$

$$V_{ave} = 16.66 \text{ V}_{dc}$$

CALCULATING AVERAGE CURRENT:

- THE VALUE OF I_{ave} can be calculated in one of two ways
 - (1) We can determine the value of V_{ave} and then use Ohm's law as follows
- $$I_{ave} = V_{ave} / R_L$$
- (2) We can convert I_{pk} to average from using same basic equations, that

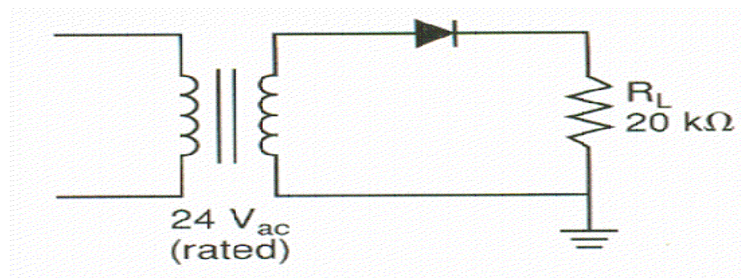
We used to convert V_{pk} to V_{ave} . The current forms of these equations are

$$I_{ave} = I_{pk} / \pi$$

And

$$I_{ave} = 0.318(I_{pk})$$

EXAMPLE: Determine the dc load current for the rectifier shown in fig.



SOLUTION:

The transformer has 24 V_{ac} rating. Thus, the peak secondary voltage is found as

$$V_{2(pk)} = 24V_{ac} / 0.707$$

$$V_{2(pk)} = 33.9 V_{pk}$$

The peak load voltage is now found as

$$V_{L(pk)} = V_{2(pk)} - 0.7$$

$$V_{L(pk)} = 33.2 V_{pk}$$

$$I_{L(pk)} = V_{L(pk)} / R_L$$

$$= 33.2 / 20k$$

$$I_{L(pk)} = 1.66mA_{pk}$$

$$I_{ave} = I_{pk} / \pi$$

$$= 1.66m / \pi$$

$$I_{ave} = 529.13 \text{ micro A}$$

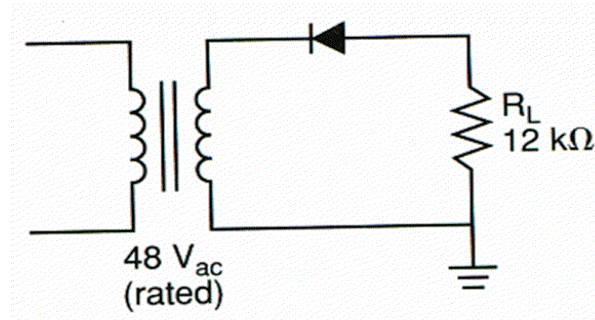
LECTURE 37

(Peak Inverse Voltage (PIV) - Full wave rectifier)

Negative Half Wave Rectifiers:

- The analysis of negative half wave rectifier is nearly identical to that of positive half wave rectifier. The only difference is that the voltage polarities are reversed. We can make a simple method for performing the mathematical analysis of negative half wave rectifier.
 - (1) Analyze the circuit as if it was a positive half wave rectifier.
 - (2) After completing calculations, change all voltage polarity signs from positive to negative.

EXAMPLE: Determine the dc output voltage for the circuit shown in the fig.



SOLUTION:

We shall start by solving the circuit as if it was a positive half wave rectifier. First,

$$\begin{aligned} V_{2(pk)} &= 48V_{ac} / 0.707 \\ &= 67.9 V_{pk} \end{aligned}$$

And

$$\begin{aligned} V_{L(pk)} &= V_{2(pk)} - 0.7 \\ V_{L(pk)} &= 67.2 V_{pk} \end{aligned}$$

Finally,

$$\begin{aligned} V_{ave} &= V_{pk} / \pi \\ &= 21.39 V_{dc} \end{aligned}$$

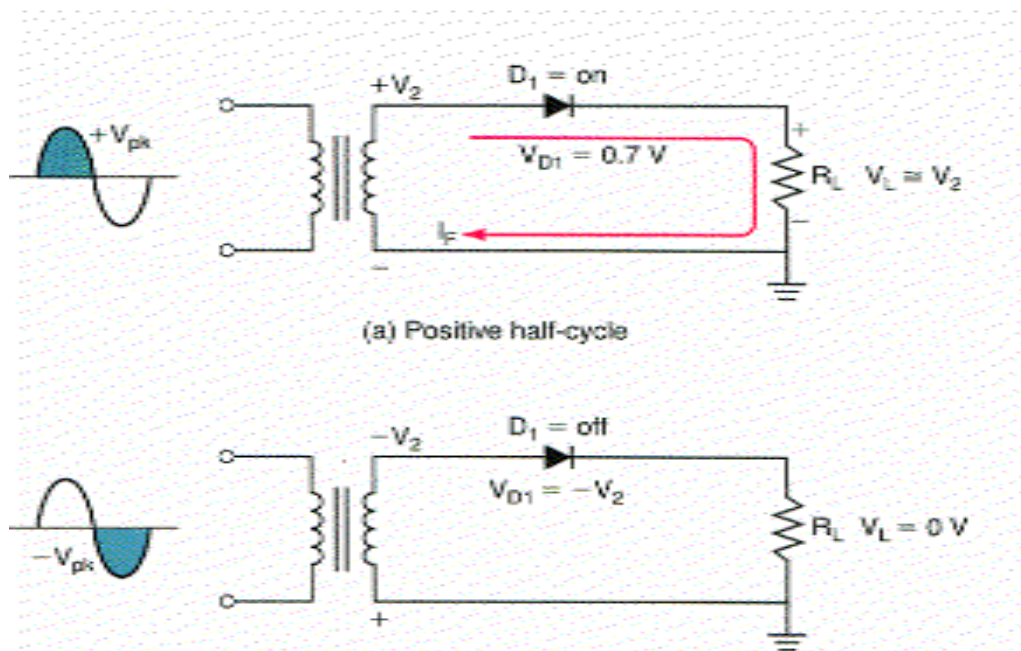
Now we will simply change all positive voltage values to negative voltage values.

$$\begin{aligned} V_{2(pk)} &= - 67.9 V_{pk} \\ V_{L(pk)} &= - 67.2 V_{pk} \\ V_{ave} &= - 21.39 V_{dc} \end{aligned}$$

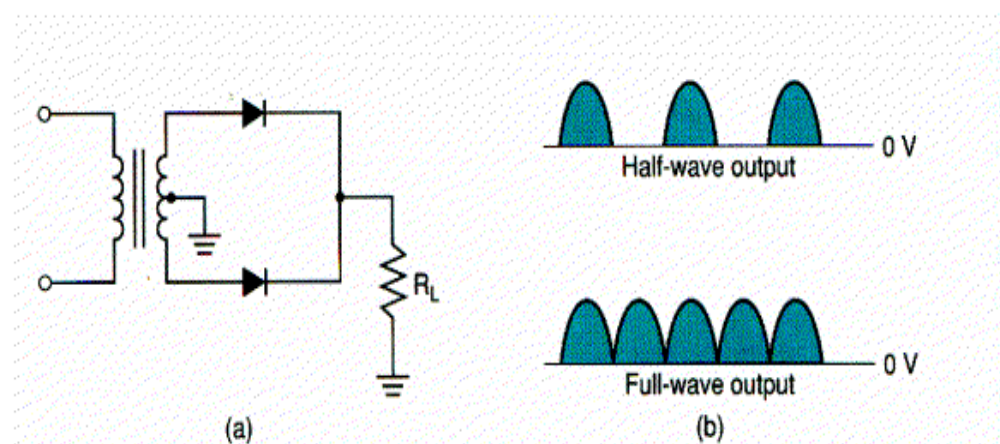
PEAK INVERSE VOLTAGE (PIV):

- The maximum amount of reverse bias that a diode will be exposed to in a rectifier is called the peak inverse voltage or PIV of the rectifier. For the half wave rectifier, the value of PIV is found as

$$PIV = V_{2(pk)}$$
- The basis for this equation can be seen by referring to figure. When diode is reversed biased, there is no voltage drop across the load.
- Therefore, all of V_2 is dropped across the diode in the rectifier.

**FULL WAVE RECTIFIER:**

- The full wave rectifier consists of two diodes and a resistor, as shown in the fig (a).



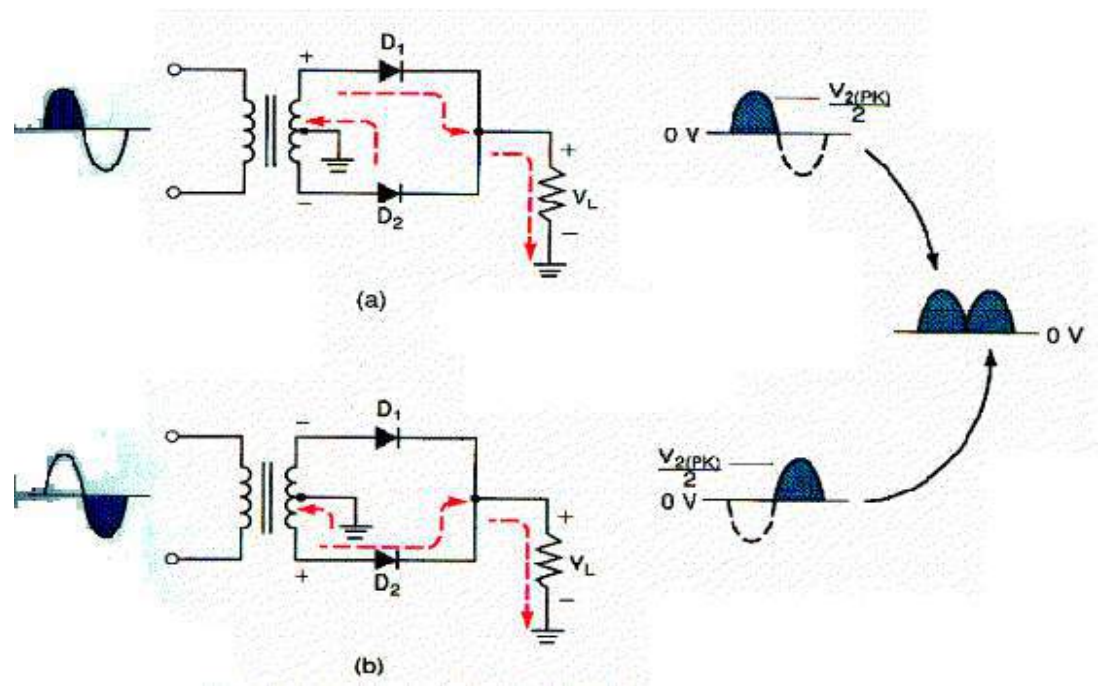
- The result of this change in circuit is illustrated in fig (b).
- In the fig (b), the output from the full wave rectifier is compared with that of a half wave rectifier.

Note that the full wave rectifier has two positive half cycle out for every one produced by the half wave rectifier.

- The transformer shown in the fig. is center tapped transformer.
- This type of transformer has a load connected to the center of the secondary winding.
- The voltage from the center tap to each of the outer winding is equal to one half of the secondary voltage. For example, let's say we have a 24V center tapped transformer. The voltage from the center tap to each of outer winding terminal is 12V.
- Center tapping of the transformer plays a major role in the operation of the full wave rectifier. For this reason, the full wave rectifier can not be line operated, that is it can not be connected directly to the ac input like the half wave rectifier can.

BASIC CIRCUIT OPERATION

- Figure shows the operation of the full wave rectifier during one complete cycle of the input signal. During the positive half cycle of the input D_1 is forward biased. Using ideal operating characteristics of the diode V_L can be found as



When the polarity of the input reverses, D_2 is forward biased, and D_1 is reverse biased. The direction of the current through the load will not change even though the polarity of the transformer secondary will. Thus another positive half cycle is produced across the load

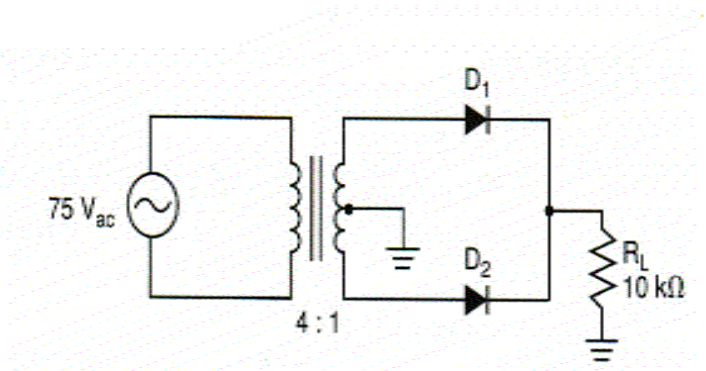
CALCULATING LOAD VOLTAGE and CURRENT VALUES:

- Using practical diode model, the peak load voltage for a full wave rectifier is found as
$$V_{L(pk)} = V_{2(pk)} / 2 - 0.7$$
- The full wave rectifier will produce twice as many output pulses as the half wave rectifier.
- For this reason, the average load voltage for the full wave rectifier is found as
$$V_{ave} = 2V_{L(pk)} / \pi$$

Or

$$V_{ave} = 0.636V_{L(pk)}$$

EXAMPLE: Determine the dc load voltage for the circuit shown in the fig.



SOLUTION:

Primary voltage is in rms so we can calculate the peak voltage

$$\begin{aligned} V_{1(pk)} &= V_{rms} / 0.707 \\ &= 75 / 0.707 \\ &= 106.08 V_{pk} \end{aligned}$$

Primary voltage and turn ratio is known so we can determine the secondary voltage

$$\begin{aligned} V_{2(pk)} &= (N_2/N_1) V_{1pk} \\ &= 1/4 (106.08) \\ &= 26.52 V_{pk} \end{aligned}$$

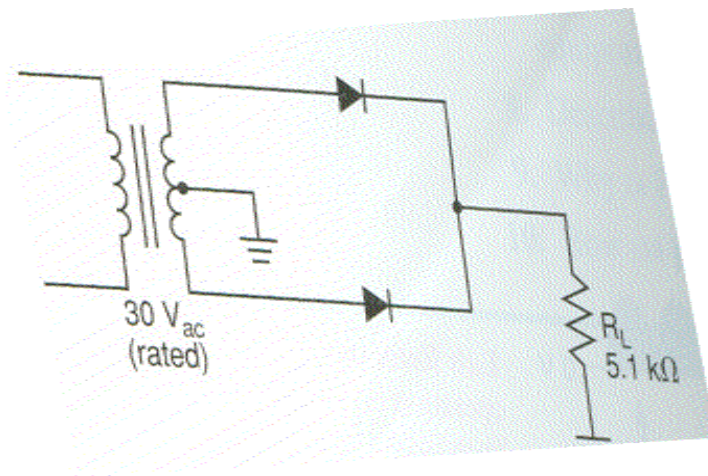
Now load voltage can be calculated as

$$\begin{aligned} V_{L(pk)} &= V_2 / 2 - 0.7 \\ &= 26.52 / 2 - 0.7 \\ &= 12.56 V_{pk} \end{aligned}$$

Now dc value of the voltage can be calculated as

$$\begin{aligned} V_{ave} &= 2V_{L(pk)} / \pi \\ &= 0.636 V_{L(pk)} \\ &= 0.636 (12.56) \\ &= 7.98 V_{dc} \end{aligned}$$

EXAMPLE: Determine the values of $V_{L(pk)}$ and V_{ave} for the circuit shown in the fig.

**SOLUTION:**

The transformer is rated at $30 V_{ac}$. Therefore, the value of $V_{2(pk)}$ is found as

$$\begin{aligned} V_{2(pk)} &= 30 V_{ac} / 0.707 \\ &= 42.2 V_{pk} \end{aligned}$$

The peak load voltage can now found as

$$\begin{aligned} V_{L(pk)} &= V_{2(pk)} / 2 - 0.7 \\ &= 21.2 - 0.7 \\ &= 20.5 V_{pk} \end{aligned}$$

Finally the dc load voltage is found as

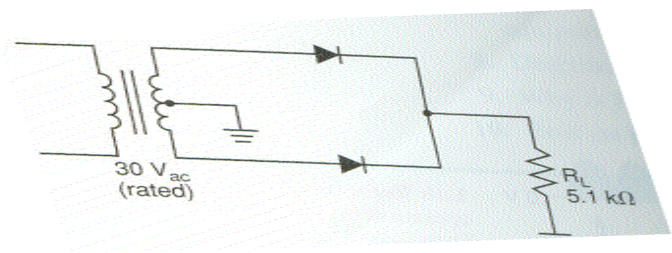
$$\begin{aligned} V_{ave} &= 2V_{L(pk)} / \pi \\ &= 41 V_{pk} / \pi \\ V_{ave} &= 13.05 V_{dc} \end{aligned}$$

LECTURE 38
(Negative full wave rectifier - Full wave bridge rectifier)

CALCULATING CURRENTS:

- Once the peak and average load voltage values are known, it is easy to determine the values of $I_{L(pk)}$ and I_{ave} with the help of Ohm's law.

EXAMPLE: Determine the values $I_{L(pk)}$ and I_{ave} for the circuit in last lecture.



SOLUTION:

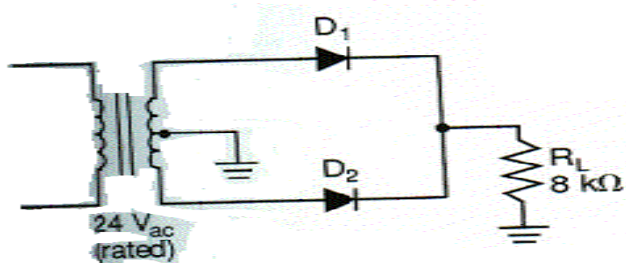
In the previous lecture, we calculated peak and average output voltages for the circuit. Using these value we can determine current as follows

$$\begin{aligned} I_{L(pk)} &= V_{L(pk)} / R_L \\ &= 20.5 / 5.1k \\ &= 4.02 \text{ mA}_{pk} \end{aligned}$$

And

$$\begin{aligned} I_{ave} &= V_{ave} / R_L \\ &= 13.05 / 5.1k \\ &= 2.56 \text{ mA}_{dc} \end{aligned}$$

EXAMPLE: Determine the values of $V_{L(pk)}$, V_{ave} , $I_{L(pk)}$ and I_{ave} for the circuit shown in the fig.



SOLUTION:

Rated value of the transformer is given so the peak value can be calculated as

$$\begin{aligned} V_{2(pk)} &= 24V_{ac} / 0.707 \\ V_{2(pk)} &= 33.94V_{pk} \end{aligned}$$

Now

$$\begin{aligned} V_{L(pk)} &= V_{2(pk)} - 0.7 \\ V_{L(pk)} &= 16.27V_{pk} \end{aligned}$$

And

$$\begin{aligned} V_{ave} &= 2V_{L(pk)} / \pi \\ &= 0.636 V_{L(pk)} \\ V_{ave} &= 10.34V_{dc} \end{aligned}$$

Now for currents

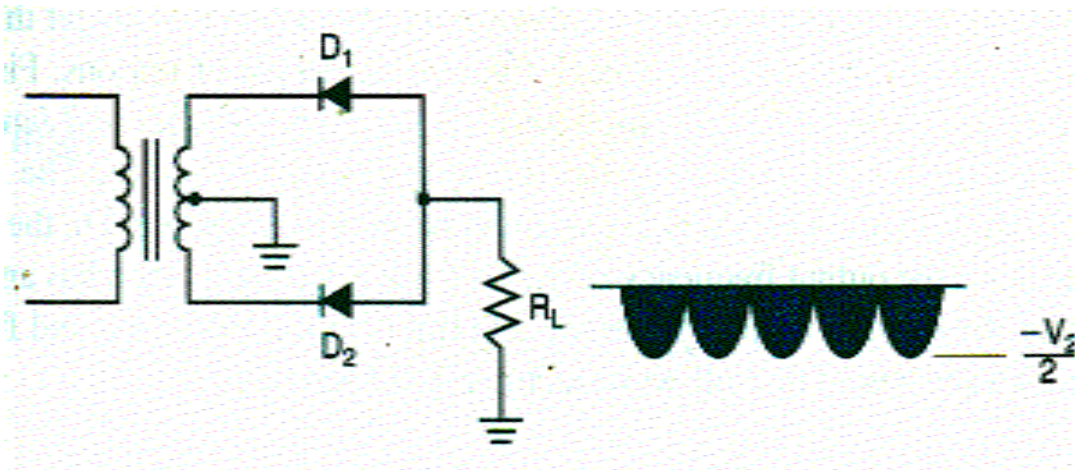
$$I_{L(pk)} = V_{L(pk)} / R_L$$

And

$$I_{L(pk)} = \frac{16.27}{8k} = 2.03mA_{pk}$$
$$I_{ave} = \frac{V_{ave}}{R_L} = \frac{10.34}{8k} = 1.29mA$$

NEGATIVE FULL WAVE RECTIFIER:

- The main differences between the positive and negative full wave rectifier are the direction that the diodes are pointing and the polarity of the output voltage.
- The analysis of the negative full wave rectifier circuit is same as negative half wave rectifier.
- If we reverse the direction of the diodes in the positive full wave rectifier, we will have a negative full wave rectifier as shown in the fig.



PEAK INVERSE VOLTAGE:

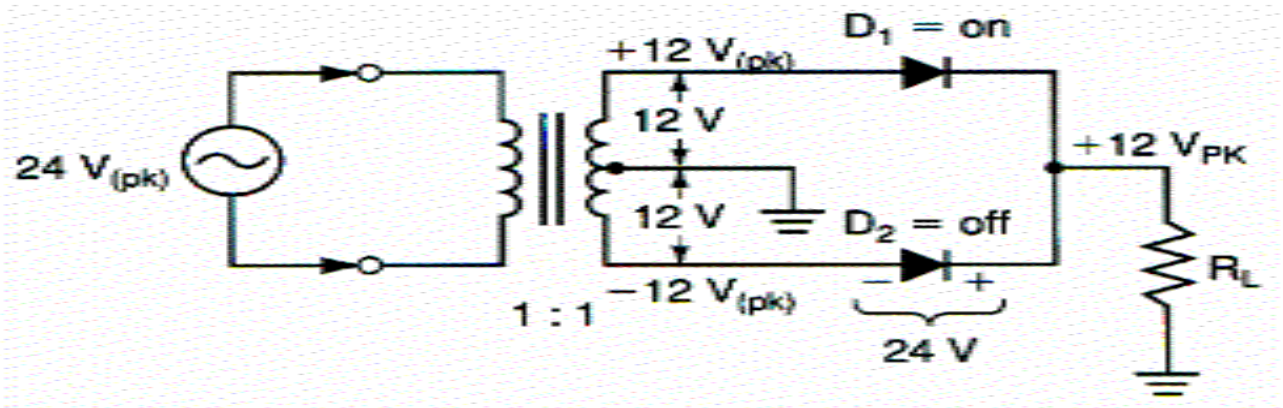
- When one of the diodes in a full wave rectifier is reverse biased, the voltage across it is approximately equal to V_2 .
- The peak load voltage supplied by the full wave rectifier is equal to the one half of the secondary voltage V_2 . Therefore, the reverse voltage will be twice the peak load voltage. By formula

$$PIV = 2V_{L(pk)}$$

Since the peak load voltage is half the secondary voltage, we can also find value of PIV as

$$PIV = V_{2(pk)}$$

- When calculating the PIV in practical diode circuit, the 0.7 drop will be also be taken into consideration. In the fig. when D_1 is on and will have voltage drop of 0.7



Since D₁ is in series with D₂ the PIV across D₂ will be reduced by the voltage across D₁. The equation

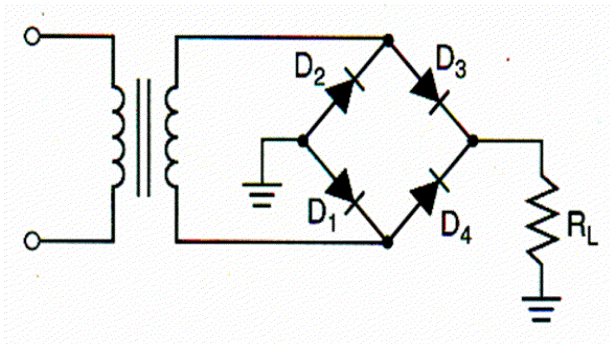
$$PIV = V_{2(pk)} - 0.7$$

Gives a more accurate PIV voltage.

Rectifier type:	Half-wave	Full-wave
Schematic diagram:		
Typical output waveform:		
Peak output voltage:	$V_{2(pk)} - 0.7 \text{ V}$	$\frac{V_{2(pk)}}{2} - 0.7 \text{ V}$
Dc output voltage:	$\frac{V_{L(pk)}}{\pi}$	$\frac{2V_{L(pk)}}{\pi}$
PIV:	Equal to $V_{2(pk)}$	$V_{2(pk)} - 0.7 \text{ V}$

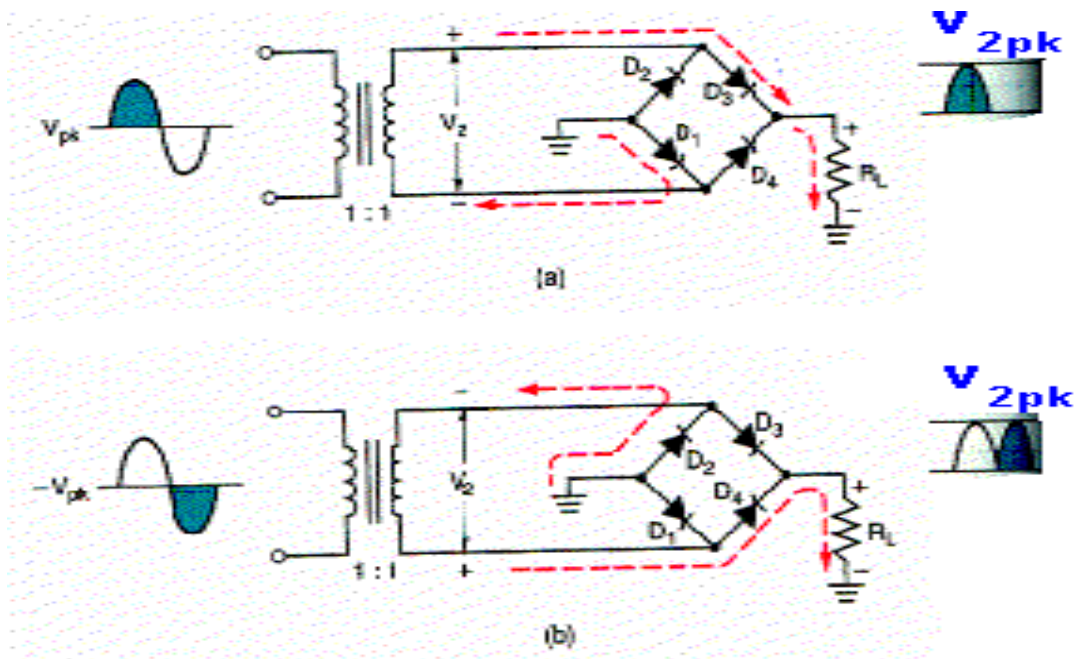
FULL WAVE BRIDGE RECTIFIER:

- The bridge rectifier is the most commonly used full wave rectifier circuit for several reasons
(1) It does not require the use of center-tapped transformer, and therefore can be coupled directly to the ac power line, if desired.
(2) Using a transformer with the same secondary voltage produces a peak output voltage that is nearly double the voltage of the full wave center-tapped rectifier. This results in the higher dc voltage from the supply.



BASIC CIRCUIT OPERATION:

The bridge rectifier shown in the fig. consists of four diodes and a resistor. The full wave rectifier produces its output by alternating the circuit conduction



between the two diodes. When one is on(conducting), the other is off(non conducting), and vice versa. The bridge rectifier works basically in the same way. The main difference is that the bridge rectifier alternates conduction between two diodes pairs.

- When \$D_1\$ and \$D_3\$ are on, \$D_2\$ and \$D_4\$ are off and vice versa. This circuit operation is illustrated in the fig.
- The current direction will not change in either condition.

CALCULATING THE LOAD VOLTAGE AND CURRENT VALUES:

- The full wave rectifier has an output voltage equal to the one half of the secondary voltage. The center-tapped transformer is essential for the full wave rectifier to work, but it cuts the output voltage to half.
- The bridge rectifier does not require the use of center-tapped transformer. Assuming the diodes to be ideal, the rectifier will have a peak output voltage of

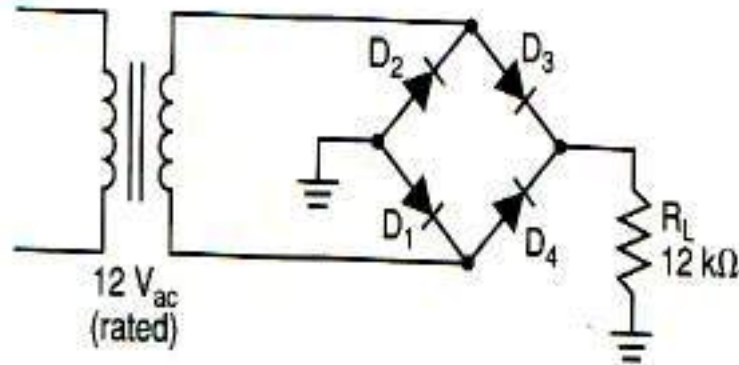
$$V_{L(pk)} = V_{2(pk)} \quad (\text{ideal})$$

- When calculating the circuit output values, we will get more accurate results if we take the voltage drops across the two conducting diodes into account. To include these values, we will use the following equation

$$V_{L(pk)} = V_{2(pk)} - 1.4$$

The 1.4 volts represents the sum of diode voltage drops.

EXAMPLE: Determine the dc load voltage and current values for the circuit shown in the fig.



SOLUTION:

With the $12V_{ac}$ rated transformer, the peak secondary voltage is found as

$$V_{2(pk)} = 12/0.707 \\ = 16.97V_{pk}$$

The peak load voltage is now found as

$$V_{L(pk)} = V_2 - 1.4 \\ V_{L(pk)} = 15.57V_{pk}$$

The dc load voltage is found as

$$V_{ave} = 2V_{L(pk)}/\pi \\ = 31.14/\pi \\ V_{ave} = 9.91 V_{dc}$$

Finally the dc load current is found as

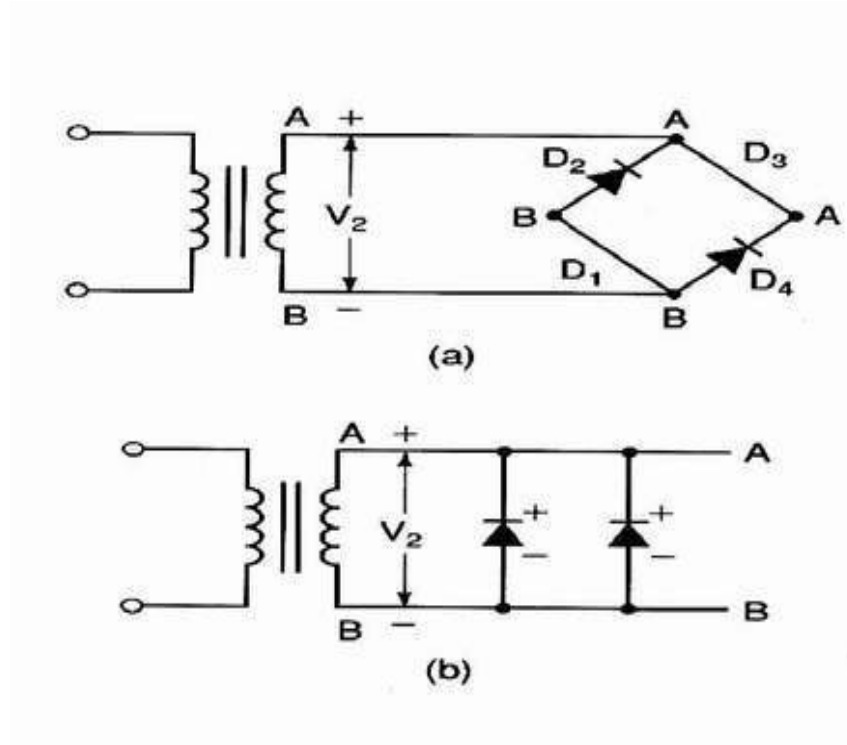
$$I_{ave} = V_{ave}/R_L \\ = 9.91/12k \\ I_{ave} = 825.8\mu A$$

LECTURE 39

(Filters - Clippers - Series clippers)

PEAK INVERSE VOLTAGE:

Using the ideal diode model, the PIV of each diode in the bridge rectifier is equal to V_2 . This is the same voltage that was applied to the diode in the full-wave center-tapped rectifier. Following fig helps us to illustrate this point. In fig a below, two things have been done :



- (1) The conducting diodes (D_1 and D_3) have been replaced by straight wires. Assuming that the diodes are ideal they will have the same resistance as wire; therefore replacement is valid.
- (2) The positive side of the secondary has been labeled **A** and the negative side has been labeled **B**.

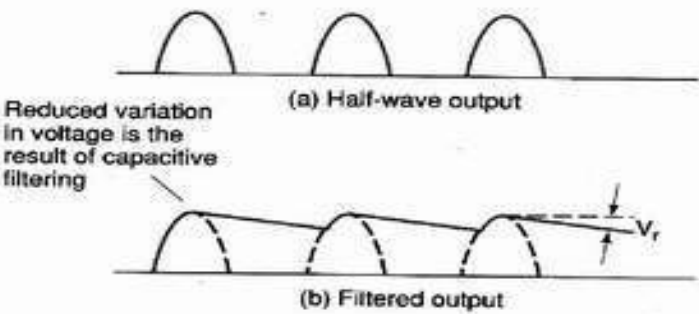
Connecting the common **A** point along a straight line and doing the same with **B** points gives us the circuit shown in the **fig b**. With the equivalent circuit, you can see that two reverse-biased diodes and the secondary of the transformer are all in parallel. Since parallel voltages are equal, the **PIV** across each diode is equal to V_2 . The same situation will exist for **D1** and **D3** when they are reverse-biased.

SUMMARY ILLUSTRATION:

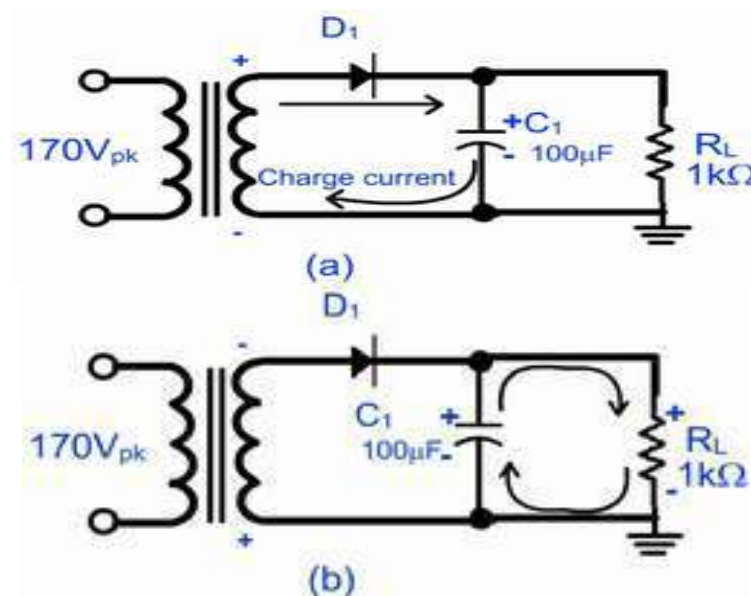
Rectifier type:	Half-wave	Full-wave	Bridge
Schematic diagram:			
Typical output waveform:			
Peak load voltage:	$V_{2(pk)} - 0.7\text{ V}$	$\frac{V_{2(pk)}}{2} - 0.7\text{ V}$	$V_{2(pk)} - 1.4\text{ V}$
dc load voltage:	$\frac{V_{L(pk)}}{\pi}$	$\frac{2V_{L(pk)}}{\pi}$	$\frac{2V_{L(pk)}}{\pi}$
dc load current:	$\frac{V_{avg}}{R_L}$	$\frac{V_{avg}}{R_L}$	$\frac{V_{avg}}{R_L}$
PIV:	Equal to $V_{2(pk)}$	$V_{2(pk)} - 0.7\text{ V}$	$V_{2(pk)} - 0.7\text{ V}$

FILTERS:

- Filters are used in power supplies to reduce the variations in the rectifier output signal. Since our goal is to produce a constant dc output voltage, it is necessary to remove as much of the rectifier output variations as possible.
- The overall result of using a filter is illustrated



The capacitor filter is the most basic filter type and the most commonly used. This filter is simply a capacitor connected in parallel with the load resistance, as shown in the fig a below. The filtering action is



based on the charge /discharge action of the capacitor .During the positive half-cycle of the input, D_1 will conduct and the capacitor will charge rapidly as shown in the **fig (a)** above. As the input starts to go negative D_1 will turn off, and the capacitor will slowly discharge through the load resistance in **fig (b)**. As the input from the rectifier drops below the charged voltage of the capacitor , the capacitor acts as the voltage source for the load. It is the difference between the charge and discharge times of the capacitor that reduces the variation in the rectifier output voltage. The difference between charge and discharge times of the capacitor is caused by two distinct RC time constant in the circuit.

CLIPPERS:

- There are variety of diode networks called clippers that have the ability to clip off a portion of the input signal without distorting the remaining part of the alternating waveform. The half wave rectifier studied earlier is a simplest form of diode clipper.
- Depending on the orientation of the diode, the positive or negative region of the input signal is "clipped" off.

IMPORTANT POINTS FOR CLIPPERS:

- (1) Make a sketch in your mind about the response of the network.
- (2) Determine the applied voltage (Transition Voltage) that causes change in the diode bias.
- (3) Be continuously aware of the defined terminal and polarity of V_o .
- (4) Sketch the input signal on the top and the output at the bottom to determine the output at instantaneous points of the input.

TYPES OF CLIPPERS:

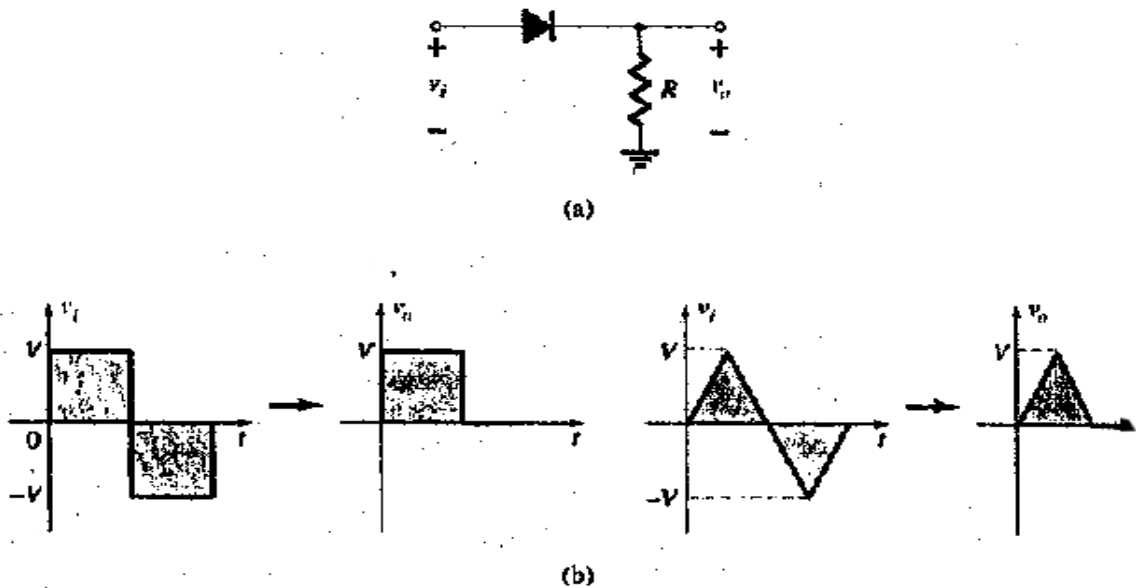
- There are two general categories of the clippers.

- (1) **Series**
- (2) **Parallel**

SERIES CLIPPERS:

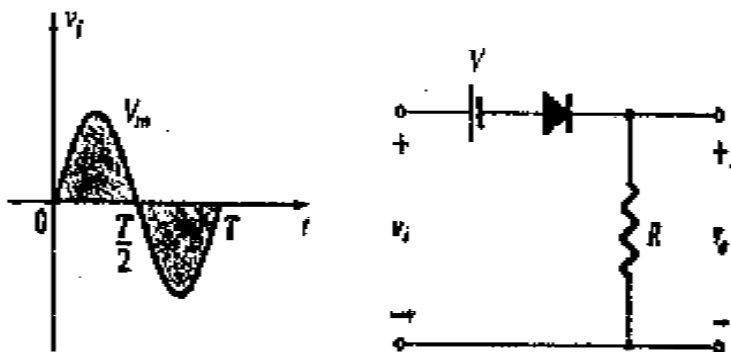
- The series configuration is defined as one where the diode is in series with the load as half wave rectifier.

EXAMPLES:

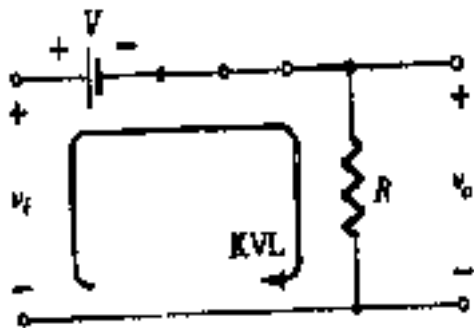


ADDITION OF A BATTERY IN THE SERIES CLIPPER CIRCUIT:

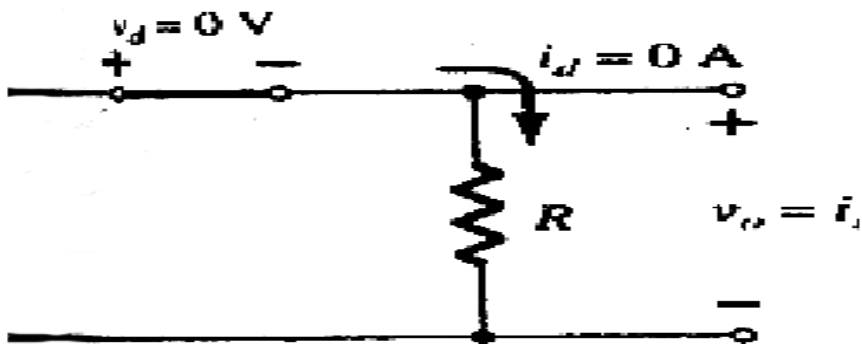
- In the fig. the direction of the diode suggests that the signal V_i must be positive to turn it on. The dc supply further requires that the voltage V_i be



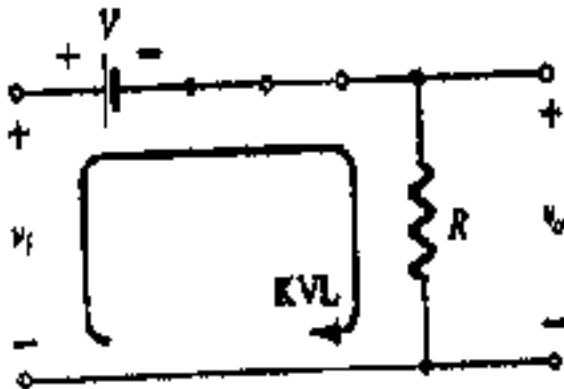
greater than V volts to turn the diode on. The negative region of the input signal is pressing the diode into off region.



- Now we determine the applied voltage that will cause a change in state for the diode. The ideal diode transition occur at the point on the characteristic where $V_d = 0$ and $I_d = 0$

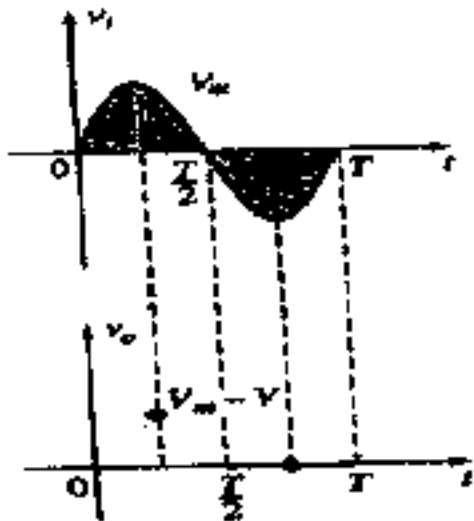


- In this case transition will occur at $V_i = V$



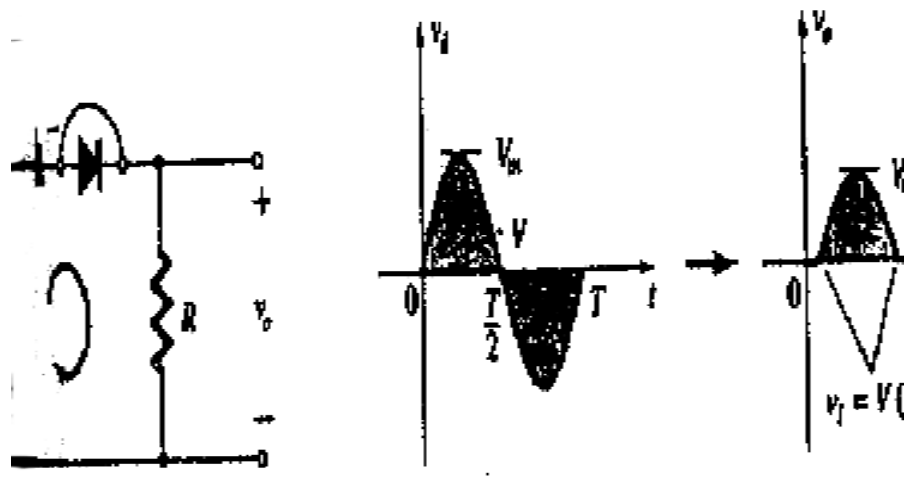
When the diode is short circuited as shown in the fig. The out put voltage V_0 can be calculated by applying KVL

$$V_i - V - V_0 = 0$$
$$V_0 = V_i - V$$

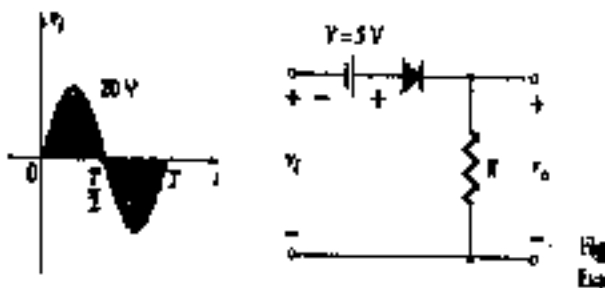


In this case

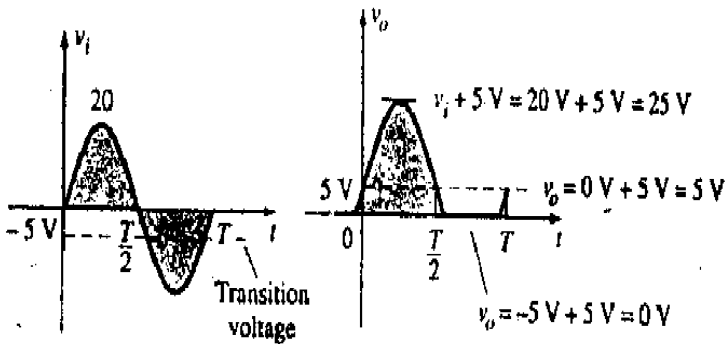
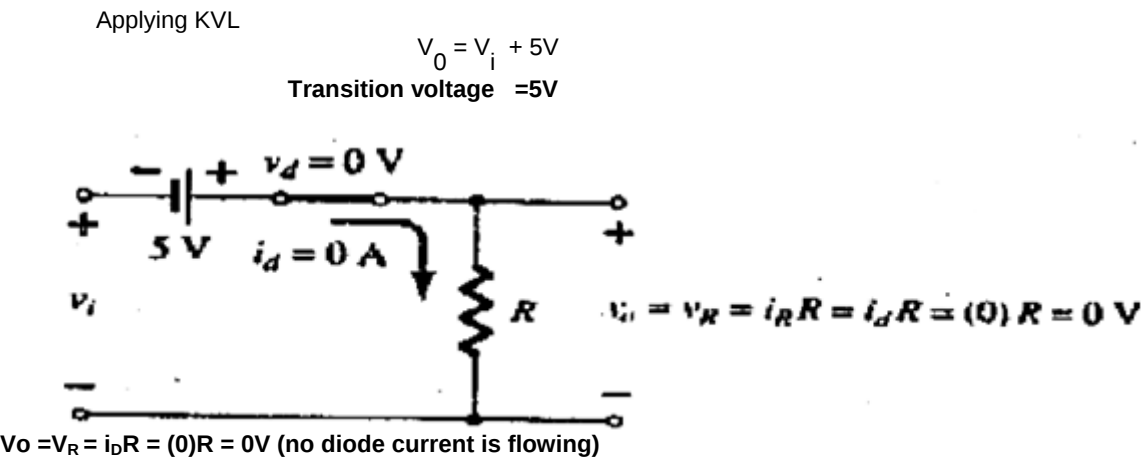
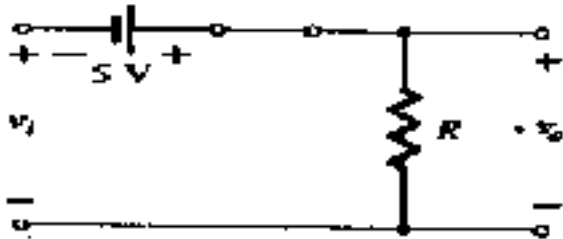
$$V_0 = V_m - V$$



EXAMPLE: Determine the output waveform for the network shown

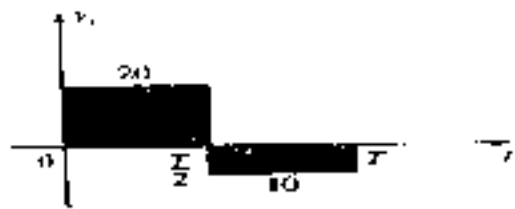


SOLUTION: The equivalent circuit will be



EXAMPLE: Determine the output for the square wave input shown in the fig.

Error!

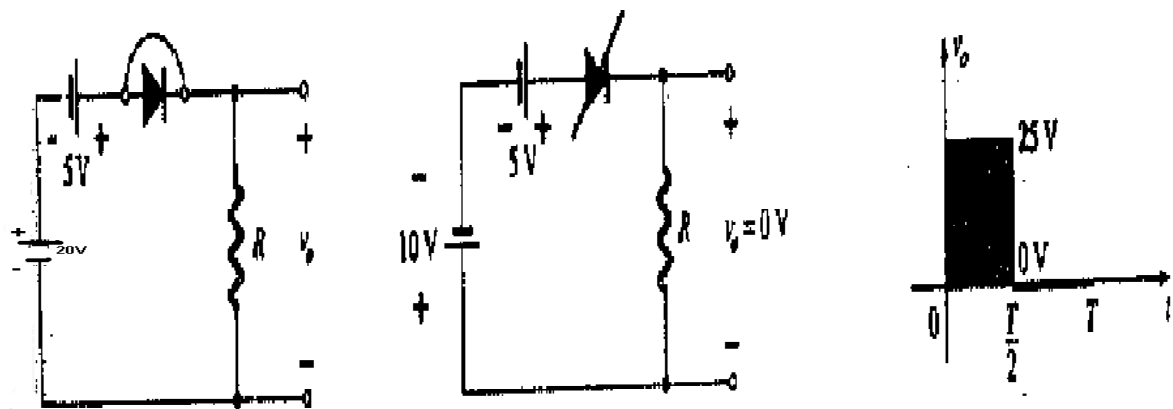


SOLUTION:

In positive half cycle. The diode is in the short circuit condition and by KVL

$$V_o = 20 + 5 = 25V$$

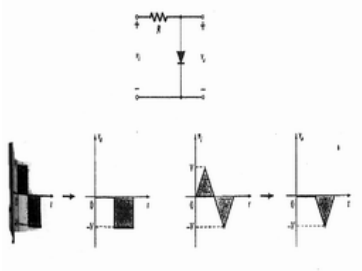
For the negative half cycle $V_i = -10V$ the result in placing the diode in the reverse condition.



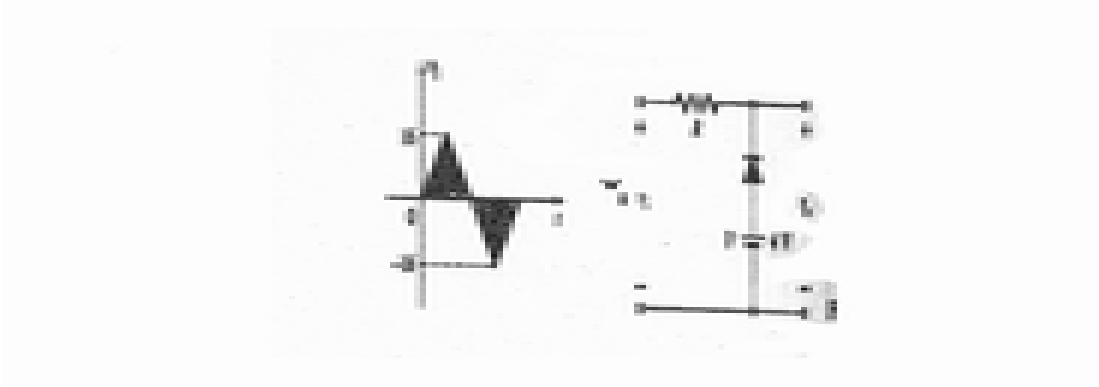
LECTURE 40
(Parallel clippers - Clamper)

PRALLEL CLIPPER:

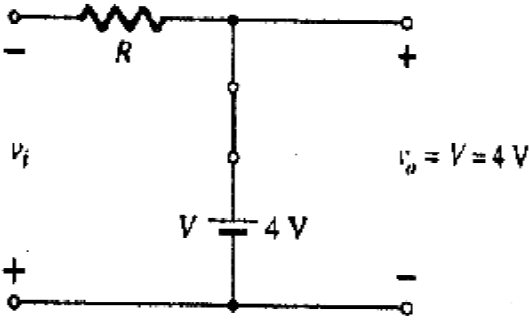
- The parallel configuration is defined as while the parallel variety has the diode in a branch parallel to the load.



EXAMPLE: Determine the output voltage for the network shown in the fig.



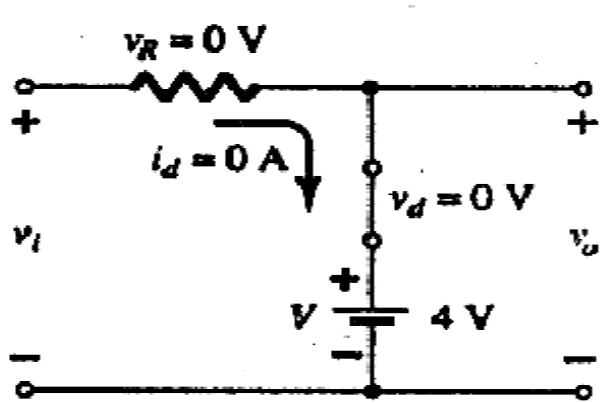
SOLUTION: Diode will be on for the $-ve$ half cycle .So the effective circuit is shown in the fig.



Apply $I_d = 0, V_d = 0$ to get transition voltage so that

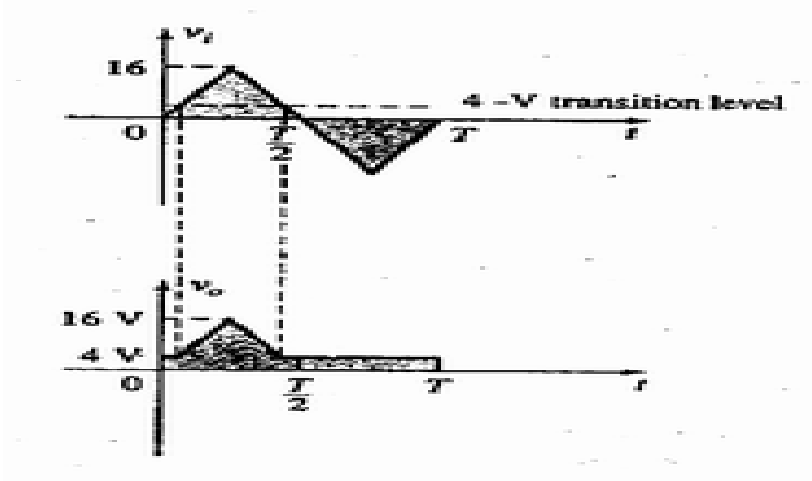
$$V_i - I_d R - 4V = 0$$
$$V_i - 0R - 4V = 0$$

$V_i = 4V$ is transition voltage.

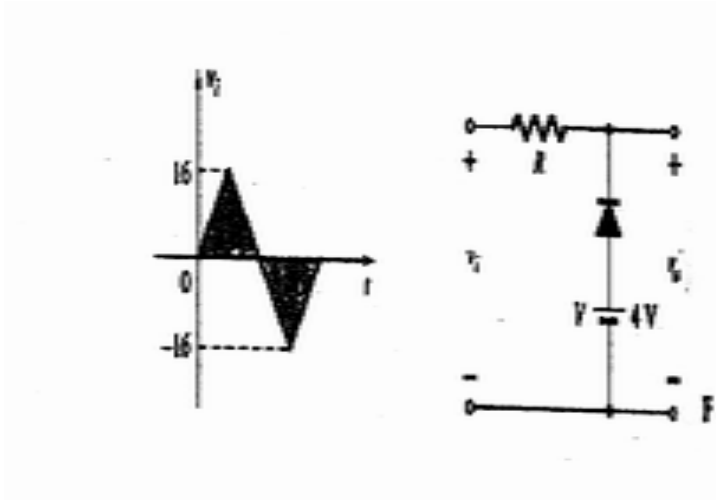


hence for the positive half cycle so that

$$V_L = V_0$$
$$V_L = 16\text{ Volts.}$$



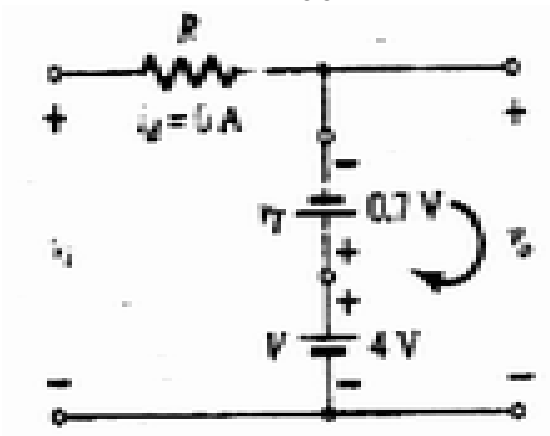
EXAMPLE: Consider in previous example the diode is silicon diode and draw its output.



SOLUTION:

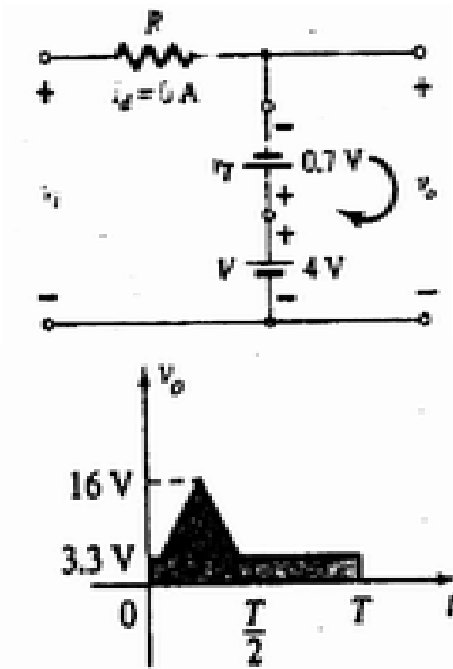
The diode will be on for the negative half cycle. Apply $I_d=0$, $V_d = 0.7$ to get transition voltage

$$\begin{aligned} V_i + V_T - V &= 0 \\ V_i &= V - V_T \\ &= 4 - 0.7 \\ &= 3.3 \text{ V} \end{aligned}$$



3.3 is the transition voltage will be 4V, hence for positive half cycle

$$\begin{aligned} V_L &= V_0 \\ V_0 &= 16\text{V} \end{aligned}$$

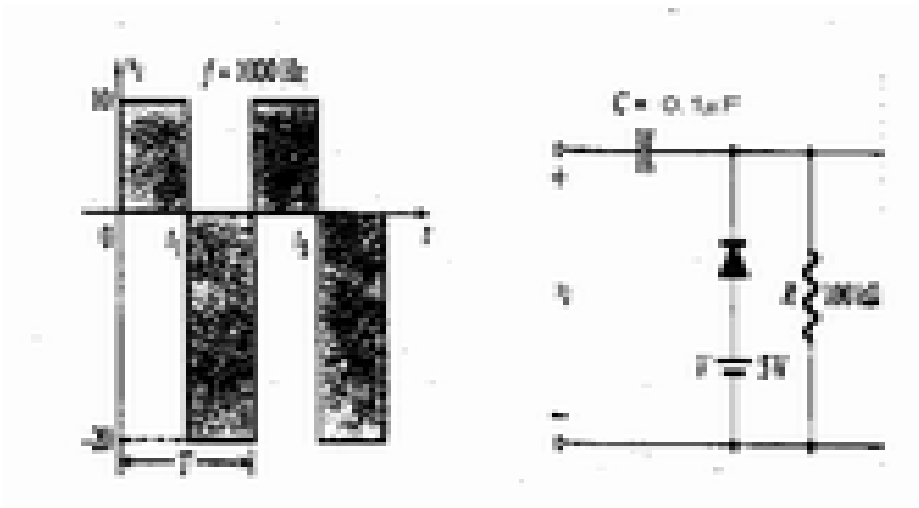


CLAMPERS:

These are the circuits which clamp the input signal to a different level depending upon the configuration of the clamper circuit. For carrying out analysis following points to be remember

- (1) The total swing of the output is equal to the total swing of the input signal.
- (2) Start the analysis by considering that part of the input which will forward bias the diode.
- (3) During the period, the diode is 'ON', assume that the capacitor will charge up instantaneously to a level determined by the network.
- (4) During the period for which the diode is 'OFF' assume that capacitor will hold its charge.
- (5) Throughout the analysis keep complete awareness of the polarity for V_0 , So that the proper value of V_0 is determined.

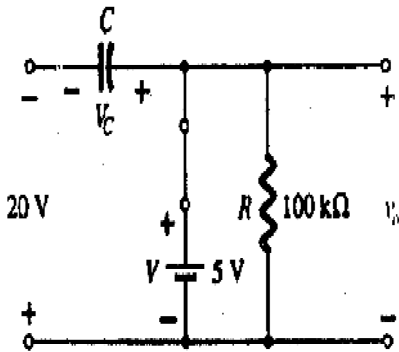
EXAMPLE: Determine v_o for the network shown in the fig.



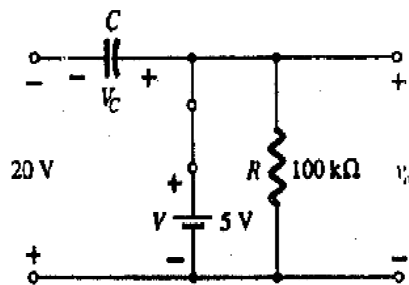
SOLUTION:

Our analysis will begin at time t_1 to t_2 .our circuit will behave as shown in the fig. Applying KVL to input loop

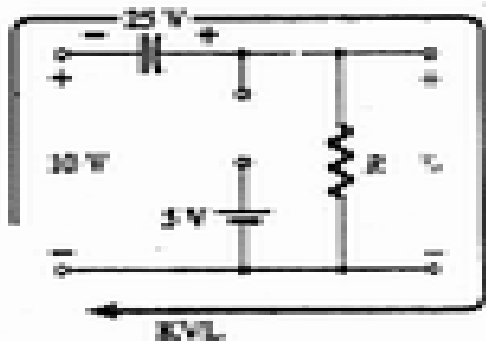
$$\begin{aligned} -20 + V_c - 5 &= 0 \\ V_c &= 25V \end{aligned}$$



The capacitor will therefore charge up to 25V. In this case the resistor will not be shorted by the diode, but a Thevenin's equivalent circuit for that portion of the network which includes the battery and resistor will result in $R_{th} = 0$, with $E_{th} = V = 5V$.



For the period t_2 to t_3 the circuit is shown in the fig.



KVL across the outer loop
 $10 + 25 - V_o = 0$

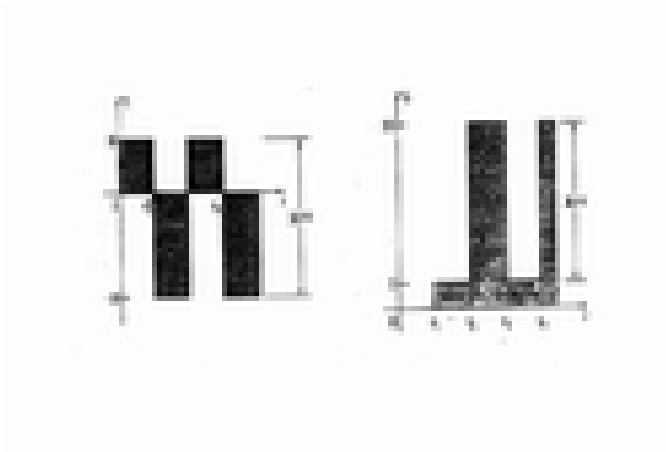
And

$$V_o = 35V$$

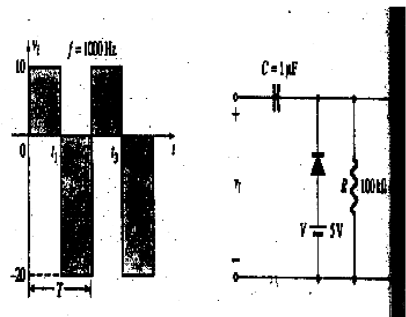
The time constant of the discharging network of fig. is determined by the product of RC and has magnitude

$$T = RC = 0.01S$$

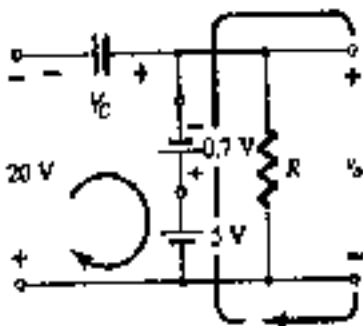
The output wave form will get the shape as



EXAMPLE: Repeat the previous example using silicon diode with $V_T = 0.7V$



SOLUTION:
For the short circuit state the network now takes the appearance of the fig. and V_0 can be determined by KVL in the output section

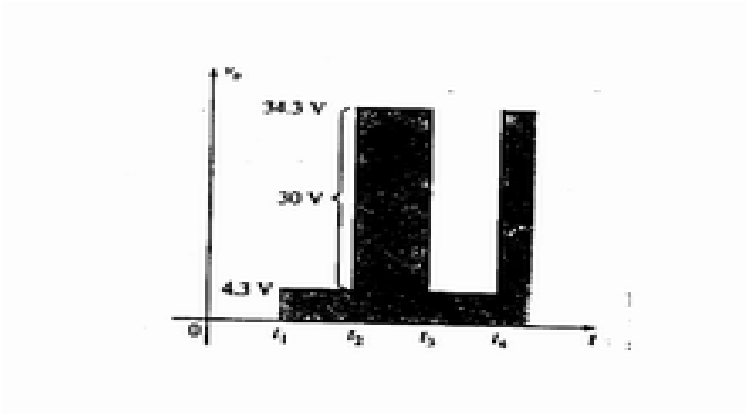


$$+5 - 0.7 - V_0 = 0$$
$$V_0 = 4.3V$$

For the input section KVL will result in

$$-20 + V_C + 0.7 - 5 = 0$$
$$V_C = 25 - 0.7$$
$$= 24.3V$$

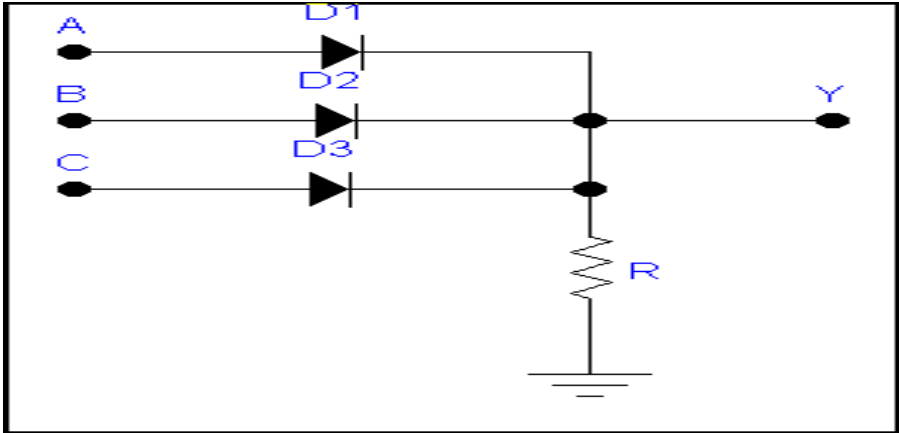
For the period t_2 to t_3 the network will now appear as in fig. By KVL

$$10 + 24.3 - V_0 = 0$$
$$V_0 = 34.3V$$


LECTURE 41

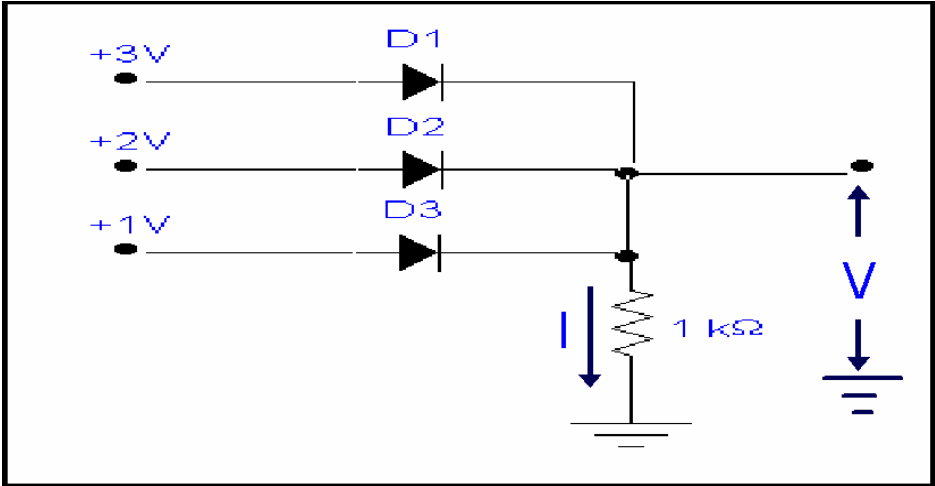
(Diode)

DIODE LOGIC GATES (OR GATE):



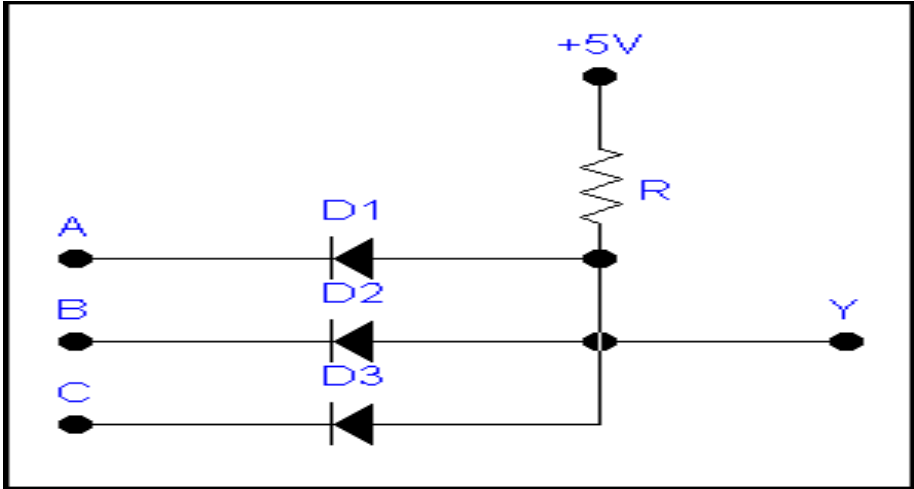
$Y=A+B+C$

EXAMPLE: Find V and I.

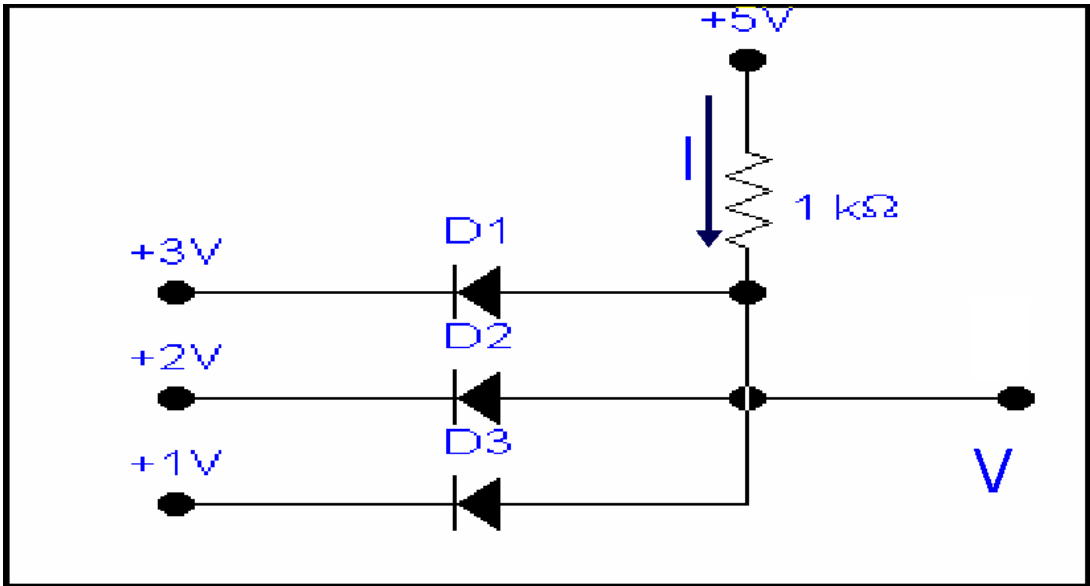


SOLUTION:
D1 is ON because it is in forward biased. As 3V potential difference is applied across diode D1 and it will go in more in conduction than D2 and D3.
D2 and D3 are off.
Therefore,
 $V= 3V$
Also by Ohm's Law
 $I = (3-0)/1k$
 $I = 3mA$

DIODE LOGIC GATES (AND GATE):



EXAMPLE: Find V and I.



SOLUTION:
D3 is ON.
In diode D3 case:
The applied voltage at the cathode terminal of the diode D3 is +1V and applied voltage at the anode terminal of the diode D3 is +5V so the potential difference is 4V. Its mean the anode terminal of the diode D3 is more positive as compared the cathode terminal of the diode the D3 so it will be in forward biased condition i.e D3 is ON.

D1 and D2 are off.
In diode D2 and D1 case:
The potential difference across the diode D2 is 3 V. Its mean the anode terminal of the diode D2 is more positive as compared the cathode terminal of the diode D2 however potential difference is less than as in case of diode D3 , so diode D2 is in forward biased condition but it is not as much forward –biased as in the case of the diode D3 because we are considering the case of ideal diodes we consider that the when diode D3 is fully forward biased and it will pass more current than diode D2 and D1.

Due to the forward biasing of diode D3 this potential difference will supply on output and this potential difference operate the diode D2 and D1 in reverse –biased.
D1 and D2 are off.
D3 is ON

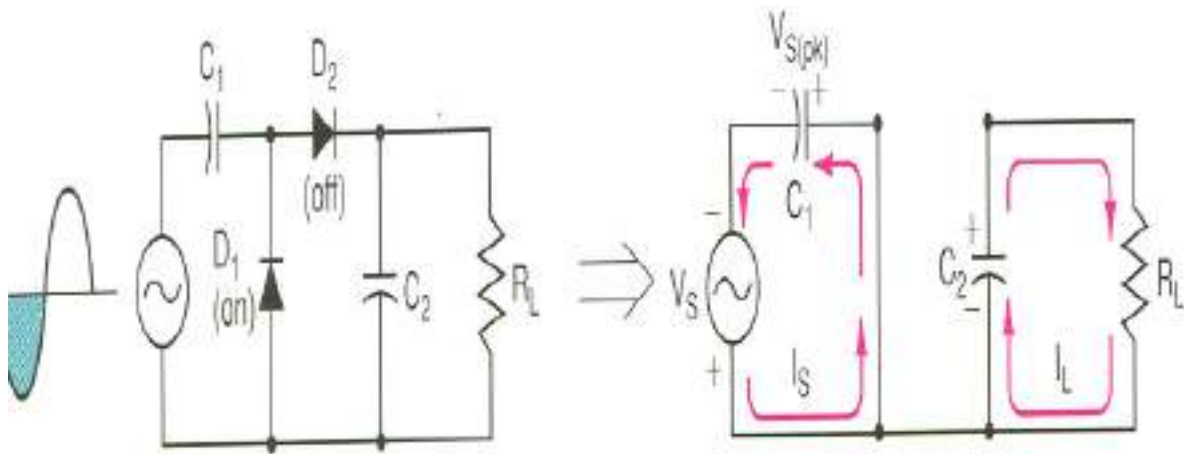
Therefore $V = +1V$
 Also by Ohm's Law
 $I = (5-1)/1k$
 $I = 4mA$

VOLTAGE MULTIPLIERS:

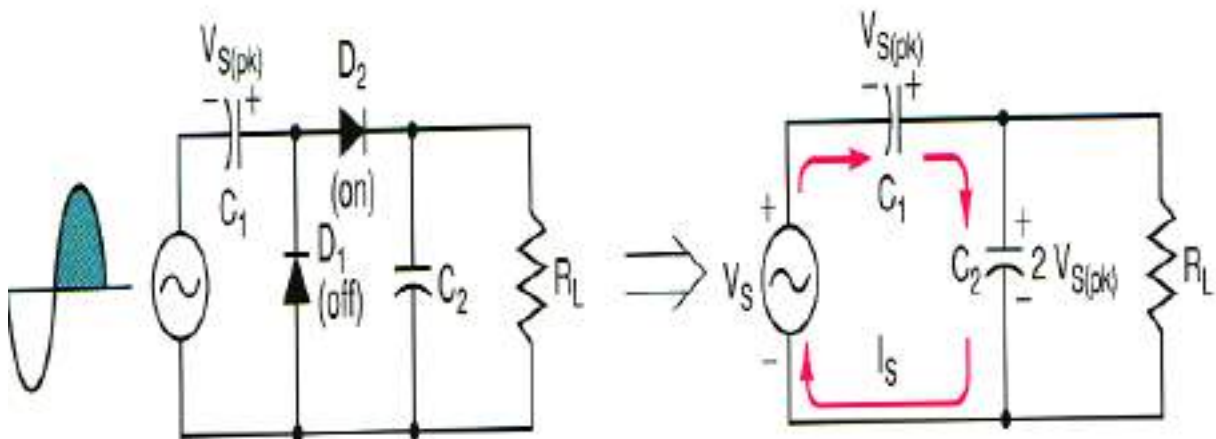
- Voltage multipliers are circuits that provide a dc output which is multiple of the peak input voltage. For example, a voltage doubler provides a dc output voltage that is twice the peak input voltage, and so on.
- While voltage multipliers provide an output voltage that is much greater than the peak input voltage, they are not power generators. When a voltage multiplier increases the peak input voltage by a given factor, the current is decreased by approximately the same factor.
- Because of this, voltage multipliers are usually used in low current high voltage applications. (As in cathode ray tube in a television).

HALF-WAVE VOLTAGE DOUBLERS:

The operation of the half-wave doubler is easier to understand if we assume that the diodes are ideal components. During the negative alternation of the input as show in the fig ,D1 is forward – biased and D2 is reverse-biased by the input signal polarity. If we represent D1 as a short circuit and D2 as open , we get the equivalent shown in the fig. As you can see the,C1 will charge until its plat-to-plate voltage is equal to the source voltage. At the same time , C2 will be in the process of discharging through the load resistance R_L .

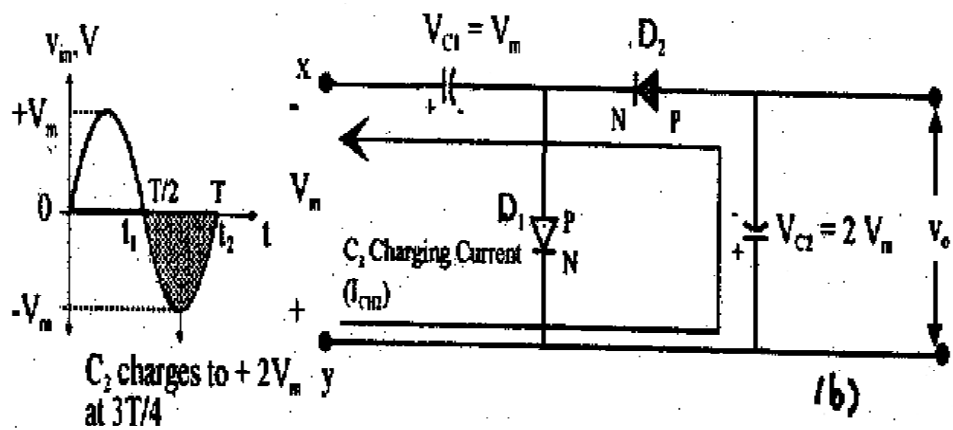


When the input polarity reverses, we have the circuit conditions show in the fig below. Since D1 is off ,it is represented as an open in the equivalent circuit. Also , D2 (which is on) is represented as a short. Using the equivalent circuit, it is easy to see that C1(which is charged to the peak value of V_s) and the source voltage now act as series –aiding voltage sources. Thus , C2 will charge to the sum of the series peak voltages. $2V_{S(pk)}$.



HALF WAVE VOLTAGE DOUBLER:

- During the first half cycle point x is positive with respect to point y of the input. A charging current will be forced through capacitor C_1 and diode D_1 which will be forward biased.
- Capacitor C_1 will charge to the peak for the time $T/4$. During the same time D_2 will remain off.



During the negative half cycle of the input point x will be negative with respect to y and voltage of C_1 (V_{C1} and V_{in}) are in series or additive.

A current will try to follow from y to x but D_1 does not allow as it is off. However D_2 will be conducting. This permits C_2 to charge up due to the current I_{ch2} will follows from point y through C_2 through D_2 and back to point x. Voltage on C_2 will be of magnitude $2V_m$.

Applying KVL we can write

$$-V_{C1} - V_{in} = V_o$$

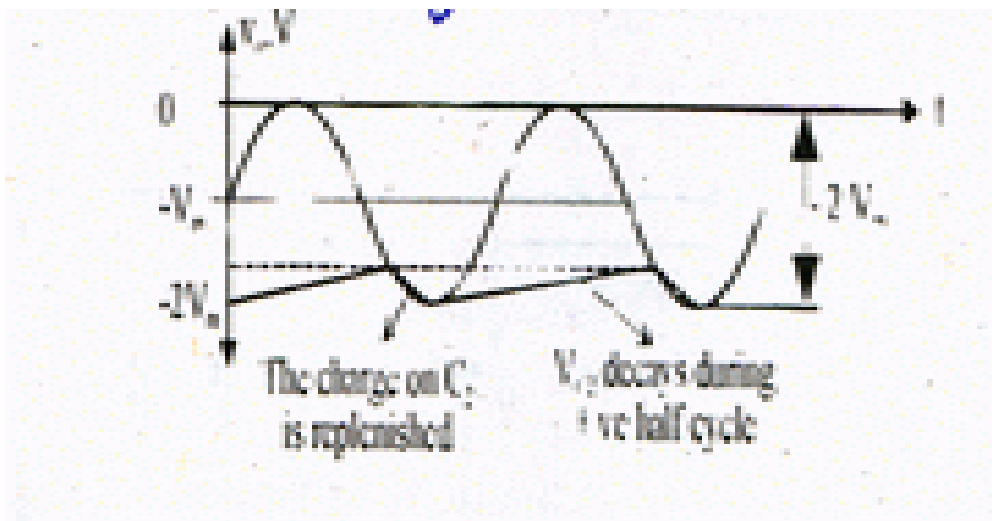
but

$$V_{C1} = V_{max}$$

Hence

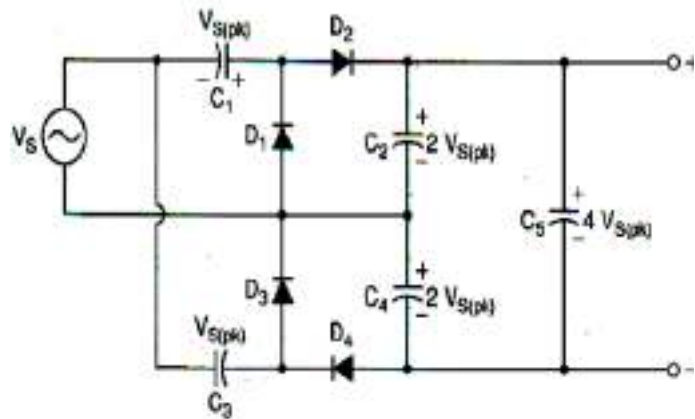
$$V_o = -2V$$

Output waveform will be like this

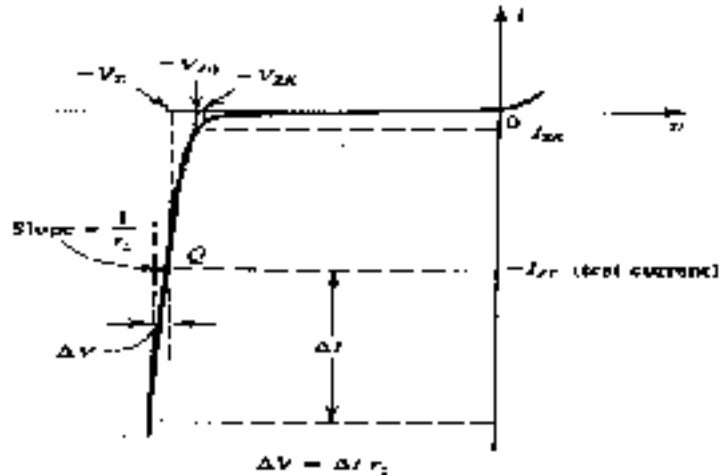


Dual Power Supply:

A dual –polarity power supply is one that provides both positive and negative dc output voltages. One such supply is show in fig below.



ZENER DIODE:

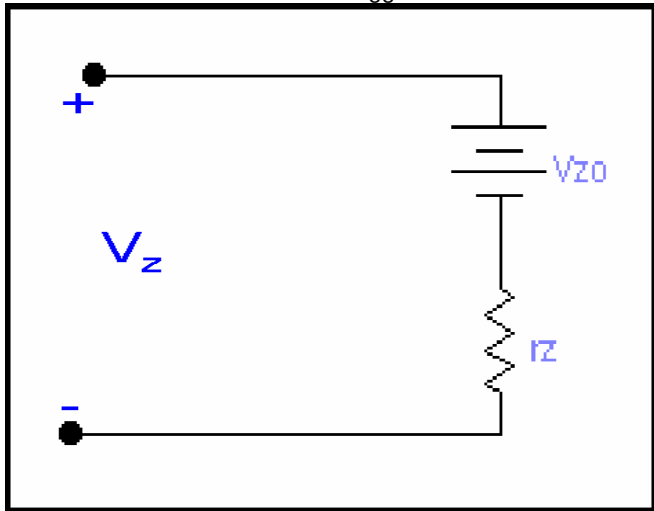


- In case of zener diode the differential voltage and current in the zener region of the characteristic curve is given by

$$\Delta V = r_z \Delta I$$

Where r_z = inverse of the slope or incremental resistance of zener diode at operating point.

- The almost linear i-v chracteristic of zener diode suggests that the device can be modeled as shown



- In practice, values of knee voltage and V_{z0} are considered same. From the model

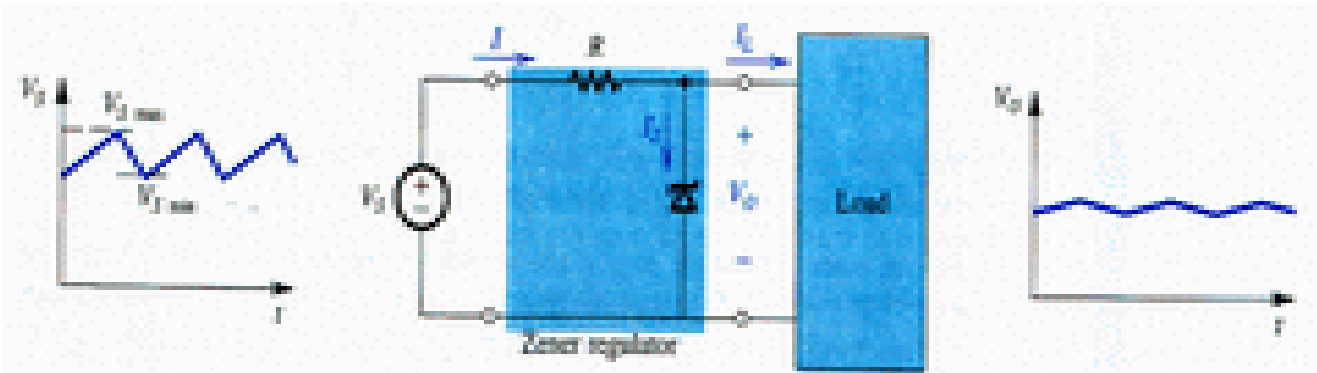
$$V_z = V_{z0} + r_z I_z$$

When $I_z > I_{zk}$

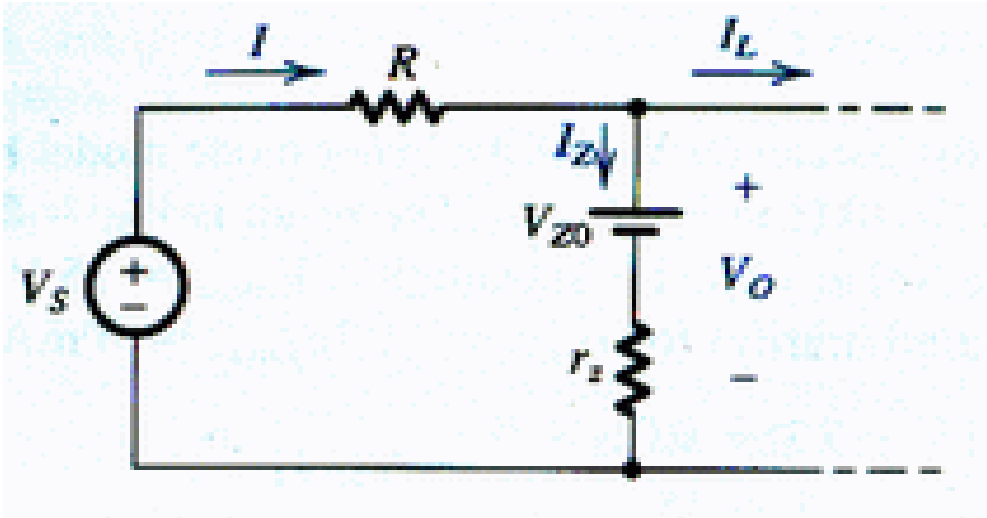
$V_Z > V_{ZO}$

ZENER DIODE SHUNT REGULATOR:

- A regulator must posses two properties
(1) No change in V_O .with any change in V_s .
(2) No change in V_O with any change in I_L . Therefore two parameters are related as



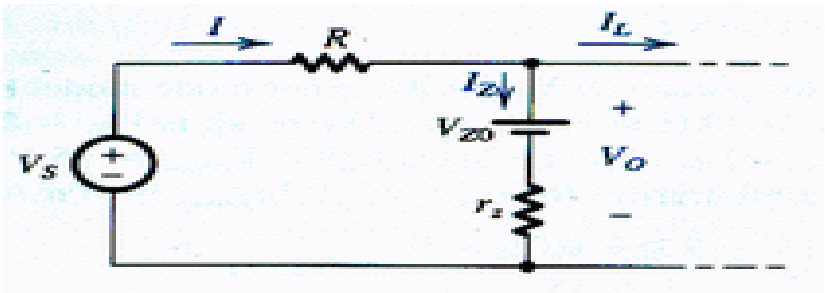
- (1) line regulation = $\Delta V_O / \Delta V_s$
(2) Load regulation = $\Delta V_O / \Delta I_L$
Another method to describe these regulations



LECTURE 42

(Zener diode-LED-PIN Photodiodes - Tunnel diode)

ZENER DIODE SHUNT REGULATOR:



From the circuit shown

$$\text{regulation} = r_z / (R + r_z)$$
$$\text{regulation} = - r_z \parallel R$$

Current in the network $= (V_s - V_{z0}) / (R + r_z)$
Voltage across r_z is

$$V_{rz} = V_{z0} + (V_s - V_{z0}) r_z / (R + r_z)$$

Therefore applying Thevenin's Theorem

$$V_o = V_{z0} + (V_s - V_{z0}) r_z / (R + r_z) \times R_L / (R_L + R \parallel r_z)$$

$$\text{Therefore } R \times (\text{Total current}) = V_{smin} - V_{z0} - r_z I_{zmin}$$

Or

$$R = (V_{smin} - V_{z0} - r_z I_{zmin}) / (I_{zmin} + I_{Lmax})$$

Example:

A zener diode whose nominal voltage is 10V at 10mA has an incremental resistance of 50Ω. What voltage Ahmad Tulaib must expect if Bilal disturbs the settings to half the diode current. What is the value of V_{z0} of the zener model.

Solution:

When the current is changed by -5mA, then voltage
 $V_z = 10 - r_z I_z = 10 - 5(0.05) = 9.75V$

When the current is doubled i.e. increased by 10mA, then
 $V_z = 10 + 10(0.05) = 10.5V$

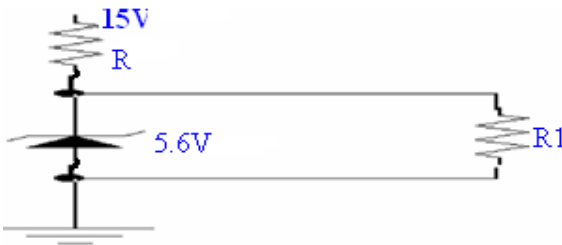
V_{z0} is the intercept of the tangent to characteristic at (10V,10mA) is the voltage change , when the current decreases by 10mA, hence
 $V_{z0} = 10 - 10(0.05) = 9.5V$

V_{z0} is the intercept of the tangent to characteristic at (10V,10mA) is the voltage change , when the current decreases by 10mA, hence
 $V_{z0} = 10 - 10(0.05) = 9.5V$

Example:

A zener diode exhibits a constant voltage of 5.6V for currents greater than five times the knee current I_{zk} is specified to be 1mA. It is to be used in the design of a shunt regulator fed from a 15V supply. The load current varies over the range 0 to 15mA. Find a suitable value for the resistance R. What is the maximum power dissipation to the zener diode.

Solution:



The minimum Zener current is

$$I_{zk}=5(1mA)=5mA$$

∴ Load current may be 15mA, so that the value of R, is such that with $I_L=15mA$, zener current of 5mA is available, therefore,

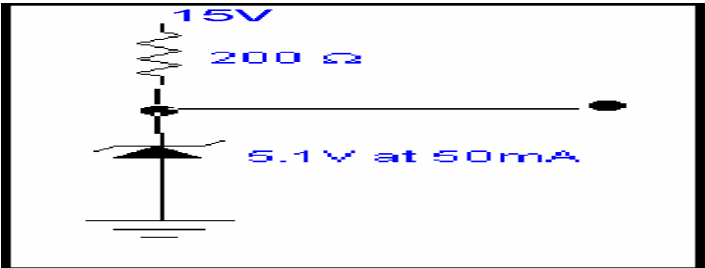
$$R= (15-5.6)/20mA$$
$$R = 470\Omega$$

The maximum power dissipated in the zener occurs when, $I_L=0$, in which case the zener current must be 20mA, therefore

$$P=20m(5.6)=112mW$$

Example: A shunt regulator utilizes a zener diode whose voltage is 5.1V at a current of 50mA and whose incremental resistance is 7Ω . The diode is fed from a supply of 15 V nominal voltages through a 200 Ω resistor. What is the output voltage at no load? Find the line regulation and load regulation.

Solution:



$$V_{zo}=5.1-7(50m)$$
$$=4.75V$$

At no load condition

$$V_o = (15-4.75)7/207+4.75$$
$$=5.1V$$
$$\text{Line regulation}=r_z/(R+r_z)$$
$$=7/207=33.8mV/V$$
$$\text{Load Regulation}= - (r_z||R)$$
$$= -(7||200)$$
$$= - 6.8\Omega$$
$$= -6.8mV/mA$$

Example: Ghania is trying to design a shunt regulator, of about 20V, for her sister Azka's toy. Two kinds of zener diodes are available: 6.8V devices with r_z of 10Ω and 5.1V devices with r_z of 30Ω . For the two choices possible, find load regulation. In this calculation neglect the effect of regulator resistance R.

Solution:

- Choices
- a). In order make up 20V using 6.8V zener, three diodes will be needed, therefore $3 \times 6.8=20.4V$, so is the resistance, $3 \times 10=30\Omega$. Therefore load regulation, neglecting load resistance = -30Ω .
- b). 4 diodes of 5.1 will make up 20.4 volts, therefore $4 \times 5.1=20.4V$, so is the resistance of the zener i.e.
- $$4 \times 30=120\Omega.$$
- Load regulation (Neglecting R) = -120Ω

Example: It is required to design a shunt regulator to provide a regulated voltage of about 10V. The available 10V, 1W zener of type 1N4740 is specified to have a 10V drop at a test current of 25mA. At this current its r_z is 7 Ω . The raw supply available has a nominal value of 20V but can vary as much as $\pm 25\%$. The regulator is required to supply a load current of 0 to 20mA. Design for a minimum zener current of 5mA.

- Find V_{zo} .
- Calculate the required value of R.
- Find the line regulation. What is the change in V_o expressed in %age, corresponding to the $\pm 25\%$ change in V_s .
- Find the load regulation. By what percentage does V_o change from no load to full load condition.
- What is maximum current that the zener in your design should be able to conduct? What is the zener power dissipation under this condition?

Solution:

The provided data is

$$V_z = 10V, I_z = 25mA, r_z = 7\Omega, V_s = 20V \pm 25\%, I_{zk} = 5mA, \Delta I_L = 20mA.$$

- $V_{zo} = V_z - I_z r_z = 10 - 25m(7) = 9.82V$
- Minimum current $I_{zk} = 5mA$ will occur when Load current I_L is maximum i.e. 20mA and V_s is at minimum, so that $V_s = 20 - 5 = 15V$.

Therefore

$$R \leq \frac{V_{smin} - V_{zo} - r_z I_{zmin}}{I_{Lmin} + I_{Lmax}}$$

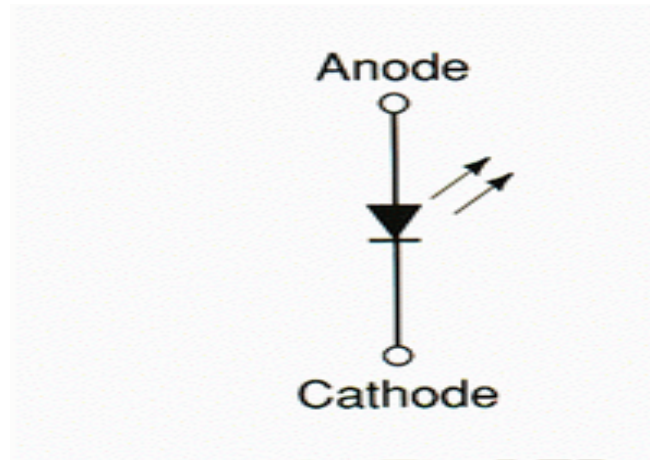
$$R \leq \frac{15 - 9.825 - (5 \times 10^{-3})7}{5m + 20m}$$

$$R < 205.6\Omega \approx 205\Omega.$$

- Line Regulation = $\Delta V_o / \Delta V_s = r_z / (R + r_z)$
 $7/212 = 33mV/V$.
 $\pm 25\%$ change in $V_s = \pm 5V$
 Therefore V_o changes by $\pm 5 \times 33m$
 $= \pm 165mV$
 Percentage = $\pm 0.165(100)/10 = \pm 1.65\%$.
- Load Regulation = $-r_z || R = -7 || 205$
 $= -6.77\Omega = -6.77V/A$
 $\Delta V_o = -6.67(20m) = -135.4mV$
 Which in % = $-135.4(100)/10 = -1.35\%$
- Maximum zener current occurs at no load and $V_s = 20(1 + 0.25) = 25V$
 Current = $(25 - 9.825)/(205 + 7) = 71.6mA$
 Zener Power Dissipation = $71.6m(10) = 716mW$
 Or more precisely
 $V_z = 9.825 + 71.6m(7) = 10.326V$
 $P_z = 71.6m(10.326) = 739.4mW$.

LIGHT EMITTING DIODES:

- LED's are diodes that will emit light when biased properly. The graphic symbol for an LED is shown in figure.



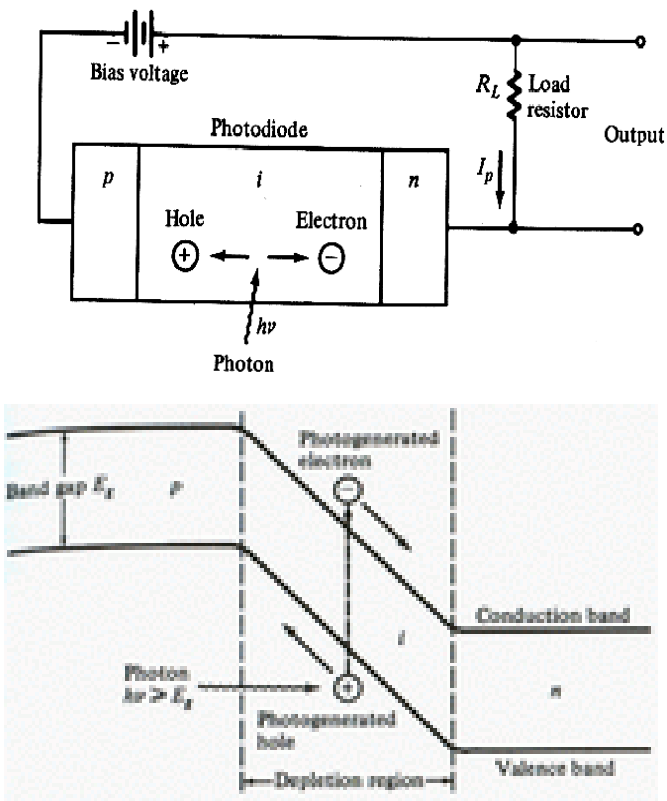
When sufficient voltage is applied to the chip across the leads of the LED, electrons can move easily in only one direction across the *junction* between the *p* and *n* regions. In the *p* region there are many more positive than negative charges. In the *n* region the electrons are more numerous than the positive electric charges. When a voltage is applied and the current starts to flow, electrons in the *n* region have sufficient energy to move across the junction into the *p* region. Once in the *p* region the electrons are immediately attracted to the positive charges due to the mutual Coulomb forces of attraction between opposite electric charges. When an electron moves sufficiently close to a positive charge in the *p* region, the two charges "re-combine". Each time an electron *recombines* with a positive charge; electric potential energy is converted into electromagnetic energy. For each recombination of a negative and a positive charge, a quantum of electromagnetic energy is emitted in the form of a photon of light with a frequency characteristic of the semi-conductor material (usually a combination of the chemical elements gallium, arsenic and phosphorus). Only photons in a very narrow frequency range can be emitted by any material. LED's that emit different colors are made of different semi-conductor materials, and require different energies to light them

- LED's are available in many of emitting colors as well as infrared (which is not visible), red, green, yellow, orange, and blue, the schematic symbol is the same for all colors. Since LED's have clear cases, there is normally no label on the case to identify the leads.
- The leads are normally in one of three ways
 - (1) The leads may have different lengths.
 - (2) One of the leads may be flattened. The flattened lead is usually the cathode.
 - (3) One side of the case may be flattened, the lead closest to the flattened side is usually the cathode.

LED CHARACTERISTICS:

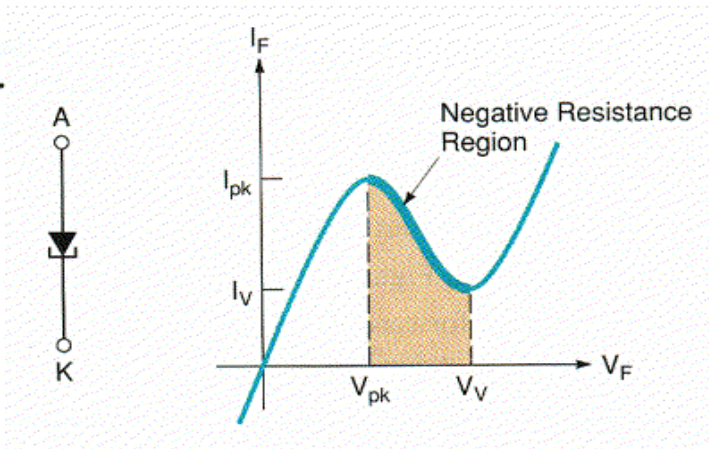
- LED's have characteristic curves that are very similar to those for pn junction diodes. However, they tend to have higher forward voltage (V_F) values and lower reverse break down voltage ratings. The typical ranges for these values are as follows
 - (1) Forward voltage: +1.2 to +4.3V
 - (2) Reverse break down voltage : -3 to -10V

PIN Photodiodes:



TUNNEL DIODE:

- Tunnel diode are used in ultra high frequency (UHF) and microwave frequency range. They have application in high frequency communication electronics .The schematic symbol and characteristic curve is shown in the fig.

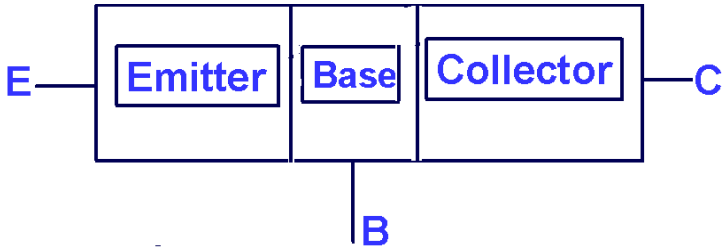


- The operating curve is a result of extremely heavy doping used in the manufacturing of the tunnel diode. In fact, the tunnel diode is doped approximately 1000 times as heavily as standard pn junction diodes.
- In the forward operating region of the tunnel diode, we are interested in the area between the peak voltage (V_{pk}) and the valley voltage (V_V).
- At $V_F = V_{pk}$ forward current is called peak current (I_{PK}). As V_F is increased to the value of V_V , I_F decreases to its minimum value called Valley current (I_V).
- The term used to describe a device whose current and voltage are inversely proportional is negative resistance.
- Thus, the region of operation between V_{pk} and V_V is called negative resistance region.

LECTURE 43

(Bipolar Junction Transistor (BJT))

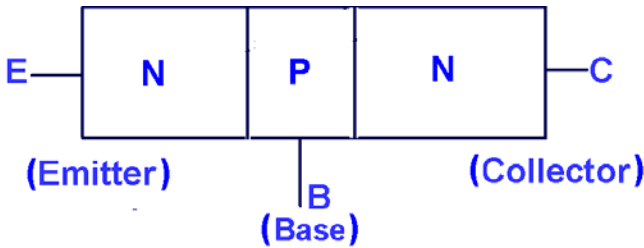
BIPOLAR JUNCTION TRANSISTOR (BJT) CONSTRUCTION:



It is three layer semiconductor device, consisting of either two n- and one p-type layers of material or two p- and one n-type layers. Since three layers are involved in this device, therefore it will have three terminals. These terminals are known as:-

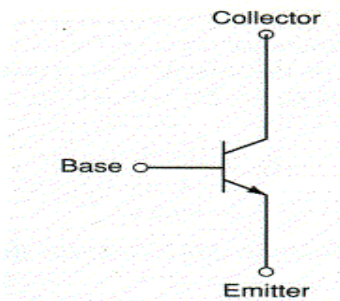
1. Emitter.
2. Base.
3. Collector.

- Emitter layer is heavily doped.
- Base layer is lightly doped.
- Collector is very lightly doped.
- The base layer is very thin as compared to other two, normally in the range of 150:1.
The designated terminology for three terminals is as follows:-
Emitter: This layer emits electrons.
Base: It helps minority carriers to transfer rapidly from emitter to collector. Electric field is created in base by non-uniform doping, so that minority carriers are accelerated from emitter to collector.
Collector: This layer collects the emitted electrons.
- Two basic types of BJTs
- These types are in equation to constructional arrangement of layers.
- The arrangement shown in the figure is called NPN Transistor.

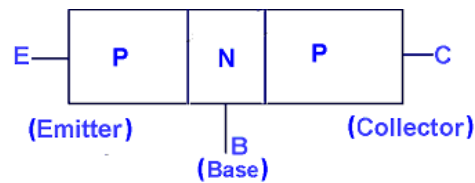


**BIPOLAR JUNCTION TRANSISTOR THE SYMBOL
FOR NPN TRANSISTOR:**

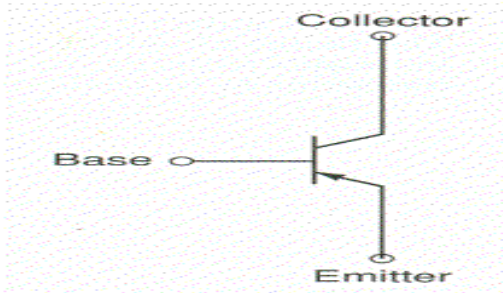
The arrow shows direction of flow of conventional current.



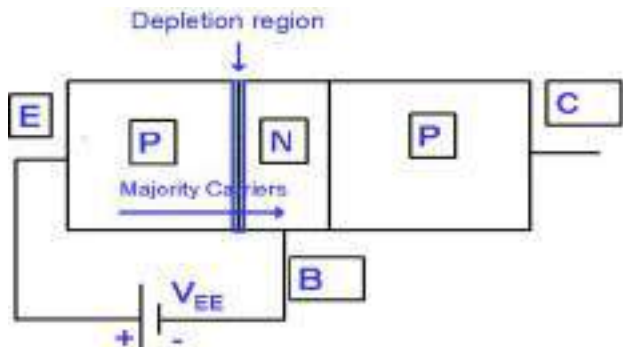
- The arrangement is known as **PNP** transistor.



BIPOLAR JUNCTION TRANSISTOR THE SYMBOL FOR PNP TRANSISTOR:

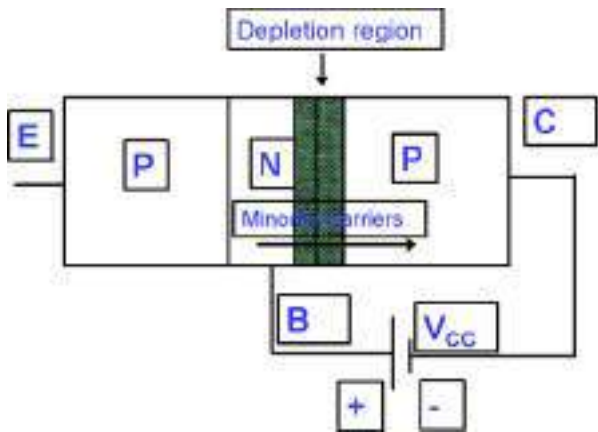


BIPOLAR JUNCTION TRANSISTOR OPERATION IN PNP TRANSISTOR EMITTER BASE JUNCTION:

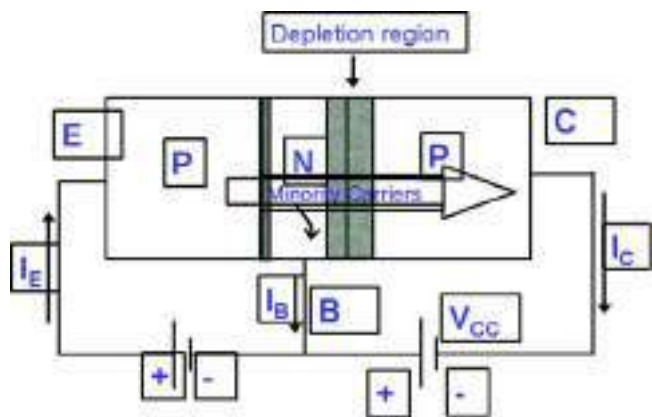


This fig shows the emitter base junction where emitter is made up of p-type material and Base is made up of N-type material .In the fig +ve terminal of the battery is connected to Emitter while –ve terminal is connected to the Base. Its mean this junction will go in forward-biased condition and depletion layer will shrink. That is majority carriers will start crossing the junction of these particular layers.

COLLECTOR BASE JUNCTION:



This fig shows the Collector-base junction where collector is made up of P-type material and Base is made up of N-type material .In the fig +ve terminal of the battery is connected to Base while –ve terminal is connected to the Collector. Its mean this junction will go in to reverse-biased mode and depletion layer will be widened and the current will flow due to minority carriers.

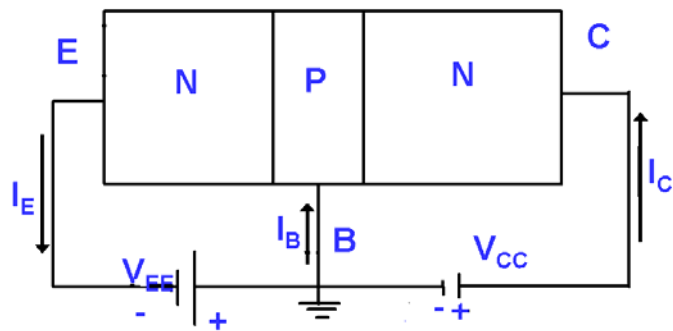


In the fig above +ve terminal of the battery is connected to Emitter while –ve terminal of V_{EE} is connected to the Base while on the right side of this fig +ve terminal of the battery V_{CC} is connected to Base while –ve terminal is connected to the the Collector. There are two depletion layers in the fig one is small while the other is bit wider and this is due to the forward biased of emitter-base junction and reverse biased due to the collector-base junction. We also see the emitter current I_E will flow inward i.e. pointing into the transistor, however the base current I_B and collector current I_C is shown going outward from the transistor.

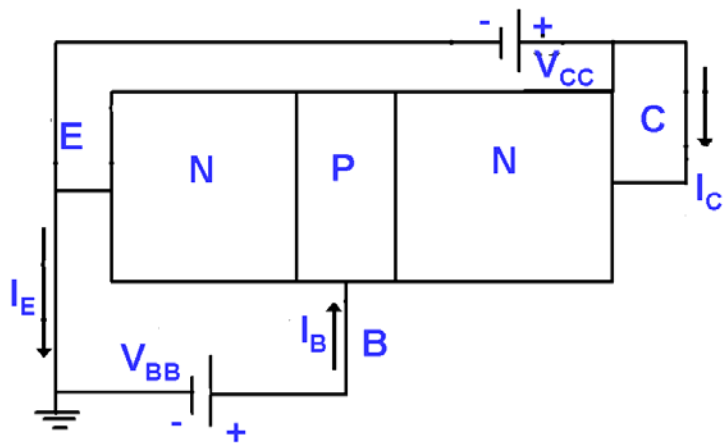
OPERATING CONFIGURATIONS OF BJT:

- There are three basic configurations under which transistor may be operated; these configurations depend upon the situation and requirements. These configurations are called
 1. Common Base Configuration.
 2. Common Emitter Configuration.
 3. Common Collector Configuration.

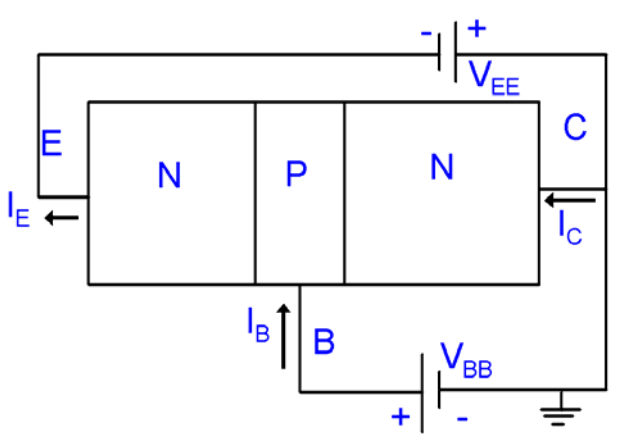
COMMON BASE CONFIGURATION:



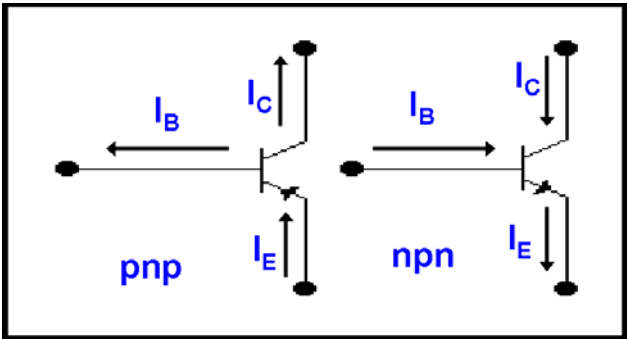
COMMON EMMITER CONFIGURATION:



COMMON COLLECTOR CONFIGURATION:



Directions of Currents in BJT:



MODES OF OPERATION OF BJT:

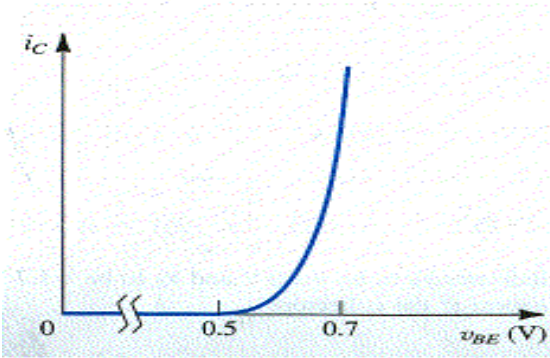
- There are three modes of operation for BJT
 - Cut Off**

When both the PN-junction of transistor are reverse –biased, the resulting mode of operation of the transistor is known as cut-off or simply as off-mode. This means that the BJT will be “OFF” and no current will flow through it. The fully supply voltage will exist across it. In everyday life an off switch will not permit any current to flow through an electric bulb and the full supply voltage will be present across the switch.
 - Active Mode**

When BEJ of a transistor is forward-biased but BCJ is reverse biased the mode of operation of a transistor is referred to as its active-mode. A bipolar junction transistor in its active-mode of operation is called as an amplifier and is able to amplify voltage or current signals both.
 - Saturation**

If both the PN-junctions , that is , the base-emitter junction (BEJ) and the base-collector junction (BCJ) are forward-biased, the transistor is said to be in its saturation (or ON) mode of operation.

Graphical Presentation of BJT:

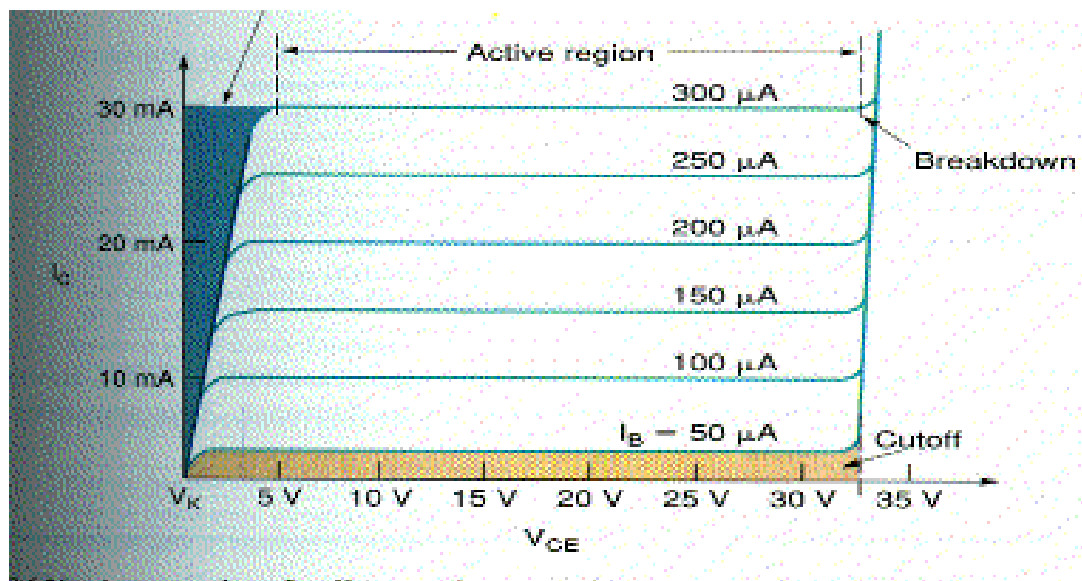


This curve has been drawn between the voltage across BEJ and the collector current. We can observe that the exponential rise in this collector current starts almost after 0.5 volts and saturated at 0.7 volts.

Input or driving point characteristics

$$I_c = I_s e^{V_{BE} / V_T}$$

GRAPHICAL PRESENTATION OF BJT OUTPUT OR COLLECTOR CHARACTERISTIC FOR COMMON BASE CONFIGURATION:



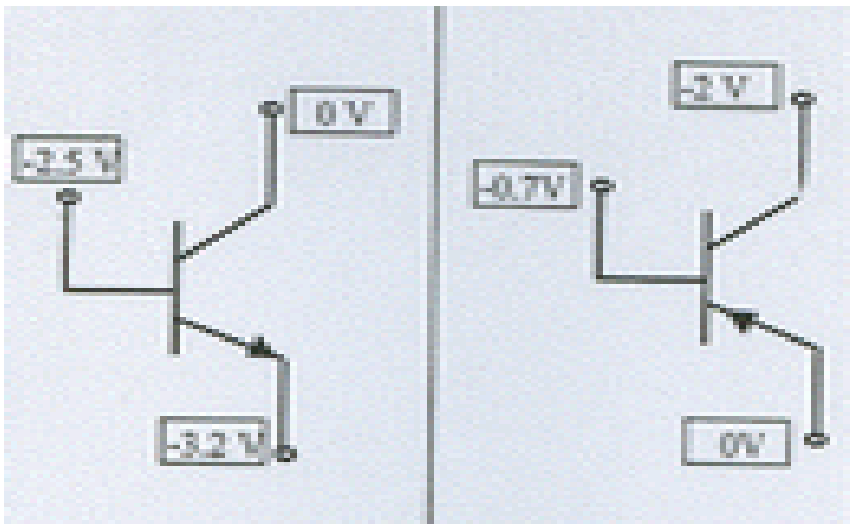
Along x-axis we are taking V_{CE} and along y-axis we are taking current collector current I_C .

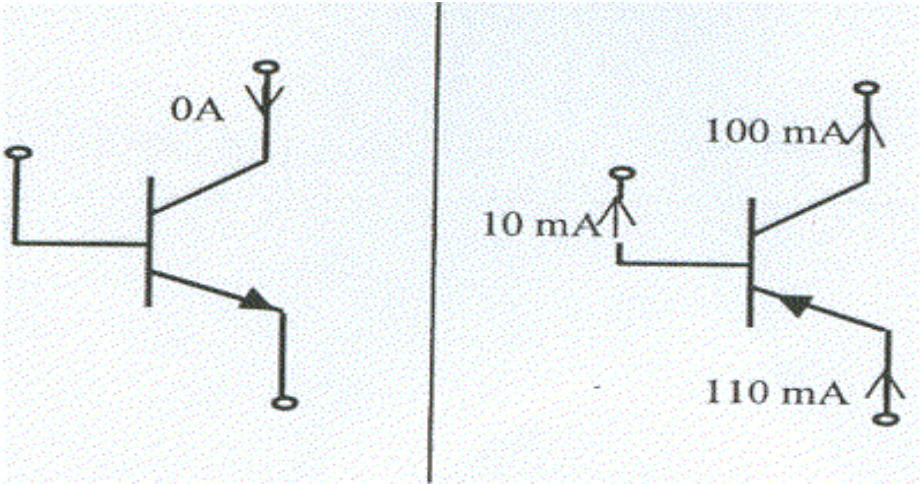
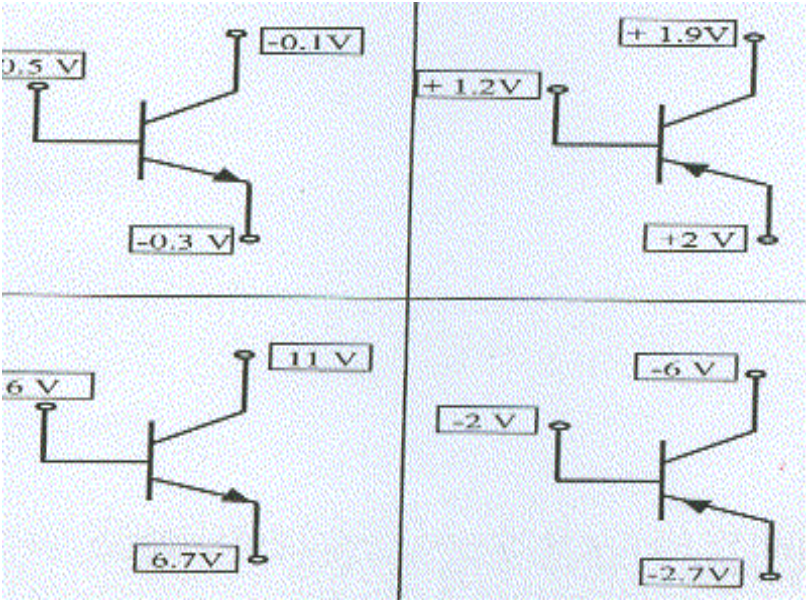
In the above fig below the 50 μA is the cut off region of the transistor .Another shaded area along y-axis this area is a saturation region in this area we have maximum value of collector current i.e. I_C exist while the value of V_{CE} is minimum so in this area transistor will go in saturation. The un shaded area is called active region and it depends upon three parameters i.e. collector current base current and the applied voltage.

MODES OF OPERATION OF BJT:

	EBJ	CBJ
• Cut Off	Reverse	Reverse
• Active	Forward	Reverse
• Saturation	Forward	Forward

For the transistors shown, state mode of operation.





LECTURE 44
(Bipolar Junction Transistor (BJT))

THE CURRENTS OF A BJT:

1. The collector current.
2. The base Current.
3. The Emitter Current.

● Collector Current

It can be expressed as $I_C = I_S e^{(V_{BE}/V_T)}$
 I_S = saturation current or current scale factor.

● The Base Current

It is given by $I_B = I_C / \beta$

Where β is a constant for particular transistor and is known as common emitter current gain, since

$$I_C / I_B = \beta$$

● The Emitter Current

Since the total current which enters a transistor must leave it, hence we may write

$$I_E = I_B + I_C$$

But $I_B = I_C / \beta$

Therefore,

$$I_E = I_C + I_C / \beta$$

Or

$$I_E = I_C (\beta + 1) / \beta$$

Or

$$I_E = I_S e^{(V_{BE}/V_T)} (\beta + 1) / \beta$$

Alternatively, we may write

$$I_C = \alpha I_E \quad \alpha = \beta / (\beta + 1)$$

Or

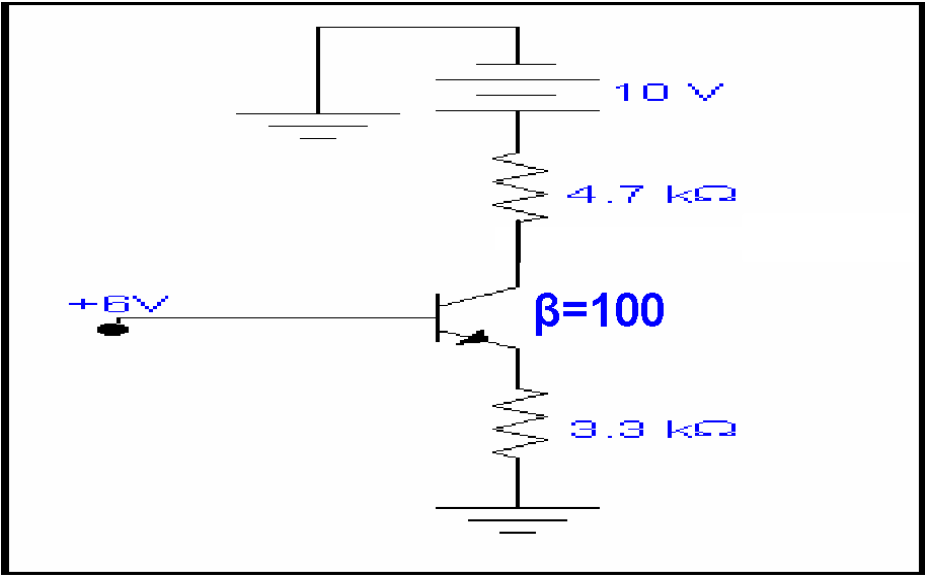
$$I_E = (I_C / \alpha) e^{(V_{BE} / V_T)}$$

Also we may write

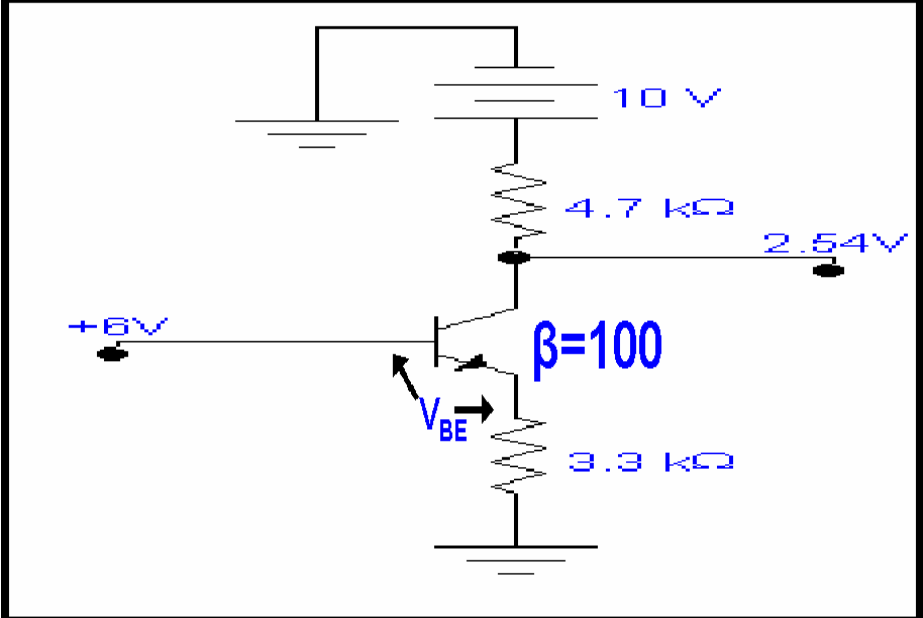
$$\beta = \alpha / (1 - \alpha)$$

α is known as **common base current gain**.

Example: Show the transistor is in saturation.



SOLUTION:



For input loop, applying KVL

$$6 - V_{BE} - I_E(3.3k) = 0$$

Rearranging

$$I_E = (6 - 0.7) / 3.3k = 1.61mA$$

Therefore

$$I_B = I_E / 101 = 1.61m / 101 = 15.94\mu A$$

$$\text{Also } V_E = I_E R_E = 1.61m(3.3k) = 5.3V$$

$$I_C = 15.94 \mu(100) = 1.59mA$$

Therefore KVL Equation for the out put loop is

$$10 - 1.59m(4.7k) - V_C = 0$$

$$\therefore V_C = 10 - 7.49 = 2.50V$$

$$\therefore V_C < V_B$$

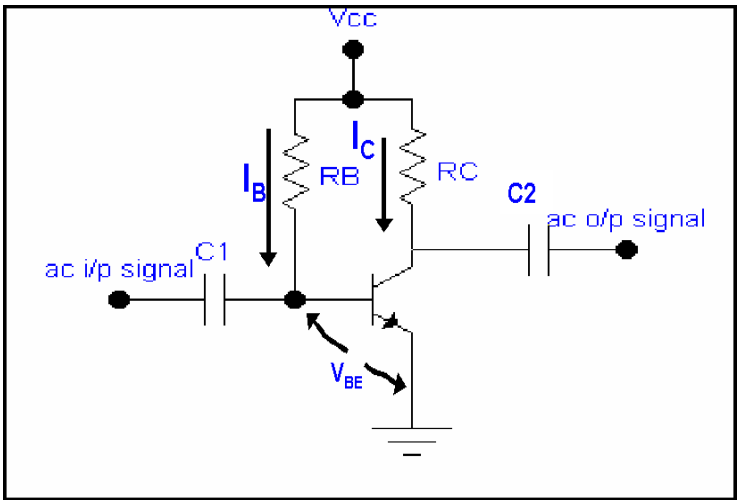
Hence the transistor is in saturation.

DC BIASING-BJT:

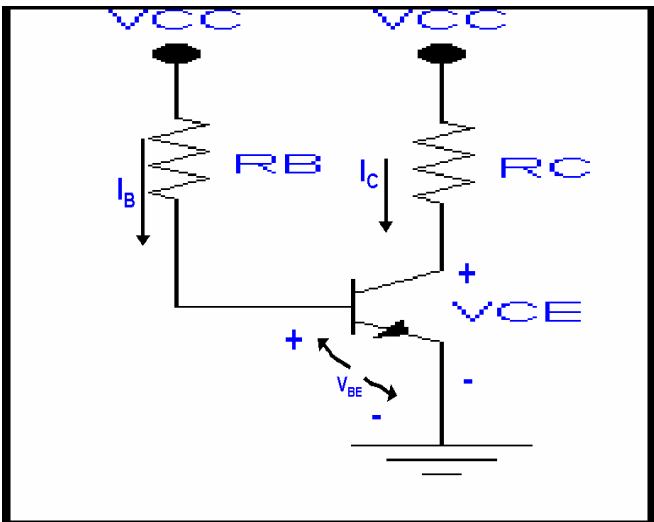
1. Fixed bias circuit.
2. Emitter stabilized biased circuit.
3. Voltage divider bias.

DC BIASING-BJT:

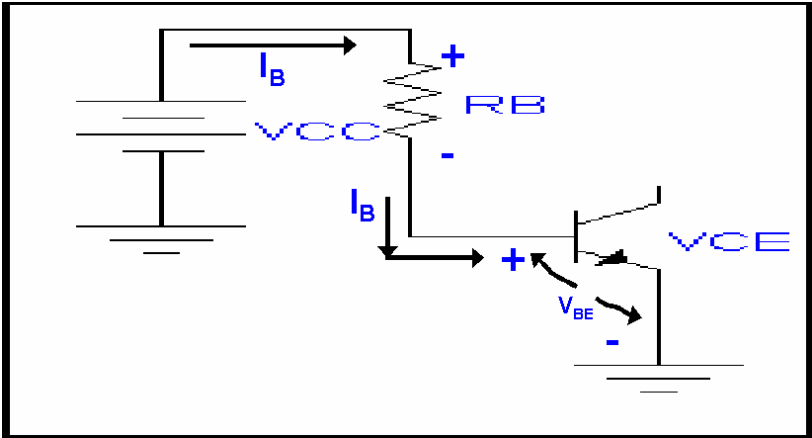
Fixed Bias Circuit



DC EQUIVALENT OF THE CIRCUIT:



Forward bias of Base Emitter:



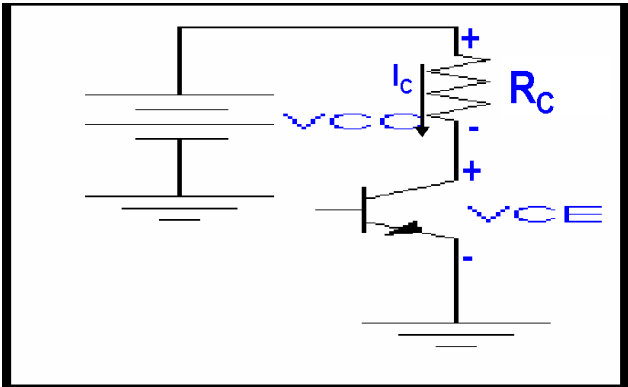
KVI Equation for the loop is

$$V_{CC} - I_B R_B - V_{BE} = 0$$

or

$$I_B = (V_{CC} - V_{BE}) / R_B$$

Collector Emitter Loop:



Applying KVL

$$V_{CE} + I_C R_C - V_{CC} = 0$$

or $V_{CE} = V_{CC} - I_C R_C$

Also we may write

$$V_{CE} = V_C - V_E$$

$\therefore V_E = 0$ in this case

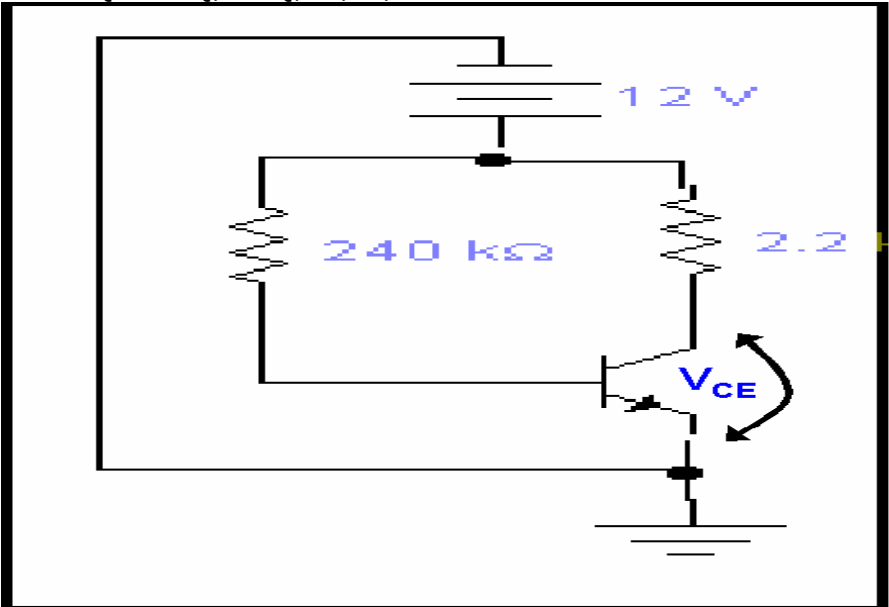
$\therefore V_{CE} = V_C$

Also $V_{BE} = V_B - V_E$

And $V_E = 0$, thus

$V_{BE} = V_B$

Example: Find I_{BQ} and I_{CQ} , V_{CEQ} , V_B , V_C , and V_{BC} . Consider $\beta = 50$.



SOLUTION:

$$I_{BQ} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{240k} = 47.9\mu A$$

Also we know that

$$I_{CQ} = \beta \cdot I_{BQ} = 90 \times 0.0019 \text{ A} = 235 \mu\text{A}$$
$$V_{CEQ} = V_{CC} - I_{CQ} R_C$$
$$V_{CEQ} = 12 - (2.35 \times 10^{-3}) (2 \times 10^3) = 6.3 \text{ V}$$
$$I_{CQ} = I_{BQ} = 0.73 \text{ mA}$$
$$I_C = I_{CQ} = 6.83 \text{ V}$$
$$I_C = I_{CQ} = I_{CQ} = 6.83 \text{ V}$$

LOAD LINE ANALYSIS:

The output equation of fixed bias circuit is given as

$$V_{CE} = V_{CC} - I_C R_C$$

Which is an equation of a straight line. The corresponding x and y axes intercepts can be calculated by inserting appropriate values equal to zero. Therefore by putting

$$I_C = 0$$

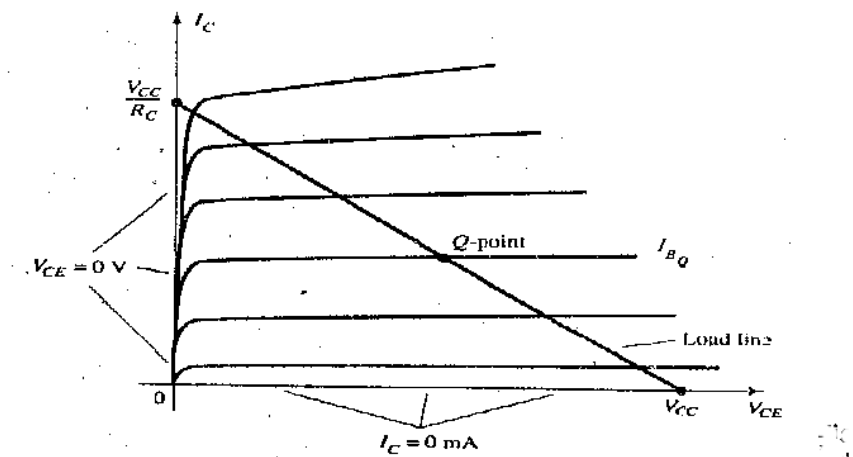
We have $V_{CE} = V_{CC} - (0)R_C$

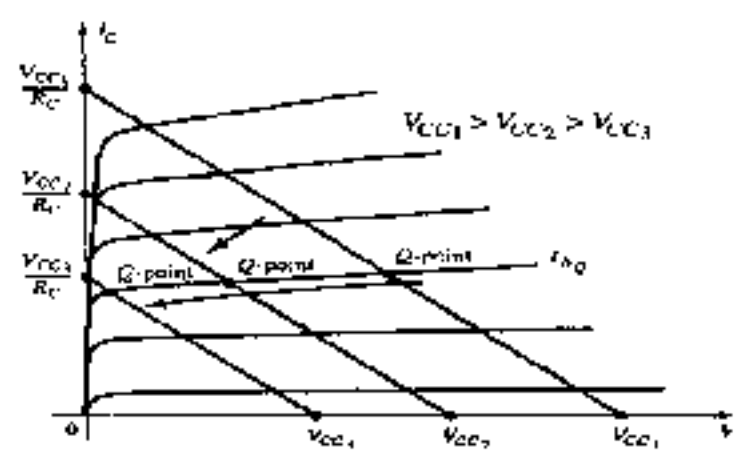
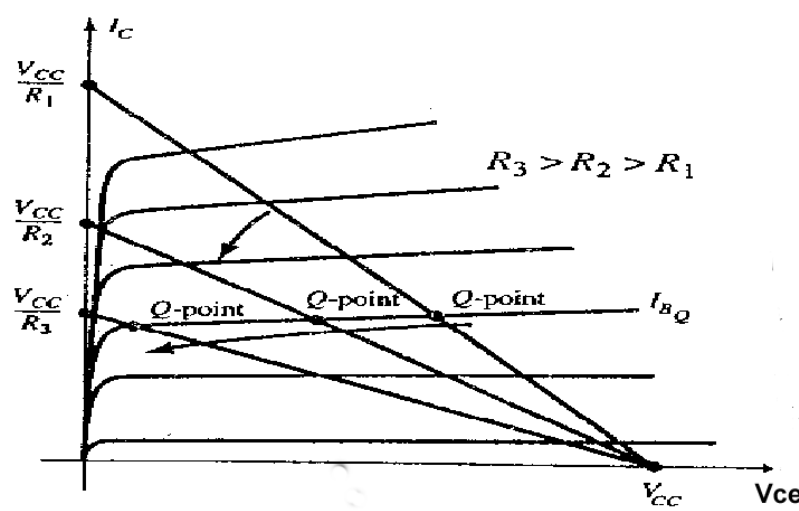
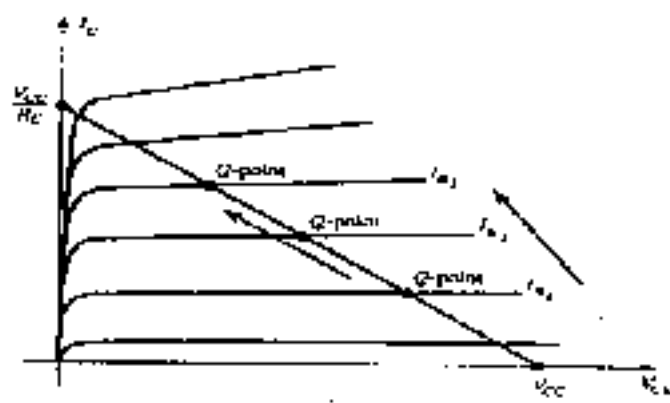
$$V_{CE} = V_{CC}$$

Y intercept can be found by putting

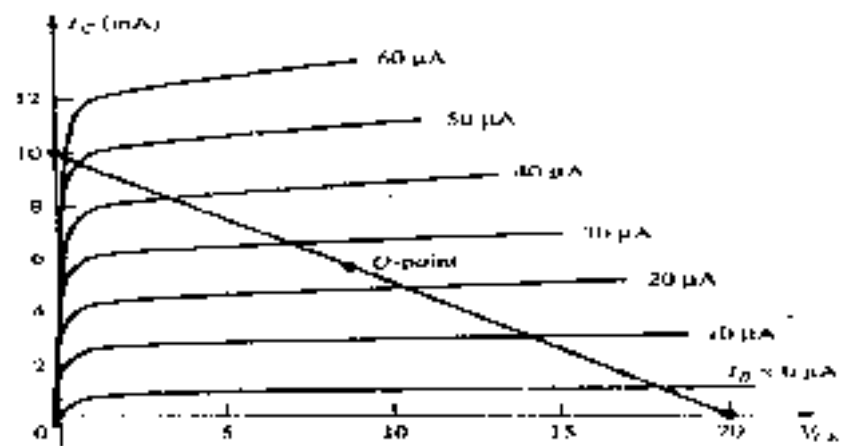
$$V_{CE} = 0 \text{ volts} \quad \text{so that}$$
$$0 = V_{CC} - I_C R_C$$
$$I_C = V_{CC} / R_C$$

Which defines the slop of the load line.





Example: Given a curve find V_{CC} , R_C and R_B for fixed bias circuit.



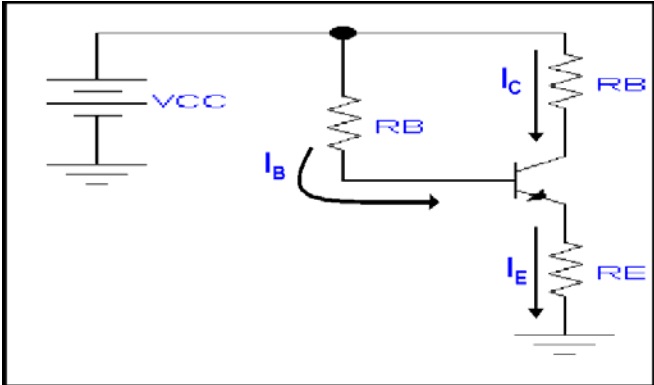
Solution:

From the graph we have, for $I_C = 0$
 $V_{CE} = V_{CC} = 20V$

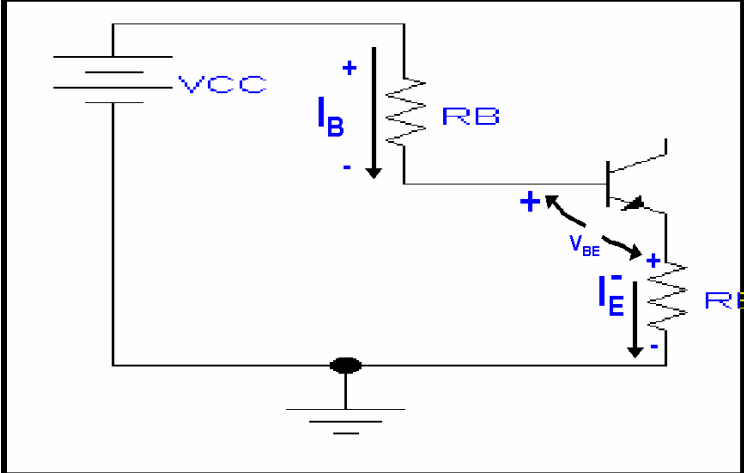
Also at $V_{CE} = 0$
 $I_C = V_{CC} / R_C$
 $R_C = V_{CC} / I_C = 20 / 10m = 2k\Omega$

Now
 $I_B = (V_{CC} - V_{BE}) / R_B$
Or
 $R_B = (V_{CC} - V_{BE}) / I_B$
 $= (20 - 0.7) / 25\mu$
 $= 772k\Omega$.

DC BIASING-BJT:
Emitter-Stabilized Bias Circuit



Base Emitter Loop



KVL Equation for the loop is

$$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$$

But

$$I_E = (\beta + 1) I_B$$

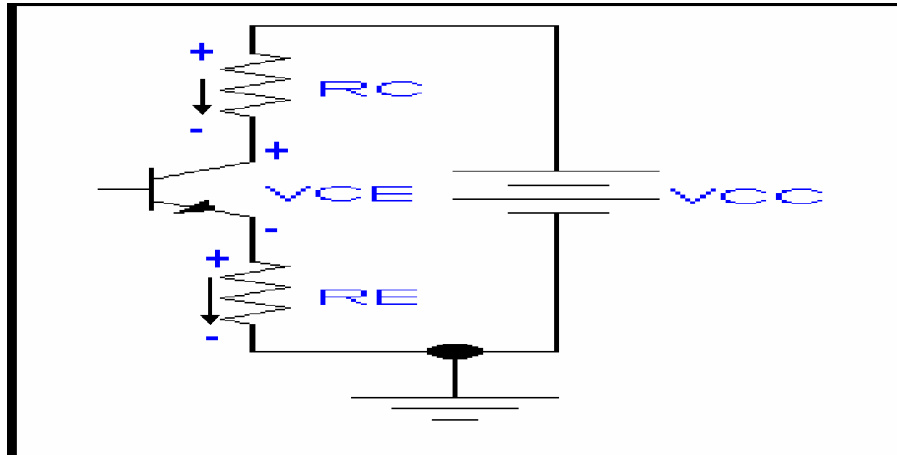
Therefore,

$$V_{CC} - I_B R_B - V_{BE} - (\beta + 1) I_B R_E = 0$$

$$-I_B (R_B + (\beta + 1) R_E) + V_{CC} - V_{BE} = 0$$

$$I_B = (V_{CC} - V_{BE}) / (R_B + (\beta + 1) R_E)$$

Collector-Emitter Loop



$$I_E R_E + V_{CE} + I_C R_C - V_{CC} = 0$$

$$\text{But } I_E = I_C$$

$$V_{CE} - V_{CC} + I_C (R_C + R_E) = 0$$

$$V_{ce} = V_{cc} - I_c (R_c + R_e)$$

Now V_E is the voltage between the Emitter terminal and the ground, given by

$$V_E = I_E R_E$$

Also Collector to ground voltage will be

$$V_{CE} = V_C - V_E$$

$$V_C = V_{CE} + V_E$$

Also

$$V_C = V_{CC} - I_C R_C$$

Voltage at the base w.r.t ground will be

$$V_B = V_{CC} - I_B R_B$$

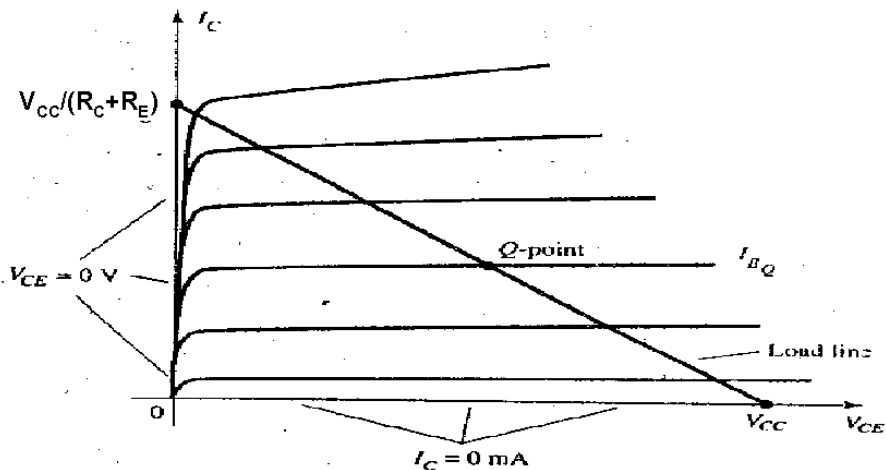
$$V_B = V_{BE} + V_E$$

Stabilization:

Due to addition of the emitter resistance the values of DC bias currents and voltages remain constant with any changes in the Temperature and transistor beta.

LECTURE 45
(Bipolar Junction Transistor BJT-circuit analysis)

Emitter-Stabilized Bias Circuit:
Load Line Analysis



The collector emitter loop equation defining load line is

$$V_{CE} = V_{CC} - I_C (R_C + R_E)$$

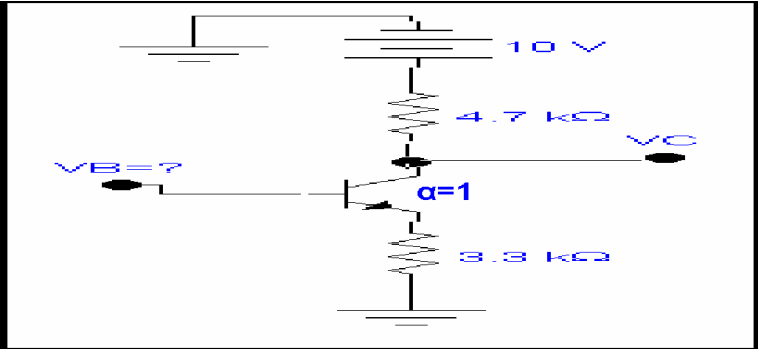
For $I_C = 0 \text{ mA}$

$$V_{CE} = V_{CC}$$

And for $V_{CE} = 0$, we get

$$I_C = V_{CC} / (R_C + R_E)$$

Example: Zammad is designing an amplifier circuit as shown in the figure. Find the highest voltage to which the base can be raised while the transistor remains in active mode. Assume $\alpha = 1$.



Solution:

Lets consider that the voltage to which base can be raised for active operation be V .

For the transistor to operate in active mode, consider $V_B = V_C = V$, therefore, for output loop, we have

$$10 - 4.7kI_C - V_C = 0 \quad (1)$$

Also for base-emitter loop we have

$$V_B - V_{BE} - I_E R_E = 0 \quad (2)$$

$$\because \alpha = 1, \text{ therefore } I_C = I_E$$

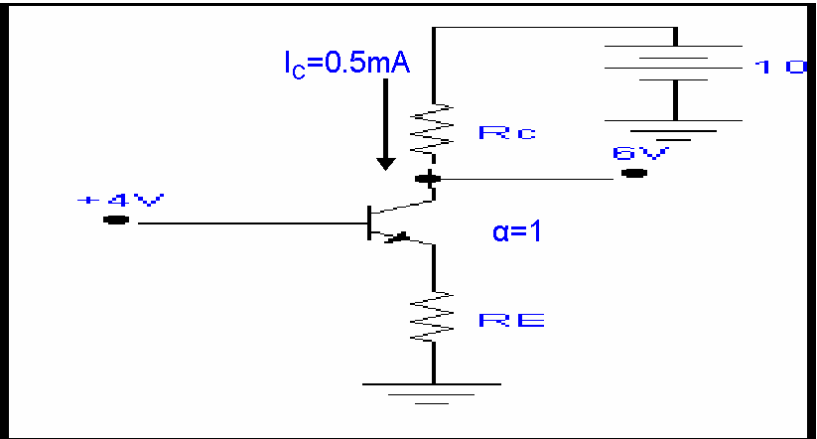
From Equation (2)

$$I_E = (V_B - V_{BE}) / R_E$$

Putting the value of I_E in equation (1)

$$10-(V_B-V_{BE})R_C/R_E-V_C=0$$
$$10-(V-0.7)4.7/3.3-V=0$$
$$3.3(10)-4.7V-3.29-V(3.3)=0$$
$$\therefore 8V=29.71$$
$$V=3.71V$$

Example: For the circuit shown find R_E & R_C .



Solution:

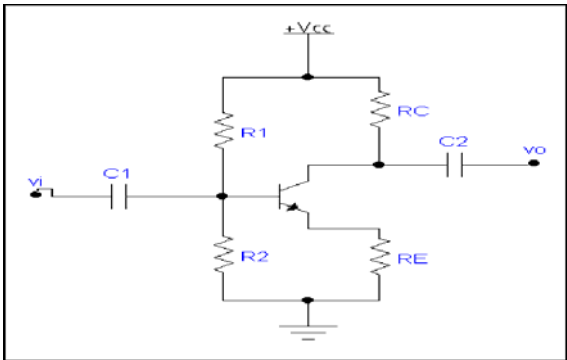
The given parameters are
 $I_C=0.5mA$ & $\alpha=1$,
also $V_{CB}=2V$

Therefore, for the output loop we have by KVL
 $(10-6)/0.5m=R_C$
 $R_C=8k\Omega$

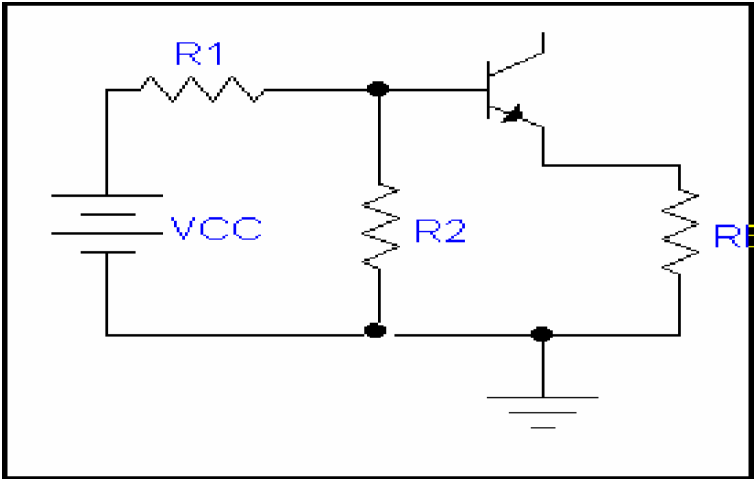
$\therefore \alpha=1$,
Therefore,
 $I_C=I_E=0.5mA$

Hence for the base emitter loop by KVL
 $(4-0.7)/0.5m=R_E$
 $R_E=6.6k\Omega$

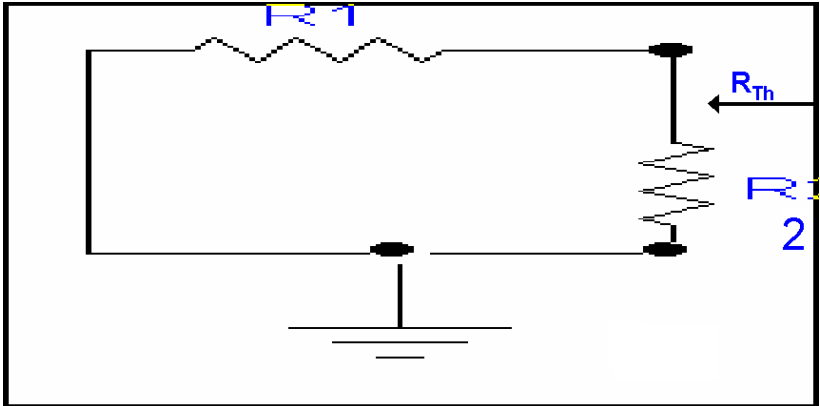
DC BIASING-BJT:
Voltage Divider Bias Circuit



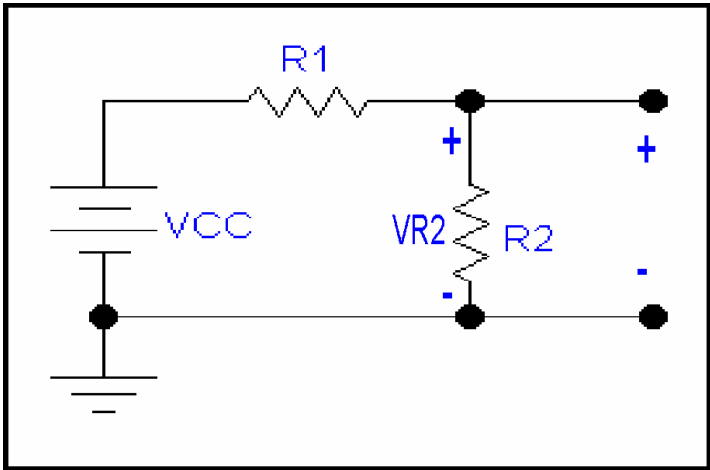
The input loop of the circuit will be



Applying Thevenin's theorem

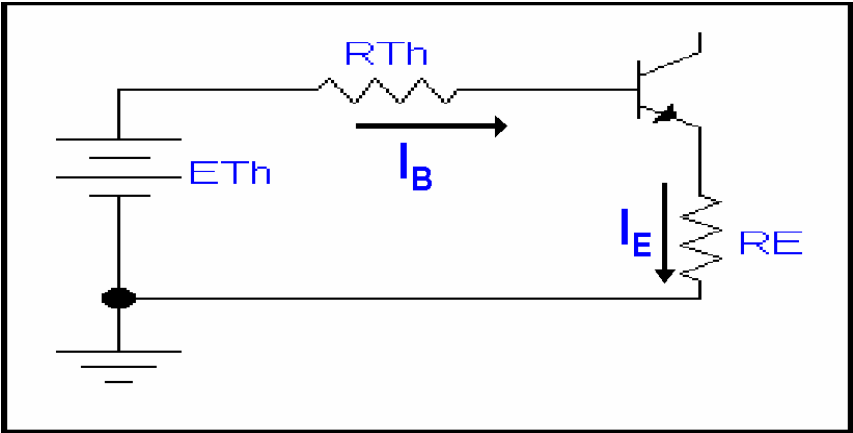


$R_{Th}=R_1\parallel R_2$



$E_{Th}=V_{R2}=\frac{R_2}{R_1+R_2}V_{cc}$

The Thévenin Equivalent circuit is



Therefore the KVL equation will be

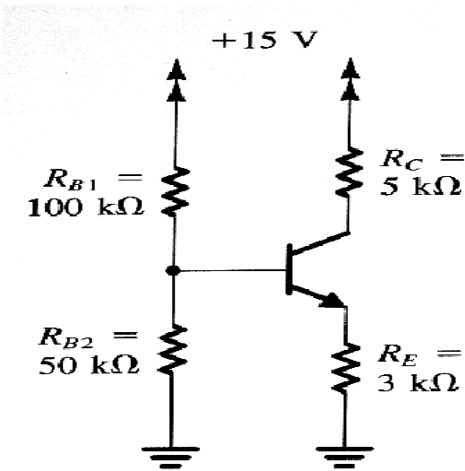
$$E_{th}-I_B R_{Th}-V_{BE}-I_E R_E=0$$

Putting $I_E=(\beta+1)I_B$ and solving for I_B

$$I_B=(E_{Th}-V_{BE})/(R_{Th}+(\beta+1)R_E)$$

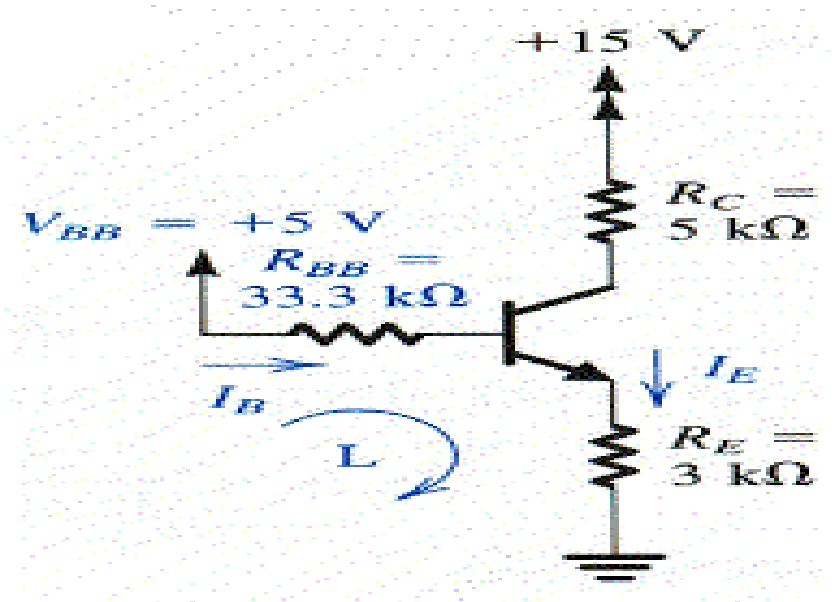
Once I_B is known the remaining problem may be solved exactly as in the previous configurations.

Example: Find voltage at all nodes and current through all branches. Assume $\beta=100$.



Solution:

Using Thevenin's theorem the circuit takes the form as shown in the figure.



By writing

$$V_{BB} = 15(R_{B2}) / (R_{B1} + R_{B2})$$
$$= 15(50) / 150 = 5V$$
$$R_{BB} = R_{B1} || R_{B2}$$
$$= 100 || 50 = 33.3k\Omega$$

Writing the KVL equation for the loop L

$$V_{BB} = I_B R_{BB} + V_{BE} + I_E R_E$$

Put $I_B = I_E / (\beta + 1)$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_{BB} / (\beta + 1)]}$$
$$I_E = \frac{5 - 0.7}{3 + (33.3 / 101)} = 1.29mA$$

The base current will be

$$I_B = 1.29 / 101 = 0.0128mA$$

Therefore the base voltage will be

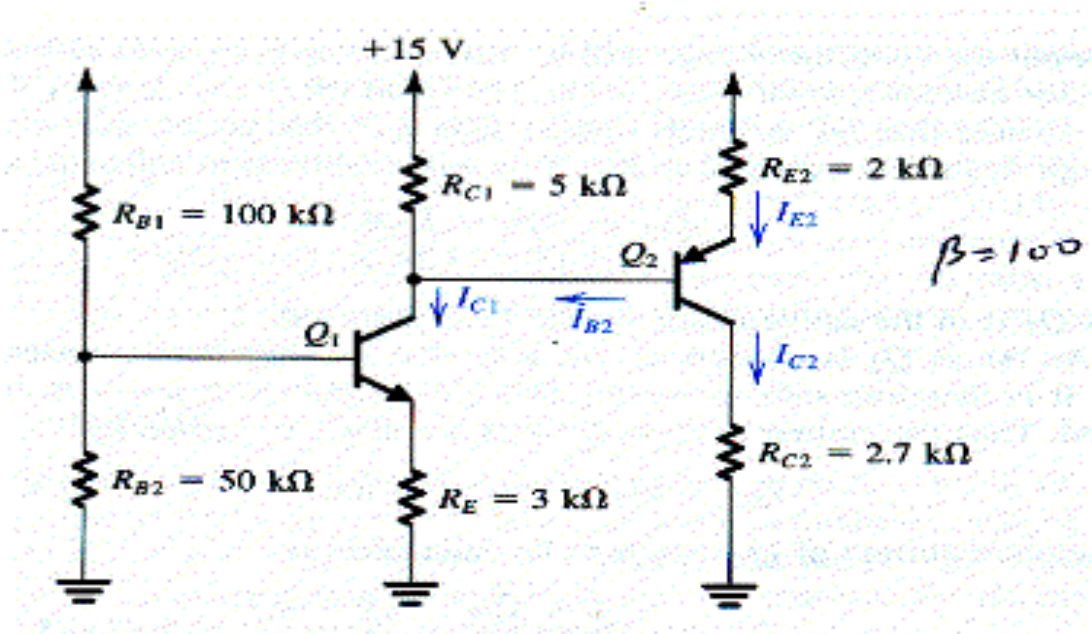
$$V_B = V_{BE} + I_E R_E$$
$$= 0.7 + 1.29(3) = 4.57V$$

Assuming active mode the collector current will be

$$I_C = \alpha I_E = 0.99(1.29) = 1.28mA$$

And $V_C = 15 - I_C R_C = 15 - 1.28(5) = 8.6V$

Example: For the circuit shown find the voltages at all nodes and current through all branches.



Solution:

Assume Q1 is in active mode,
Therefore, from previous example

$$V_{B1} = 4.57V$$
$$I_{E1} = 1.29mA$$
$$I_{B1} = 0.0128mA$$
$$I_{C1} = 1.28mA \text{ (assuming } I_{B1} \text{ negligibly small)}$$

Hence

$$V_{C1}=15-I_{C1}R_{C1}=15-(1.28)5=8.6V$$

Now assume that emitter base junction of Q2 is forward bias, hence

$$V_{E2}=V_{C1}+V_{EB}$$
$$=8.6+0.7=9.3V$$

Also $I_{E2}=(15-V_{E2})/R_{E2}$

$$=(15-9.3)/2k$$

$$I_{E2}=2.85mA$$

Now grounding of Q2 collector through R_{C2} shows, BC junction reverse bias

$$I_{C2}=\alpha I_{E2}$$
$$=0.99(2.85m)$$
$$=2.82mA \ (\beta=100)$$
$$\therefore V_{C2}=I_{C2}R_{C2}=7.62V$$

The value of V_{C2} is lower than $V_{C1}=V_{B2}$ by 0.98V, therefore, Q2 is active.

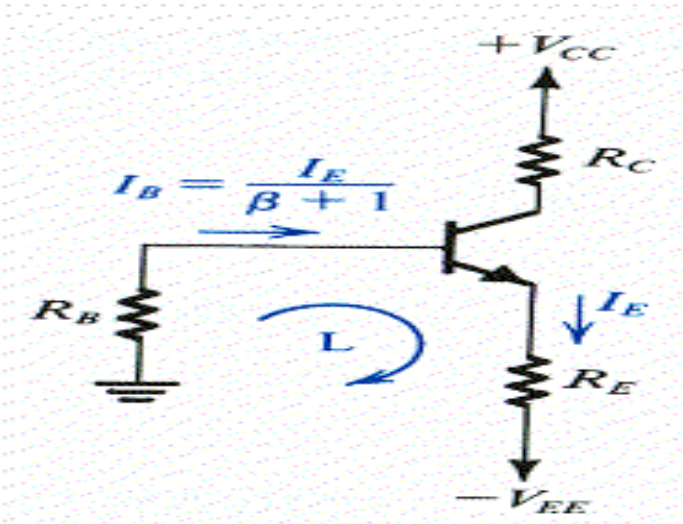
Now errors

$$I_{B2}=I_{E2}/(\beta_2+1)$$
$$=2.85m/101$$
$$=0.028mA$$

Now doing iterations for I_{C1} and I_{B2}

$$I_{Rc1}=I_{C1}-I_{B2}=1.25mA$$
$$V_{C1}=15-5(1.252m)8.74V$$
$$V_{E2}=8.74+0.7=9.44V$$
$$I_{E2}=(15-9.44)/2k=2.78mA$$
$$I_{C2}=0.99(2.78m)=2.75mA$$
$$V_{C2}=2.75(2.7)=7.43V$$
$$I_{B2}=2.78/101=0.0275mA$$

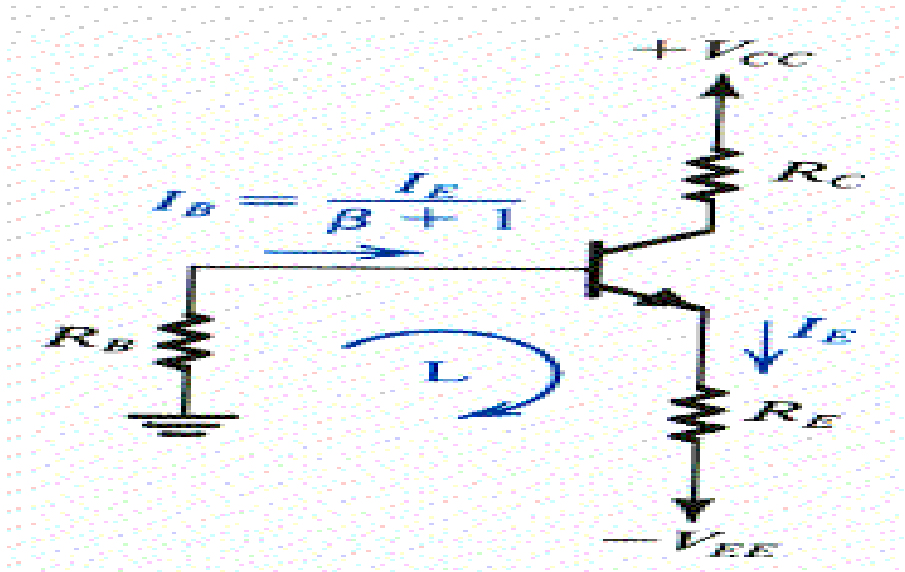
Biasing using two power supplies:



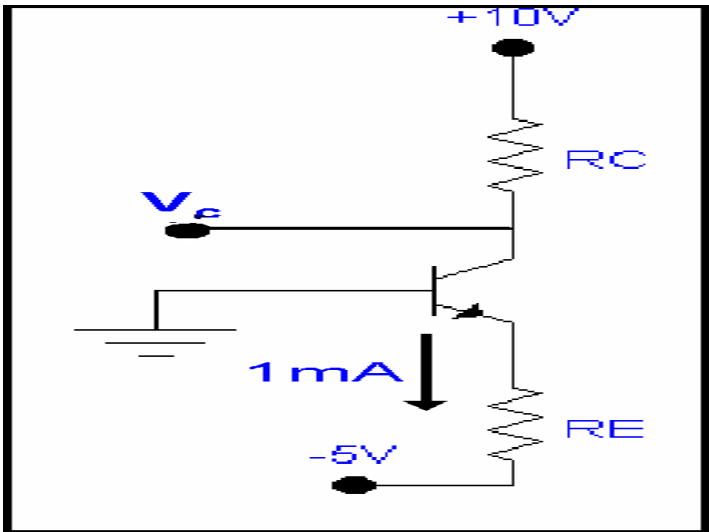
The loop equation for the loop marked as L is

$$I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B/(\beta + 1)}$$

Example: The bias arrangement shown in the figure is to be used in common base amplifier. Design the circuit to establish a dc emitter current of 1mA and provide the highest possible gain while allowing for a maximum signal swing of $\pm 2V$ at the collector. Use +10V and -5V power supplies.



Solution: Since the amplifier is to use in common base configuration, therefore the circuit will take the form as shown.



hence $R_B = 0$

Considering the base-emitter junction forward biased, the emitter resistance can be found as
 $R_E = (-0.7 - (-5))/1m = 4.3k\Omega$

In order to allow for $\pm 2V$ signal swing at the collector, while choosing as larger value of R_C as possible, set $V_C = +2V$, therefore -2V signal would not saturate the BJT. Thus

$$\begin{aligned} R_C &= (10 - 2)/I_C \\ &= 8/1mA \\ R_C &= 8k\Omega \end{aligned}$$